Invariance of Maxwell's Equations under Galileo's Transformations

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Abstract

A proof of the invariance of Maxwell's equations for inertial reference frames, making use of the Galilean Transformations.

Text

Galileo's Transformations were formulated before by Galileo and then confirmed by Newton even if with some difference. They are

$$x' = x - vt \qquad (1)$$

$$y' = y \qquad (2)$$

$$z' = z \qquad (3)$$

$$t' = t \qquad (4)$$

in which x,y,z, are three space coordinates and t is the time coordinate of the reference frame S[O,x,y,z,t], supposed at rest; x',y',z' are space coordinates and t' is time coordinate of the reference frame S'[O',x',y',z',t'], supposed in motion with constant velocity v with respect to S along the same direction x=x'. Axes y' and z' are instead parallel to axes x and y.
O and O' are origins respectively of S and S'.
The fundamental difference between Galileo and Newton consists in the fact that for Galileo the relation (4), relative to time, means only the two reference frames S and S' have the same time while for Newton the relation (4) represents the absolute time, that is there is an unique time for all the universe. This apparently insignificant difference allows instead to understand difficulties of classical physics to explain a few experimental facts.
Maxwell's Equations that are valid in the Theory of Reference Frames are:

$$\text{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_o} \qquad \text{(Poisson-Gauss law)} \qquad (5)$$

$$\text{rot}\,\mathbf{E_t} = -\frac{\delta\,\mathbf{B}}{\delta\,t} \qquad \text{(Faraday-Neumann-Lenz law)} \qquad (6)$$

$$\text{rot}\,\mathbf{B} = \mu_o\,\mathbf{J} + \frac{1}{c^2}\,\frac{\delta\,\mathbf{E_t}}{\delta\,t} \qquad \text{(Ampere-Maxwell law)} \qquad (7)$$

$$\mathbf{E_t} = \mathbf{E} + \mathbf{u}\Lambda\mathbf{B} \qquad \text{(Lorentz law)} \qquad (8)$$

In ME that have been considered, the irrelevant equation div**B**=0 has been replaced with Lorentz's law.
In order to semplify calculations I will consider here the case of the propagation of the e.m. field into in the "physical vacuum". The physical vacuum isn't the nothing, because the nothing doesn't exist, but the physical vacuum is a medium with known physical properties (electric permittivity, magnetic permeability, mechanical resistance) in which e.m. waves can propagate while for instance sound waves cannot propagate. The physical vacuum isn't ether because supporters of ether claim ether is a strange and unknown material substance while the physical vacuum is absence of known matter.
If medium isn't the physical vacuum but another medium, it will need to consider only different values of permittivity and permeability.
Let us suppose that these equations are valid with respect to the reference frame S that has its origin O in the physical system of the source, supposed at rest. Fields **E** and **B** that are generated by source propagate with velocity c with respect to S. Supposing to consider the propagation of fields **E** and **B** in a zone of the physical vacuum in which there aren't sources (ρ=J=0) and supposing then there is no particle with velocity u (u=0), Maxwell's Equations reduce only to two significant equations:

$$\text{rot}\,\mathbf{E} = -\frac{\delta\,\mathbf{B}}{\delta\,t} \qquad (9)$$

$$\text{rot}\,\mathbf{B} = \frac{1}{c^2}\frac{\delta\,\mathbf{E}}{\delta\,t} \qquad (10)$$

These two equations describe the e.m. field and the propagation of e.m. waves through the considered medium with respect to the reference frame S[O,x,y,z,t] supposed at rest.
Ordinary calculations prove if e.m. wave moves along the axis x with velocity c, then longitudinal components E_x and B_x are constant and hence they aren't significant while transverse components E_y, E_z, B_y, B_z are described by the so-called relation of plane waves. That is with respect to the reference frame at rest S[O,x,y,z,t] every transverse component satisfies a relation of the type

$$\frac{\delta^2\Phi}{\delta x^2} = \frac{1}{c^2}\frac{\delta^2\Phi}{\delta t^2} \qquad (11)$$

where Φ represents each time one of transverse components and c is the velocity of light with respect to S.
In concordance with the Principle of Relativity, the relation of plane waves with respect to any other reference frame S'[O',x',y',z',t'] that moves with constant velocity v with respect to S, is

$$\frac{\delta^2\Phi'}{\delta x'^2} = \frac{1}{c'^2}\frac{\delta^2\Phi'}{\delta t'^2} \qquad (12)$$

in which, assuming Galileo's Transformations for inertial reference frames, it is

$$x' = x - vt$$
$$t' = t \qquad (13)$$
$$c' = c - v$$

Making use of (13) it is possible to prove (12) is in concordance with (11) because

$$c't' = (c-v)t = ct-vt = x-vt = x' \qquad (14)$$

It doesn’t happens if we apply Lorentz’s Transformations or any other transformation of space-time different from Galileo’s Transformations.
It proves effectively laws of electromagnetism, represented by so-called Maxwell’s Equations, are invariant with respect to Galilean Transformations for inertial reference frames.
Hence it is altogether untrue the statement by postclassical physicists and postmodern physicists that Maxwell’s Equations prove the validity of Lorentz’s Transformations.

References

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