A hidden variable solution to the EPR paradox

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Abstract

The hidden variable solution to the EPR paradox proposes that correlation of measurements of entangled particles is due to variables that get decided when the entangled particles get created. It is shown that the correlation of spin measurements in Bell's form of the EPR paradox can be explained as deriving from spin of the entangled particles in the x-direction. This spin parameter is not hidden as it is included in the standard quantum mechanical formulation.

Albert Einstein, Boris Podolsky and Nathan Rosen proposed a thought experiment where conjugate properties are measured from two entangled particles in a paper [1] published in 1935. The authors claimed that the position could be measured precisely from one particle and the momentum from another. By Heisenberg's uncertainty principle position and momentum cannot both be precisely measured, thus the authors concluded that the measurements of entangled particles must be correlated: if the position is precisely measured from one particle, the momentum cannot be precisely measured from the other particle because of some mechanism. The authors suggested that the mechanism of such correlation can either be faster than light transfer of information from one particle to the other one, which Einstein naturally rejected, or that the particles have in some way agreed on how to answer to future measurements. This agreement would be stored in some variables in the particles. As such variables did not seem to exist in quantum mechanics, they become called hidden variables in the EPR paradox.

In 1964 John Bell reformulated the paradox as a measurement of the spin of two entangled particles and proved Bell's Teorem in [2]. His proof seemed to show that local hidden variables could not be a solution to the EPR paradox. Recently I made an elementary argument in [3] showing that Bell's proof is not valid: Bell's inequality violations are caused by an incorrect normalization of detector directions. Experiments that have confirmed Bell's Theorem use the same incorrect scaling of detector directions and naturally get the same result as Bell.

The present short note shows that the EPR paradox in the form proposed by Bell can be solved by what he called hidden parameters, but these parameters are already in the formulation of quantum mechanics and threfore not in any way hidden. Notably it will be shown that the entangled particle system studied in the EPR paradox may be in two mixed states after breaking up, but in both states each particle has a definite spin angular momentum value.

In one of these states the first particle has the spin + in the x-direction and the second has -, while in the second mixed state the spins in the x-direction are the opposites. Assuming, as in the hidden parameter solution, that the particles do have definite spins, the wave functions are not mixes of these two states but one or the other.

As spin is conserved, spin directions are not changed by the following measurement in either particle. This simple mechanism gives the observed anticorrelation of the spins of the two particles. The argument does not show that this is the way it happens, but it shows that this is a possible logical explanation to the EPR paradox.

The entangled wave function studied in the EPR paradox in [2] is of the type

$$|\phi\rangle = c_1|\psi_{z+}\rangle \otimes |\psi_{z-}\rangle + c_2|\psi_{z-}\rangle \otimes |\psi_{z+}\rangle$$

where $|\psi_{j+}\rangle$ and $|\psi_{j-}\rangle$, $j \in \{x, y, z\}$, are the eigenvectors of the Pauli matrices σ_j corresponding to the eigenvalues 1 and -1 respectively and c_1, c_2 are complex numbers. The wave function $|\phi\rangle$ is created by a break-up of a spin zero single state and must have spin zero. We have to derive the spin zero subspace.

The first particle is

$$|\psi\rangle = c_1|\psi_{z+}\rangle + c_2|\psi_{z-}\rangle.$$

As

$$\langle\psi_{x+}|\psi\rangle = \frac{1}{\sqrt{2}}(c_1 + c_2)$$

$$\langle\psi_{x-}|\psi\rangle = \frac{1}{\sqrt{2}}(c_1 - c_2)$$

$$\langle\psi_{y+}|\psi\rangle = \frac{1}{\sqrt{2}}(c_1 - ic_2)$$

$$\langle\psi_{y-}|\psi\rangle = \frac{1}{\sqrt{2}}(c_1 + ic_2)$$

$$\langle \psi_{x+} | \psi \rangle = c_1$$

$$\langle \psi_{z-} | \psi \rangle = c_2$$

the probabilities of measuring the eigenvalues $|\psi_{n\alpha}\rangle$, $m \in \{x, y, z\}$, $\alpha \in \{+, -\}$, are the real numbers

$$|\langle \psi_{x+} | \psi \rangle|^2 = \frac{1}{2}(1 + (c_1^* c_2 + c_1 c_2^*))$$

$$|\langle \psi_{x-} | \psi \rangle|^2 = \frac{1}{2}(1 - (c_1^* c_2 + c_1 c_2^*))$$

$$|\langle \psi_{y+} | \psi \rangle|^2 = \frac{1}{2}(1 - i(c_1^* c_2 - c_1 c_2^*))$$

$$|\langle \psi_{y-} | \psi \rangle|^2 = \frac{1}{2}(1 + i(c_1^* c_2 - c_1 c_2^*))$$

$$|\langle \psi_{z+} | \psi \rangle|^2 = c_1^* c_1$$

$$|\langle \psi_{z-} | \psi \rangle|^2 = c_2^* c_2.$$

The wave function $|\psi\rangle$ can be expressed in a basis $|\psi_{m+}\rangle$, $|\psi_{m+}\rangle$ as

$$|\psi\rangle = \langle \psi_{m+} | \psi \rangle |\psi_{m+} + \langle \psi_{m-} | \psi \rangle |\psi_{m-}.$$

The second particle

$$|\psi'\rangle = c_2 |\psi_{z+}\rangle + c_1 |\psi_{z-}\rangle.$$

has similar formulas with c_1 and c_2 interchanged, but as it is moving in the opposite direction, the spin is inverse. Thus, the total spin + in the x-direction for the two particles is

$$\frac{1}{2}\hbar \frac{1}{2}(1 + (c_1^* c_2 + c_1 c_2^*)) + \frac{1}{2}\hbar \frac{1}{2}(1 - (c_2^* c_1 + c_2 c_1^*)) = \frac{1}{2}\hbar$$

The total spin - in the x-direction for the two particles is

$$-\frac{1}{2}\hbar \frac{1}{2}(1 - (c_1^* c_2 + c_1 c_2^*)) - \frac{1}{2}\hbar \frac{1}{2}(1 + (c_2^* c_1 + c_2 c_1^*)) = -\frac{1}{2}\hbar.$$

The total spin of the two particles in the x-direction is the sum of these numbers, i.e., it is zero for any c_1, c_2.

The total spin + in the y-direction for the two particles is

$$\frac{1}{2}\hbar\frac{1}{2}(1 - i(c_1^*c_2 - c_1c_2^*)) + \frac{1}{2}\hbar\frac{1}{2}(1 + i(c_2^*c_1 - c_2c_1^*))$$

$$= \frac{1}{2}\hbar(1 - i(c_1^*c_2 - c_1c_2^*))$$

The total spin - in the y-direction for the two particles is

$$-\frac{1}{2}\hbar\frac{1}{2}(1 + i(c_1^*c_2 - c_1c_2^*)) - \frac{1}{2}\hbar\frac{1}{2}(1 - i(c_2^*c_1 - c_2c_1^*))$$

$$= -\frac{1}{2}\hbar(1 + i(c_1^*c_2 - c_1c_2^*))$$

The total spin of the two particles in the y-direction is the sum of these numbers,

$$\frac{1}{2}\hbar i(-c_1^*c_2 + c_1c_2^*).$$

This number is zero if

$$c_2 = \pm\frac{c_1}{|c_1|}|c_2|.$$

The total spin + in the z-direction for the two particles is

$$\frac{1}{2}\hbar(c_1^*c_1 + c_2^*c_2) = \frac{1}{2}\hbar$$

if the norm of the wave function is set to one. The total spin - in the z-direction for the two particles is

$$-\frac{1}{2}\hbar(c_2^*c_2 + c_1^*c_1) = -\frac{1}{2}\hbar.$$

Summing the numbers shows that the total spin in the z-direction is always zero.

The total spin must be zero to all directions. Therefore $c_1^*c_2 - c_1c_2^* = 0$. It implies that

$$|\langle\psi_{y+}|\psi\rangle|^2 = |\langle\psi_{y-}|\psi\rangle|^2 = \frac{1}{2}$$

i.e., the spin in the y-direction is zero for each particle.

We can set

$$c_1 = \frac{1}{\sqrt{2}}$$

as a basis vector can be multiplied by any complex number. We do not necessarily need to have $|c_1| = |c_2|$ but can select the basis wave functions so that this is true. Thus,

$$c_2 = \pm \frac{1}{\sqrt{2}}.$$

As a conclusion, there are two basis vectors for the subspace spin zero:

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}(|\psi_{z+}\rangle \otimes |\psi_{z-}\rangle - |\psi_{z-}\rangle \otimes |\psi_{z+}\rangle)$$

and

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|\psi_{z+}\rangle \otimes |\psi_{z-}\rangle + |\psi_{z-}\rangle \otimes |\psi_{z+}\rangle).$$

Both of these basis vectors give total spin zero to all directions for the two particle system and both give spin zero in y and z directions to each particle separately, but they do not give spin zero in the x-direction for each particle. Indeed, for $|\phi_1\rangle$ the first particle has the spin

$$\frac{1}{2}\hbar\frac{1}{2}(1 + (c_1^* c_2 + c_1 c_2^*)) - \frac{1}{2}\hbar\frac{1}{2}(1 - (c_2^* c_1 + c_2 c_1^*))$$
$$\frac{1}{2}\hbar(c_1^* c_2 + c_1 c_2^*) = -\frac{1}{2}\hbar$$

in the x-direction and the second particle has the opposite spin. For $|\phi_2\rangle$ it is inversely.

In the hidden parameter solution each particle has a definite spin after the single state has broken up. Thus, $|\phi\rangle$ is not a linear combination of the basis vectors $|\phi_1\rangle$ and $|\phi_2\rangle$. It is one or the other with equal probabilities.

The total spin of a particle cannot change from + to - in a measurement, thus, if the first particle has the spin + in the x-direction, it is also the total spin of this particle, and the first measurement can only collapse it to $|\psi_{m+}\rangle$ eigenvectors. It follows that the second particle must have - spin in the x-direction and the second measurement can only collapse it to $|\psi_{m-}\rangle$ eigenvectors. A similar conclusion is true if the first particles has spin -.

The basis vectors $|\phi_1\rangle$ and $|\phi_2\rangle$ are both mixed states and measurements collapse these mixed stated to pure states of eigenvectors $|\psi_{m\alpha}\rangle$, thus measurements have this somewhat mysterious property of collapsing wave functions, but the correlation is caused by the conservation of angular momentum, in this case, spin angular momentum. This mechanism gives the observed anticorrelation between the measurements without assuming neither instantaneous long distance information transfer nor any missing variables in quantum mechanics.

The original EPR paradox in [1] has a different solution. The authors of [1] mistakenly assume that both position and momentum could be measured precisely from two entagled particles unless there is some mechanism causing correlation of measurements. There is such a mechanism in the case of spin or polarization measurements, but with conjugated properties in Heisenberg's uncertainty principle no such correlation mechanism is needed: particles are waves and they do not have precise values for conjugated properties.

A point mass has both a precise position and a precise momentum. The momentum can be obtained by measuring the mass and averaging the velocity over a long distance. Because the momentum is conserved, the velocity measured over any short distance equals the average over a long distance.

A wave packet, instead, has some minimum distance over which velocity can be measured so that it still closely equals the average velocity over a long distance. Trying to measure velocity over a shorter distance faces the problem that the wave extends outside this distance, i.e., the mass is not the whole mass. Therefore the momentum measurement becomes necessarily unprecise if the location is very precise. Indeed, a wave packet does not have a precise position in the sense a point mass has.

References:

[1] Albert Einstein, Boris Padolsky, and Nathan Rosen, *Can Quantum-Mechanical Description of Physical Reality be Considered Complete? Physical Review.* 47 (10):777-780, 1935.

[2] John Bell,*On the Einstein Podolsky Rosen Paradox. Physics.* 1 (3):195-200, 1964.

[3] Jorma Jormakka, *On Bell's Theorem. arxiv.* 2018.