Qubit state represented by pendulum oscillations

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Abstract: Dirac's misconception that classical and quantum superpositions are different because absolute phase is quantum physically meaningless is, even today, seemingly blindly believed by many physicists. However, there is no such difference because, time translation symmetry means absolute time and, thus, absolute phase are classical physically meaningless, too. As a qubit can be a polarization division multiplexed (PDM) quadrature amplitude modulated (QAM) symbol of light, its state has direct correspondence with polarization state of classical light. Even more intuitively, the state may be represented by two-dimensional mechanical oscillations of a pendulum, which enables intuitive understanding of not-really-quantum superpositioned states of a qubit even by elementary school pupils.

1. Introduction

Dirac, on page 14 at the end of section 4 in his text book [1], stated "It is important to remember, however, that the superposition that occurs in quantum mechanics is of an essentially different nature from any occurring in the classical theory, as is shown by the fact that the quantum superposition in order to be capable of a sensible physical interpretation. The analogies are thus liable to be misleading.", which is still seemingly blindly believed by so many physicists as is exemplified by the following explanation: 'According to Dirac: "*the superposition that occurs in quantum mechanics is of an essentially different nature from any occurring in the classical theory*."' in [2] (though correctness of the wikipedia entry is not assured by formal peer review, it is not a problem to refer the entry as an example of incorrect common sense) that they take it granted that quantum state cannot be represented classically.

Reasoning of Dirac on pages 17 and 18 in [1] are:

1) "All the states of the dynamical system are in one-one correspondence with all the possible directions for a ket vector, no distinction being made between the directions of the ket vectors $|A\rangle$ and $-|A\rangle$,"

and

2) "Again while there exists a classical state with zero amplitude of oscillation everywhere, namely the state of rest, there does not exist any corresponding state for a quantum system, the zero ket vector corresponding to no state at all."

Point of reasoning 1) is that quantum states only differ in absolute phase are the same quantum state, which cannot be, he believed, represented classically. But, as he explained it with non-zero ket vectors $|\mathrm{A}\rangle$ and $-|\mathrm{A}\rangle$, which are different ket vectors, he is actually saying that quantum states cannot be represented by ket vectors, which are used extensively to represent quantum states by himself. It is merely that quantum states may be represented by ket vectors specifying absolute phase as long as we remember that absolute phase difference must be ignored. Point of reasoning 2) is that no quantum state is represented by zero ket vector, which, again, denies ket vector representations of quantum states. But, that ket vectors represent quantum state means there is correspondence between all the quantum states and a subset of ket vectors. Thus, zero ket vector may be excluded from the subset.

Moreover, time translation symmetry means absolute time and, thus, absolute phase, are physically, regardless of quantum or classical, meaningless. Though Dirac might have thought that he could have observed absolute phase of a classical oscillation, what is actually observed is relative phase relative to the time of observation or relative to Dirac, an observer. If the same oscillation is observed at different time, observed phase will, in general, be different. Thus, even with usual classical state representations specifying absolute phase, we must remember that absolute phase difference must be ignored.

As the misconception by Dirac is related to phase, some even thinks "the channel that randomizes the phases of input states" "transmits classical information perfectly, but transmits no quantum information at all" [3]. That is, some thinks classical states could not have phase, even though we, communication engineers and scientists, know that phase of classical radio waves, as solutions of Maxwell's equations, may be modulated to carry classical information, not because absolute phase is classically meaningful, but because the phase is relative one between the radio waves, the observed, and us, the observer.

A ket vector representation (thus, specifying physically meaningless classical absolute phase) of superpositioned state of a qubit is a unit vector in two dimensional Hilbert space over **C** as $a_0 e^{i\theta_0}|0\rangle + a_1 e^{i\theta_1}|1\rangle \quad (a_0, a_1, \theta_0, \theta_1 \in \mathbf{R},\ {a_0}^2 + {a_1}^2 = 1, 0 \leq \ \theta_0, \theta_1 < 2\pi)$.

Classical polarization state, on the other hand, is represented (also specifying physically meaningless classical absolute phase) as $a_0 e^{i\theta_H} E_H + a_1 e^{i\theta_V} E_V \quad (a_0, a_1, \theta_H, \theta_V \in \mathbf{R},\ 0 \leq \theta_H, \theta_V < 2\pi)$, where E_H and E_V are vectors of horizontally and vertically polarized electric field, respectively. For electric field with unit strength, ${a_0}^2 + {a_1}^2 = 1$. Space of classical polarization state becomes a Hilbert space by introducing inner product as $E_H \cdot E_H = E_V \cdot E_V$.=1 and $E_H \cdot E_V = 0$.

Thus, it is obvious that there is one to one correspondence between superpositioned state of a qubit: $a_0 e^{i\theta_0}|0\rangle + a_1 e^{i\theta_1}|1\rangle$ and classical polarization state: $a_0 e^{i\theta_H} E_H + a_1 e^{i\theta_V} E_V$ of unit strength. Quantum superposition is no different from classical one.

As a qubit can be superposition of a vertically and horizontally polarized photon state with various relative amplitude and relative phase, which are analog, and we can use QAM (Quadrature Amplitude Modulation) to modulate amplitude and phase of a photon, such a qubit is a QAM PDM (Polarization Division Multiplexed) symbol of light. While quantum physicists often state "the unit of quantum information is the quantum bit or qubit" [3], it is as metaphysical as stating "the unit of quantum energy is quantum Joule or quJ". Just as quantum energy is energy, quantum information is information, unit for which is "bit". A qubit is not a bit but an analog symbol. The number of bits encoded into a symbol of a qubit or qubits depends on noise and can be calculated by quantum extension of theory of Shannon, which is, ironically enough, [3].

If "superposition" means multiplications of some coefficients followed by additions, elements of modules over coefficient groups, in general, can be superpositioned. For example, velocity toward northeast is superposition of velocity toward north and velocity toward east and, in a sense, simultaneously represents velocity toward north and velocity toward east.

2. Intuitive representation of superpositioned state of a qubit or classical polarization state

Binary quantum state of oscillations, such as superposition of polarization modes of photons, has classical representation as polarization state of classical radio waves. Because the state is binary, it is in a two dimensional vector space. However, polarization state is very poorly intuitive, which should be the reason why some physicists misunderstood it something classically impossible.

If, instead of radio waves, visible light is used, we can see polarization state by our eyes. However, what we see is very remote that we can merely see changes in brightness of light through various polarizers. We cannot see something oscillating or, in case of circular polarization, circulating.

On the other hand, vectors representing position or velocity are mechanical and, thus, much more intuitive than polarization state of radio waves or light. However, as vector space of position or velocity is over **R**, it is not appropriate to represent binary quantum states of oscillations, which needs a vector space over **C**. To have such a vector space, we need something with phase, that is, something oscillating.

The simplest mechanically oscillating apparatus should be a pendulum. Fortunately, in the three dimensional real world, a pendulum has two orthogonal directions of oscillations, north/south and east/west (assuming we are not at north or south pole), which means vector space of its oscillating state is two dimensional over **C**, which is just enough to represent binary quantum state of oscillations.

Qubit state $a_0 e^{i\theta_0}|0\rangle + a_1 e^{i\theta_1}|1\rangle$ can be represented by pendulum oscillations of $\left(\mathrm{Re}\left(a_0 e^{i(\omega t+\theta_0)}\right),\ \mathrm{Re}\left(a_1 e^{i(\omega t+\theta_1)}\right)\right) = (a_0 \cos(\omega t + \theta_0),\ a_1 \cos(\omega t + \theta_1))$. Linear oscillations of a pendulum represent linearly polarized qubit states. Oscillations of a pendulum in northeast/southwest direction is superposition of oscillations of the pendulum in north/south and east/west directions. So are left and right circular oscillations, though north/south and east/west oscillations are superpositioned at different relative phase from northeast/southwest linear oscillations.

With a pendulum, we can actually see the pendulum oscillating and, in case of circular oscillations, which represents circular polarization, circulating. Elliptic oscillations of the pendulum, which represents elliptic polarization, can also be seen.

If a pendulum oscillating in northeast/southwest direction or left or right circularly is viewed from north and east, we can see as if the pendulum is oscillating in east/west and north/south directions, respectively, with smaller amplitude, which corresponds to projection of quantum state. If north/south component of the oscillations of a diagonally or left or right circularly oscillating pendulum is dumped, the pendulum will oscillate linearly in east/west direction with smaller amplitude, which corresponds to passing diagonally or left or right circularly polarized photons through a horizontal linear polarizer and square of amplitude is the probability to obtain horizontally polarized photon.

3. Conclusions

It is shown that, despite Dirac's misconception, because of time translation symmetry, absolute phase is meaningless both in quantum and classical physics, which means quantum and classical superpositioned states have direct correspondences. It is also noted that qubit is not a unit of quantum information but a QAM PDM symbol of light, state of which can be represented by classical polarization state or, even more intuitively, two dimensional mechanical oscillations of a pendulum.

With the pendulum representation, even elementary school pupils without knowledge on vectors or complex numbers, should be able to understand what not-really-quantum superposition is.

References

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[2] https://en.wikipedia.org/wiki/Superposition_principle, as of July 10th. 2019.
[3] S. Lloyd, “The capacity of the noisy quantum channel”, Phys. Rev. A55 (1997) 1613, https://arxiv.org/abs/quant-ph/9604015.