Phase Inversion of the 3S2 Oscillation of the Earth

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Abstract: The analysis of the gravitational data from 2004 and 2011 consistently shows regular phase reversal at intervals of around 50 hours. The four-leaf directional pattern rotates in 200 hours westwards around the N-S axis of the earth. The supposed "anomalous splitting" of 3S2 is a measurement error, caused by wrongly chosen parameters of the Fourier transformation.

Introduction

Earthquakes stimulate the earth to vibrate. The measured frequencies can provide information about the internal structure, even about the Earth's core itself. It is simple to analyze strong natural resonances such as 0S0, but the information content is quite low. Some normal modes like 3S2 are much more difficult to measure and obviously cause special measuring problems, in [1] it is even said: "*A still elusive mode is 3S2 for which no reliable observations of the m= 0 line exist, not even after the Sumatra–Andaman event. The reason for this is not fully understood*."

In previous measurements[2][3], the spectrum shows a double peak with a minimum exactly where a maximum should be. Subsequently, it is shown that this is a miscalculation. Since the phase of this spectral line interchanges at regular intervals, the duration of the measurement must be adjusted. If chosen correctly, we get a one-peak result. In all cases, the oscillation frequency of 3S2 (1106.2 ± 0.1 µHz)[4] deviates strongly from the actual frequency (1102.7 ± 0.09 µHz).

For several reasons the measurement of the 3S2 oscillation is nor easy: It is excited by very strong earthquakes only and it can be detected during a few days before the signal disappears in the noise. A particular obstacle is the unexpected phase reversal that misleads FFT when used carelessly.

Earlier investigations[5] have shown that the temporal change in amplitude deviates noticeably from a slow exponential decay. At regular intervals, the phase of the signal reverses and the three-dimensional pattern of nodes is not rigidly bound to the Earth's body. In order to rule out that this was a singular event triggered by the earthquake in 2004, the available measurements from 2011 have also been evaluated and compared. The broad agreement shows that characteristic and previously unknown properties of the normal mode 3S2 were discovered. Apparently, in the calculation models of eigenmodes at least one essential property of the earth was overlooked.

The Preparation of the Data

The raw data of the SG instruments, which are stored every second, form the database. The data can be obtained here. The pre-processed minute files do not always have the necessary quality. Processing takes place in two stages. First, the frequency range 400 µHz to 1800 µHz is emphasized by applying a comb filter twice. The extremely strong tidal signals at about 22 µHz are so much attenuated that they no longer bother. The subsequent low-pass is realized by a gentle convolution, followed by the decimation to a sampling interval of 60 seconds. This coarse data (see Figure 1) can also be used for other examinations in the mentioned frequency range. Now it is shown in a sequence how the signal content of 3S2 is determined.

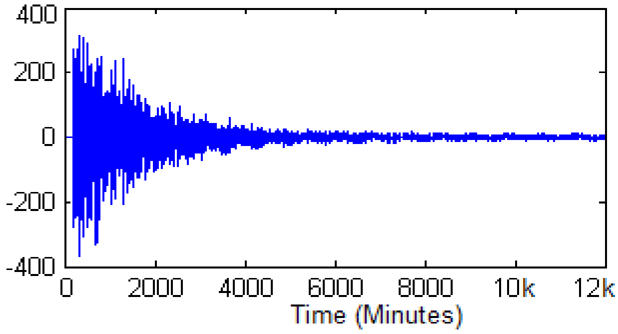

Fig. 1: The filtered SG-signal from S1 during the first 200 hours after the earthquake.

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The wanted spectral line of 3S2 is filtered out of the coarse data. Here are some facts of the signal processing to be considered: Each signal is modulated, so it occupies a certain minimum frequency range, called bandwidth. If the bandwidth is chosen too small, the "message content" will be distorted. Since the signal is unknown, one must gradually reduce the bandwidth step by step until the signal changes fundamentally.

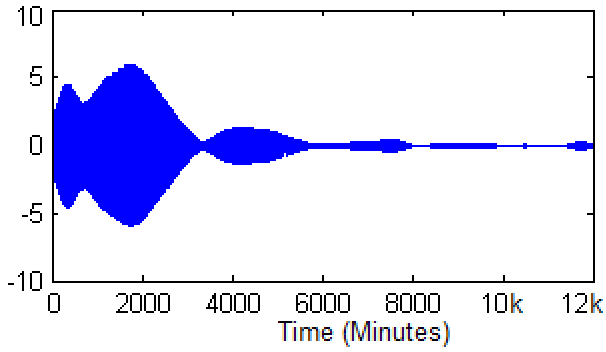


Fig 2: After reducing the bandwidth to 40 µHz, it can be seen that the amplitude of the 3S2 signal does not simply decrease exponentially over time. Apparently, a periodic change is superimposed. Changing the bandwidth between 14 µHz and 120 µHz does not significantly affect the signal.

In the middle of the spectrum is the so-called carrier, which contains no information. Any information must be searched in the accompanying sidebands. The Fourier analysis shows the relationship between the spectrum and time-dependent modulation. A temporally exponential change in amplitude fills the sidebands without any apparent structure and it's almost the same as increasing the noise level. Therefore, it makes sense to compensate for the exponential attenuation of the signal. This creates welcome side effects: hidden information in the sidebands can be more easily recognized and analyzed; the quality factor Q can be determined; the early and late signal components are weighted equally.

Figure 3 shows the remaining modulation of 3S2 after the exponential reduction of the amplitude is removed (corresponding to $Q \approx 400$). As the signal gain increases over time, disturbing noise also increases and the signal-to-noise ratio decreases. In the case of 3S2, at the latest after 12000 minutes, the noise predominates and it is time to finish the analysis.

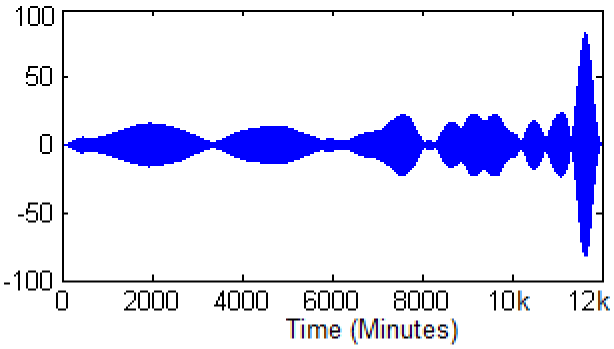


Fig. 3: After compensation of the attenuation, the periodic modulation remains. Over time, the noise is increasingly emphasized.

A reduction in bandwidth is a good way to improve the signal-to-noise ratio. The bandwidth must not be reduced too much, otherwise the modulated signal will be deleted. It has been determined experimentally that the signal from 3S2 requires a minimum bandwidth of about 12 µHz.

After narrow-band filtering of the signal, it becomes obvious that the measurable amplitude of 3S2 changes periodically. The mean amplitude depends on the geographical location of the SG station, but not the modulation frequency. What causes this enigmatic periodicity? The solution of this riddle is probably buried deep inside our earth. But that is not the end of the story, because now the spectrum of the almost noiseless signal will be considered.

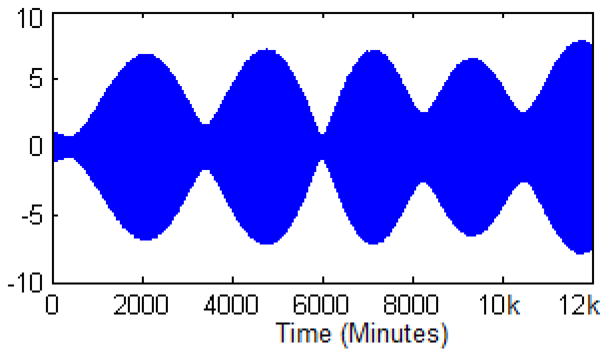


Fig. 4: By reducing the bandwidth, much noise can be removed, leaving the 3S2 signal.

The signal of 3S2, as seen in Figure 4, is not due to amplitude modulation as used in the early days of radio engineering to transmit speech and music. Because then the spectrum would have to contain mainly *three* frequencies: the carrier frequency and two symmetric sideband frequencies. But the spectrum is quite different and contains only two frequencies, as Figure 5 shows. What is wrong?

The key to solving this problem is shortening the signal length to the environment of a maximum before determining the frequency. If one chooses, for example, the period between 500 s and 3400 s or 3500 s and 6000 s, the Fourier analysis delivers the expected spectrum. In both cases, the carrier frequency is 1103 µHz, plus the two weaker sideband frequencies. Just as you expect.

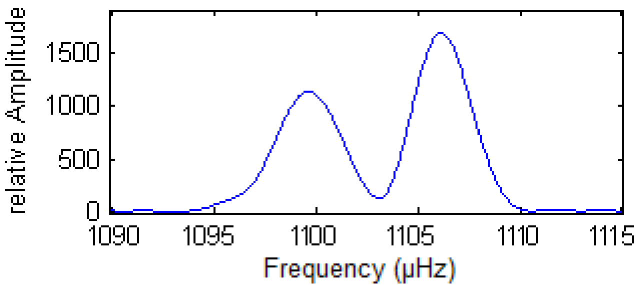


*Fig. 5: The (**wrong**) spectrum of the 3S2 signal shown in Figure 4.*

But if you choose the entire interval 500 s to 6000 s, you get a completely different picture – a double peak with a pronounced minimum near 1103 µHz (Figure 5). It would be grossly negligent to overlook this alarm signal. The structure of the spectrum *must not* change fundamentally by doubling the file length!

The rule is: The shorter the file length, the larger the half width of the spectral lines. And vice versa: With increasing file length, the half-width of the spectral lines decreases.

The Elimination of Phase Switching

Considering all the individual results of the previous section, there is only one explanation for the unexpected change in the spectrum of 3S2: At regular time intervals, the phase of the received signal is reversed. This can be explained with a simple model: Imagine that inside the earth a huge loudspeaker rotates around the north-south axis. The sound is preferably radiated in the equatorial plane and in two opposite directions. For an observer on the earth's surface, one revolution takes about 100 hours (after the 2004 earthquake). Triggered by an earthquake, the loudspeaker produces a damped oscillation of the frequency 1103 µHz, which is heard particularly loud in Canberra or Sutherland. (For an unknown reason, less volume is measured in the northern hemisphere.)

The SG stations measure the sound produced by the "front" of the loudspeaker for 50 hours, followed by 50 hours of reverse phase sound produced by the "back". (After the 2011 earthquake, the duration of rotation was only 86 hours). Below, this model is specified more precisely.

This phase inversion can be easily detected with "direct conversion". The filtered signal (yS1 from Fig. 4) is multiplied by an oscillator signal y(ref) of the same frequency. With correctly selected initial phase, the picture in Figure 6 results. During a certain period of time, the two oscillations are *in phase* and the product is positive, then they are *out of phase* and the result is negative. After 100 hours, the whole repeats itself; this interval is called 2*P1. Due to the low quality factor Q of 3S2, the signal is no longer identifiable 170 hours after the earthquake.

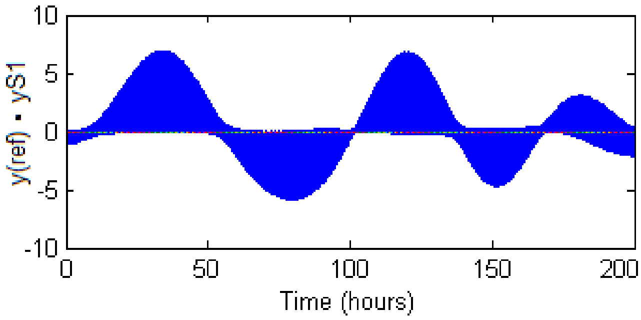


Fig. 6: The multiplication of two periodic functions reveals the phase relationship. The colored zero line shows that the frequency of 3S2 is constant. The phase of the oscillation changes periodically.

After the two major earthquakes in 2004 and 2011, all SG stations around the world registered these signals with varying delays and varying quality.

There is another way to detect periodic phase inversion. Interchange the signs of the measured values (yS1 from Fig. 4) in the ranges 3430 s — 6020 s and 8270 s — 10480 s, before you carry out the Fourier analysis. The result in Figure 7 shows the carrier frequency at 1102.7 µHz, accompanied by the symmetrical sideband frequencies 1097 µHz and 1109 µHz. Their distance from the carrier frequency corresponds to the modulation period of about 47 hours. (This applies only to the *changed* data. The actually measured geophysical data have a twice as long period of 94 hours.)

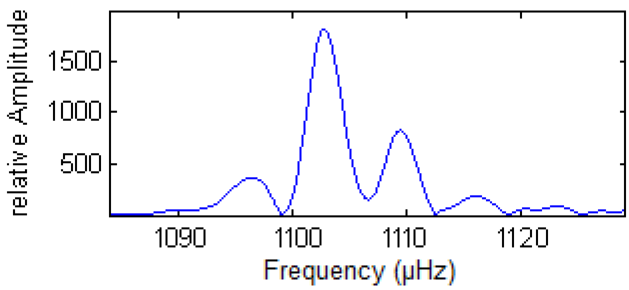


Fig. 7: The spectrum of the modified 3S2 signal clearly shows the frequency of the 3s2 self-resonance. Since the amplitude changes periodically, the two sideband frequencies arise.

Mathematical description of 3S2

The two strong earthquakes of 2004 and 2011 stimulated many normal modes to resonate whose different frequencies can be measured at the Earth's surface. In most normal modes analyzed so far, the analysis of the measured values showed particularities that are not explained by any theoretical model.

- The trivial 0S0 mode can be tracked for months and often reduces its frequency during this period. There are strange sub-modes that, according to theory, should not exist [[6]].
- All five spectral lines of the 0S2 resonance clearly change their frequency in a 60-hour rhythm. The amplitude decreases exponentially, as expected, and without any periodic components[[7]][[8]].
- The same applies to the 10S2 resonance, but here, the modulation frequency changes every 12 hours[[9]].

The normal mode 3S2 examined here can only be observed for about seven days until the signal disappears in the noise. Despite this short time, a completely different phenomenon can be detected here: in contrast to the above examples, it is not the frequency that changes, but the amplitude, combined with a periodic phase inversion.

The mathematical representation of the 3S2 signal is: $y = A e^{-\alpha t} \cdot \sin(2\pi f_c t) \cdot \cos(2\pi t/T + \Phi)$

- A is the initial amplitude, which does not affect the following examinations.
- α is the attenuation of the oscillation over time. There is a simple correlation with the quality factor Q: $\alpha = \pi f_c / Q$. (Q ≈ 400)
- f_c = 1102.7 µHz is the oscillation frequency of the normal mode 3S2.
- The $\cos(2\pi t/T + \Phi)$ factor is not a property of the 3S2 normal mode, but it describes the motion of the knot model relative to a fixed point on the surface of the earth. The time constant is T ≈ 94 hours (measured by an observer on the surface of the earth).
- The phase shift Φ depends on the geographical position of the SG-station.
- t is the time after the earthquake, measured in seconds.

The appendix includes a short program (MATLAB) that allows you to study the effect of interval phase inversion on the spectrum of 3S2. If one eliminates the next-to-last line, one obtains the *wrong* spectrum depicted in [2, Figure 3] and in Figure 5 above.

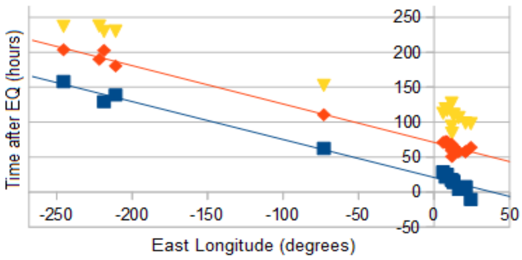
Results for 2004

After the earthquake on December 26, 2004, the stations CB, ES, H1, H2, M1, MA, MB, MC, ME, NY, S1, S2, ST, TC, VI, W1, W2 and WU registered data every second. These were evaluated using a uniform method, which was described in detail above. The results:

- The frequency is 1102.756 ± 0.090 µHz, calculated with the jacknife method.

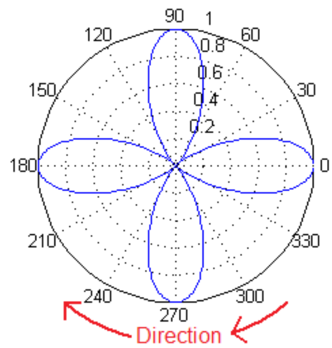
Presumably, the oscillation of normal mode 3S2 starts immediately after the earthquake. Due to overloading of the instruments, data can only be recorded several hours later. Nevertheless, the initial phase can be clearly reconstructed by direct conversion.

- The reference function for MB, ME, NY, TC and WU is $y=\sin(2\pi t f_{ref}+\varphi)$ with φ = 0.26 ± 0.30. 3S2 begins to oscillate immediately after the earthquake.
- The reference function for the stations CB, ES, H1, H2, M1, MA, MC, S1, S2, ST, VI, W1 and W2 is $y=\sin(2\pi t f_{ref}+\varphi)$ with φ = 3.07 ± 0.09. Here, antiphase oscillations are measured.
- The phase of the 3S2 signal alternates at regular intervals (see Fig. 6). The first two zero crossings (blue and red) can be defined relatively accurately, the third one (yellow) can only be determined inaccurately due to the low signal-to-noise ratio. After the 2004 earthquake, the first intervall, called $P_{1,}$ lasts 49.78 ± 0.41 hours. This is the vertical distance between red and blue lines.



- The timings of the zero crossings depend on the geographical length of the measuring station and apparently migrate slowly westward. The time required is about -33.3 ± 0.5 minutes per degree or 200 ± 3 hours per 360° (= P_2). The latitude is apparently irrelevant.

The comparison of P_1 and P_2 leads to an interesting consequence. From a fixed point on Earth, the nodal structure (= radiation pattern) of 3S2 requires 200 hours for one turn (westward). During this period, each SG station measures four zero crossings $4 \cdot P_1 = P_2$. Viewed from the Earth's North Pole, the 3S2 radiation pattern looks like a four-leaf shamrock spinning slowly in a clockwise direction. At a fixed observation site, a signal maximum is measured every 50 hours.



Results for 2011

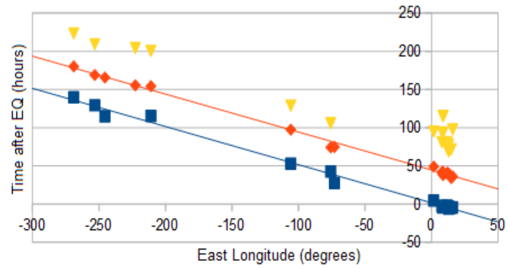
After the earthquake on March 11, 2011, the stations AP, B1, B2, CA, CB, CI, CO, DJ, H3, H4, H5, KA, LH, MC, ME, NY, OS, PE, ST, TC, W3, W4 and WU registered data every second. These were evaluated using a uniform method, which was described in detail above. The results:

- The frequency is 1102.700 ± 0.063 µHz, calculated with the jacknife method.

Presumably, the oscillations of normal mode 3S2 start immediately after the earthquake. Due to overloading of the instruments, data can only be recorded several hours later. Nevertheless, the initial phase can be clearly reconstructed by direct conversion.

- The reference function for the stations AP, CA, CB, CI, LH, TC and WU is $y=\sin(2\pi t f_{ref}+\varphi)$ with φ = 0.37 ± 0.22.

- The reference function for the stations B1, B2, CO, DJ, H3, H4, H5, KA, MC, OS, PE, ST, W3 and W4 is $y=\sin(2\pi t f_{ref}+\varphi)$ with φ = 3.70 ± 0.09.
- The phase measured by NY and ME can not be clearly determined. This may be due to the high latitude of the geographical position.
- The phase of the 3S2 signal alternates at regular intervals (see Fig. 6). After the 2011 earthquake, a period lasts 42.89 ± 0.12 hours (=P1). Since the signal-to-noise ratio is fairly modest for all European stations, the times of the third phase switching can only be determined inaccurately.



- The timings of the zero crossings depend on the geographical length of the measuring station and apparently migrate slowly westward. The time required is about -27.9 ± 1.1 minutes per degree or 167.4 ± 6.6 hours per 360º (=P2). The latitude is apparently irrelevant.

Just as after the 2004 event, we have $4 \cdot P_1 = P_2$ and again, 3S2 generates a four-sheet directional pattern. Strangely, in the year 2011 it rotates 16% slower than in 2004. After the 2011 event and at a fixed observation site, a signal maximum is measured every 43 hours.

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[10].

Programs

```
%Show the spectrum of function y
function [erg f]=zeig_sp2(y,Ts,w) %Ts in s
[z,p,k] = cheby1(6,0.1,0.95); [sos,g] = zp2sos(z,p,k);
y=g(1,1)*sosfilt(sos,y);
if Ts<0, Ts=-Ts;
    y=y.*hamming(length(y));
end
if isnumeric(w)
  if w<0, w=-w; a=1; else a=0; end
    verg=round(10*(w-floor(w)));
    NFFT = 2^(nextpow2(length(y))+verg);% z.B. +3
    f = 1/(Ts*2)*linspace(0,1,NFFT/2+1);
    Y = fft(y,NFFT);
    erg=abs(Y(1:NFFT/2+1));
    w=floor(w);
    w=min(2^verg*floor(w),length(f));
    if a==0
      plot(f(1:w),erg(1:w)) %Ausschnitt vergrößern
      %semilogy(f(1:w),erg(1:w)) %Ausschnitt vergrößern
      title('Spektrum'), xlabel('Frequenz in Hz')
      ylabel('relative Amplitude')
    end
else
    NFFT = 2^(nextpow2(length(y)));
    f = 1/(Ts*2)*linspace(0,1,NFFT/2+1);
    Y = fft(y,NFFT);
    erg=abs(Y(1:NFFT/2+1));
```

```
    if strcmp(w,'lin'),plot(f,erg)
          title('Spektrum'), xlabel('Frequenz in Hz')
          ylabel('relative Amplitude')
    end %Ausschnitt vergrößern
    if strcmp(w,'log'),semilogy(f,erg)
      title('Spektrum'), xlabel('Frequenz in Hz')
      ylabel('relative Amplitude')
    end
end

%simulate 3S2
fc=1103e-6; %Hz
T=94*3600; %s rotation time rel Earth
Q=480; alpha=pi*fc/Q;
Ts=60; %s time step
t=1:Ts:8*24*3600; %s observation time = 8 days
y=exp(-alpha*t).*sin(2*pi*fc*t).*cos(2*pi*t/T); plot(y)
%now switch phase in certain segments
y(2806:5637)=-y(2806:5637); y(8480:11267)=-y(8480:11267);
zeig_sp2(y',-Ts,1200.5); %now show spectrum
```

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