Two simples proofs of Fermat 's last theorem and Beal conjecture

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Abstract :If after 374 years the famous theorem of Fermat-Wiles was demonstrated in 150 pages by A. Wiles [2], the puspose of this article is to give a simple demonstration and deduce a proof of the Beal conjecture.

Résumé : Si après 374 ans le célèbre théorème de Fermat-Wiles a été démontré en 150 pages par A. Wiles [2], le but de cet article est de donner une simple démonstration et d'en déduire une preuve de la conjecture de Beal.

Keywords : Fermat, Fermat-Wiles theorem, Fermat's great theorem.

The Subject Classification Codes : 11D41 - 11G05 - 11G07 - 26B15 - 26B20 - 28A10 - 28A75 -

1-Introduction :

Set out by Pierre de Fermat [2], it was not until more than three centuries ago that Fermat's great theorem was published, validated and established by the British mathematician Andrew Wiles [4] in 1995.

In mathematics, and more precisely in number theory, the last theorem of Fermat [2], or Fermat's great theorem, or since his Fermat-Wiles theorem demonstration [3], is as follows : There are no non-zero integers a, b, and c such that : $a^n+b^n=c^n$, as soon as n is an integer strictly greater than 2 ".

The Beal conjecture is the following conjecture in number theory : If $a^x+b^y=c^z$ where a, b, c, x, y and z are positive integers with x, y, z > 2, then a, b, and c have a common prime factor. Equivalently, There are no solutions to the above equation in positive integers a, b, c, x, y, z with a, b and c being pairwise coprime and all of x, y, z being greater than 2.

If the famous Fermat-Wiles theorem has been demonstrated in **150 pages** by A. Wiles [4], the purpose of this article is to give a simple proof and deduce a proof of the Beal conjecture.

2. The proof of *Fermat 's last theorem*

Théorème :

There are no non-zero integers a, b, and c such that: $a^n+b^n=c^n$, with n an integer strictly greater than 2

Lemma 1 :

If n, a, b and c are a non-zero integers with and $a^n+b^n=c^n$ then:

$$\int_0^b x^{n-1}-\left(\frac{c-a}{b}x+a\right)^{n-1}\frac{c-a}{b}dx=0$$

Proof :

$$a^n+b^n=c^n \Leftrightarrow \int_0^a n\,x^{n-1}dx+\int_0^b n\,x^{n-1}dx=\int_0^c n\,x^{n-1}dx$$

But as :

$$\int_0^c n\,x^{n-1}dx=\int_0^a n\,x^{n-1}dx+\int_a^c n\,x^{n-1}dx$$

So :

$$\int_0^b n\,x^{n-1}dx=\int_a^c n\,x^{n-1}dx$$

And as by changing variables we have :

$$\int_a^c n\,x^{n-1}dx=\int_0^b n\left(\frac{c-a}{b}y+a\right)^{n-1}\frac{c-a}{b}dy$$

Then :

$$\int_0^b x^{n-1}dx=\int_0^b\left(\frac{c-a}{b}y+a\right)^{n-1}\frac{c-a}{b}dy$$

It results:

$$\int_0^b x^{n-1}-\left(\frac{c-a}{b}x+a\right)^{n-1}\frac{c-a}{b}dx=0$$

Corollary 1 : If N, n, a, b and c are a non-zero integers with and $a^n+b^n=c^n$ then :

$$\int_0^{\frac{b}{N}} x^{n-1}-\left(\frac{c-a}{b}x+\frac{a}{N}\right)^{n-1}\frac{c-a}{b}dx=0$$

Proof : It results from the proof of lemma 1 by replacing a, b and c respectively by $\frac{a}{N}$, $\frac{b}{N}$ and $\frac{c}{N}$.

Lemma 2 :

If $a^n+b^n=c^n$, where n, a, b and c are a non-zero integers with $n>2$ and $a\le b\le c$. Then for an integer N big enough we have :

$$f(x,a,b,c,N)=x^{n-1}-\left(\frac{c-a}{b}x+\frac{a}{N}\right)^{n-1}\frac{c-a}{b}\le 0\quad \forall x\in\left[0,\frac{b}{N}\right]$$

Proof :

We have : $\frac{\partial f}{\partial x}=(n-1)x^{n-2}-(n-1)\left(\frac{c-a}{b}x+\frac{a}{N}\right)^{n-2}\left(\frac{c-a}{b}\right)^2$, $f(0,a,b,c,N)<0$ and $\frac{\partial f}{\partial x}|_{x=0}<0$ so, by continuity, $\forall y\in\left[0,\frac{b}{N}\right]$ with N an integer big enough, we have $\frac{\partial f}{\partial x}|_{x=y}<0$. So the function f is decreasing in $\left[0,\frac{b}{N}\right]$ and we have :

$$f(x,a,b,c,N)\le 0\quad \forall x\in\left[0,\frac{b}{N}\right]$$

Proof of Theorem:

If $a^n+b^n=c^n$, where n, a, b and c are a non-zero integers with $n>2$ and $a\le b\le c$. Then for an integer N big enough, it results from the **lemma 2** that we have :

$$f(x,a,b,c,N)=x^{n-1}-\left(\frac{c-a}{b}x+\frac{a}{N}\right)^{n-1}\frac{c-a}{b}\le 0\quad \forall x\in\left[0,\frac{b}{N}\right]$$

And by using the **corollary 1**, we have $\int_0^{\frac{b}{N}} x^{n-1} - \left(\frac{c-a}{b}x + \frac{a}{N}\right)^{n-1} \frac{c-a}{b} dx = 0$.

So : $x^{n-1} - \left(\frac{c-a}{b}x + \frac{a}{N}\right)^{n-1} \frac{c-a}{b} = 0 \quad \forall x \in \left[0, \frac{b}{N}\right]$

And therefore $\frac{c-a}{b} = 1$ because f(x, a, b, c, N) is a null polynomial as it have more than n zeros. So $c = a + b$ and $a^n + b^n \neq c^n$ which is absurde .

3- The proof of Beal conjecture :

Corollary : **[Beal conjecture]**

If $a^x + b^y = c^z$ where a, b, c, x, y and z are positive integers with x, y, z > 2, then a, b, and c have a common prime factor.

Equivalently, there are no solutions to the above equation in positive integers a, b, c, x, y, z with a, b and c being pairwise coprime and all of x, y, z being greater than 2.

Proof :

Let $a^x + b^y = c^z$

If a, b and c are not pairwise coprime, then by posing $a = ka'$, $b = kb'$, and $c = kc'$.

Let $a' = u'^{yz}$, $b' = v'^{xz}$, $c' = w'^{xy}$ and $k = u^{yz}$, $k = v^{xz}$, $k = w^{xy}$

As $a^x + b^y = c^z$, we deduce that $(uu')^{xyz} + (vv')^{xyz} = (ww')^{xyz}$.

So : $k^x u'^{xyz} + k^y v'^{xyz} = k^z w'^{xyz}$

This equation does not look like the one studied in the first theorem. But if a, b and c are pairwise coprime, we have $k = 1$ and $u = v = w = 1$ and we will have to solve the equation : $u'^{xyz} + v'^{xyz} = w'^{xyz}$

The equation $u'^{xyz} + v'^{xyz} = w'^{xyz}$ have a solution if at least one of the equations : $(u'^{xy})^z + (v'^{xy})^z = (w'^{xy})^z$, $(u'^{xz})^y + (v'^{xz})^y = (w'^{xz})^y$, $(u'^{yz})^x + (v'^{yz})^x = (w'^{yz})^x$, have a solution .

So by the proof given in the proof of the first Theorem we must have : $z \leq 2$ or $y \leq 2$, or $z \leq 2$.

We therefore conclude that if $a^x + b^y = c^z$ where a, b, c, x, y, and z are positive integers with $x, y, z > 2$, then a, b, and c have a common prime factor.

4- Important notes :

1- If a, b, and c are not pairwise coprime, someone, by applying the proof given in the corollary like this : $a=u^{yz}$, $b=v^{xz}$, $c=w^{xy}$ will have $u^{xyz}+v^{xyz}=w^{xyz}$, and could say that all the x,y and z are always smaller than 2. What is false: $7^3+7^4=14^3$.
The reason is sipmle: it is the common factor k which could increase the power, for example if $k=c'^r$ in the proof, then $c^z=(kc')^z=c'^{(r+1)z}$. You can take the example : $2^r+2^r=2^{r+1}$ where $k=2^r$.

2- These techniques do not say that the equation $a^n+b^n=c^n$ where $a,b,c\in\mathbb{R}\cap]0,+\infty[$ has no solution since in the proof the equation $X^2+Y^2=Z^2$ can have a sloution. We take $a=X^{\frac{2}{n}}$, $b=Y^{\frac{2}{n}}$ and $C=Z^{\frac{2}{n}}$.

3 – In [3] I proved the abc conjecture which implies only that the equation $a^x+b^y=c^z$ has only a finite number of solution with a, b, c, x, y, z a positive integers, a, b and c being pairwise coprime and all of x, y, z being greater than 2.

5- Conclusion :

The techniques used in this article have allowed to prove both the Fermat' last theorem and the Beal' conjecture and have shown that the Beal conjecture is only a corollary of the Fermat' last theorem.

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