Refutation of the Blok-Esakia theorem for universal classes

Abstract: Grzegorczyk (gzr) algebras as used for support and the Blok-Esakia theorems are not confirmed as tautologies and hence refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r: a *or* p *or* P, c, g;
~ Not; & And, ∧; + Or, ∨; > Imply, →; < Not Imply, less than, ∈; = Equivalent;
% possibility, for one or some, <>; # necessity, for every or all, [];
~(y<x) (x≤y); ~(y>x) (x≥y).

From: Stronkowski, M.M. (2018). On the Blok-Esakia theorem for universal classes. arxiv.org/pdf/1810.09286.pdf m.stronkowski@mini.pw.edu.pl

Remark 1.0: Eqs. are keyed to the text sections and sequential order if not specifically numbered. Grzegorczyk algebras are grz.

Introduction to the Blok-Esakia theorem
[]([](p → []p) → p) → p (1.0.1)

#(#(p>#p)>p)>p ; CTCT CTCT CTCT CTCT (1.0.2)

A modal algebra M is called an interior algebra if for every a ∈M it satisfies
[][]a=[]a ≤ a (3.0.1.1)

##p=~(p<#p) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.0.1.2)

An interior algebra M is a Grzegorczyk algebra if it also satisfies
[]([](a → []a) → a) ≤ a (3.0.2.1)

~(p<(#(#(p>#p)>p)))=(p=p) ; TNTN TNTN TNTN TNTN (3.0.2.2)

Proposition
[](x → []x) → x (3.2.1)

#(p>#p)>p ; CTCT CTCT CTCT CTCT (3.2.2)

Remark 3.2.2: Eqs. 1.0.2 and 3.2.2 as rendered are equivalent with identical truth table results.

Appendix Blok theorems: For every c subset of C define Pc=[](g∨c)∧~c.
Then (Pc≤((g∨c)∧~c))=((g∧~c)≤ g), and [](g∨c)≥[](Pc∨c)=[]((([](g∨c)∧~c)∨c)
=[]([](g∨c)∨c)≥[][](g∨c)=[](g∨c). Thus (P) [](Pc∨c)=[](g∨c). (5.10.1)

(((p&q)=(#(r+q)&~q))>((~(((r+q)&~r)<(p&q))=~(r<(r&~q)))
&((~(#((p&q)+r)>#(r+q))=#((#(r+q)&~q)+q))=(~((r+q)=#(r+q))>
#(#(r+q)+q))))))>((#(p&q)+q)=#(r+q)) ; TTTN CCTN TTTN CCTN (5.10.2)

Eqs. for paper sections 1, 3, and 5 are *not* tautologous. This means that gzr algebras as used for support and the Blok-Esakia theorems are refuted.