The Theories Of The Graviton
Part Two: The Quantum Framework of the Particles Nature and Mechanics

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(Dated: 24 October 2018)

Gravitons are the quanta of gravity that, if proven to exist, would potentially connect quantum mechanics with gravitation. The second part of the Graviton Theory entity focuses on the quantum side of the graviton's mechanics and nature (which were first proposed in the classical framework in Part One). They will be explained in depth, as to how gravitons act as quantum particles, and how they can act as both strings in Sting Theory and as loops in Loop Quantum Gravity. This analysis is to propose how gravitons behave, not only in our set of four-dimensional spacetime, but also in higher dimensional sets of spacetime.

PACS numbers: 14.70.Kv, 04.20.-q, 04.30.Nk, 04.60.-m
Keywords: Gravitons, Quantum Gravity, QFT, Relativity.

INTRODUCTION

The graviton is the theoretical quanta of the gravitational field. Theorized by Soviet scientists Dmitrii Blokhintsev and F. M. Gal'perin in 1934, the particle is thought to harness the gravitational force in the quantum scale, attempting to formulize a gravitational quantum field theory[1]. The main goal of this paper, and the work within, is to describe how gravity functions on the Planck scale, described by quantum mechanics. In addition, this paper will discuss how gravitons can travel into higher dimensional sets of spacetime as closed string particles. Spacetime will be geometrically treated as if it were Hilbert space.

The reason why gravitons are of great importance is because quantum gravity will construct a new way of seeing the universe, where the force that holds the entire universe together is all because of these quantum particles, and that there would eventually be a theory of everything.

The paper begins with the derivation of the graviton's Poynting vector, whose curl is evaluated in Part One, discussing its significance to the classical gravitational field. Furthermore, the path strand metric, also derived in Part One, is analyzed in the complex case that oscillating graviton paths are not irrotational (as is the classical gravitational field), but rather rotational, due to the interactions with other oscillating gravitons. In the case that gravitonic paths are irrotational, its time-dependent and -independent actions are analyzed and verified to have symmetry.

Following the path strand analysis is the discussion of gravitonic interactions within the energetic lattice, describing how they are essential to the inertial nature of unbent spacetime - yet giving verification of traces of quantum foaming. A second order differential equation was drafted, in order to supply a conditionary solution, and model the specific conditions.

Afterwards, gravitons in higher dimensions are addressed by their interactions with the gravitino (their anti-particle), as well as their transitions into higher dimensional sets and universal states. A transitionary wave function dependent on all space and all time is derived, and the probability density of "stronger gravity" in parallel universes was determined by analytical models.

I. THE GRAVITON'S POYNTING VECTOR $|S_h\rangle$

When it comes to composing a quantum field theory for gravitation, the greatest problems depend on the quantization of Einstein's General Relativity. Most successful quantum field theories are central around linear fields acting upon spacetime, as if it were actors performing upon a stage. However, in general relativity, spacetime (the stage) is the actor, which causes non-linearity - the worst problem in formulating a gravitational QFT[a].

When Einstein formulated General Relativity, he disregarded gravity as a field, but as a warped manifold. However, if it were possible to "create" a field that corresponds with the manifold, it would be a step closer to a formulizable gravitational QFT.

In the case of graviton particles, which in Part One were hypothesized to compose Einstein's fabric of spacetime and oscillate within a system of mass/energy, the particles that undergo relativistic motions create the classical gravitational field. It is their vector of motion - their Poynting vector - that is the field upon the manifold - the actor upon the gravitational stage.

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It is not proclaimed that the following work is the solution to a century-long problem, but it is hypothesized that the following work would help benefit the cause in formulating a gravitational QFT.

It is stated in Part One that the graviton's Poynting vector S_h is the quantum identity of the classical gravitational field $\vec{g}$, let there be a direct or approximate proportionality[2]. Only the curl of the vector was derived, under the pretense that S_h was an unknown vector.

As long as the graviton's path is irrotational, meaning no other oscillating gravitons are interacting with a single reference oscillating graviton, the mathematical description of the S_h field will be possible to derive.

The Poynting Vector for gravitons is adopted by the gravito-electromagnetic (GEM) Poynting Vector, for gravitational and electrostatic mechanics are equal through the interactions between gravitons and photons[3].

Classically, the Poynting Vector is derived as follows:

$$S_h = \frac{-hc}{2\gamma^*}\frac{1}{|r_1|^3}\hat{r_1} \times \frac{4\pi Gh}{c^2}\frac{1}{|r_2|^3}\hat{r_2} \tag{1}$$

where $\gamma^* = 26.889$, the Lorentz Constant in which v is the exact value of the speed of light $299,792,458$m/s.

which is simplified into:

$$|S_h| = \frac{-hc}{2\gamma^*}\frac{\hat{r_1}}{|r_1|^3}\frac{4\pi Gh}{c^2}\frac{\hat{r_2}}{|r_2|^3}\sin(\theta)$$

The two components of the Poynting vector are the following: a relativistic quantum component $\frac{hc}{\gamma}$, which is subject to relativistic length contraction ($hc \propto l$, and $l' = l/\gamma$), and a quantum gravitational component $\frac{4\pi G}{c^2}h$.

Each of the components is an essential identity to the graviton particle itself: it is the quanta of gravity that must obey both relativity and quantum mechanics.

For gravitons are quantum particles, the Poynting Vector must be quantized. Reassessing the entire classical Poynting vector into quantum operators, hc would become $\hat{\mathbf{E}}\hat{r}$ for energy and displacement, $-\sin(\theta)$ would be seen as the ratio between the spin projection and magnitude $m_s/\sqrt{6}$ (As derived in Part 1 [4]), and Gh/c^2 would become $\hat{\mathbf{p}}\hat{r}^2/m$: momentum and the displacement-squared divided by the mass.

After further derivation, setting $1/r_{1,2} = 1/r = R$, making one of the R functions equal to the oscillating gravitonic radius, the quantized equivalent to the graviton's Poynting Vector is as follows:

$$\Big|S_h\Big\rangle = \frac{4\pi G}{c^2}\frac{-\hbar^2}{2\gamma^*}\nabla^2 R \cdot \frac{m_s}{\sqrt{6}}\hat{R} \tag{2}$$

The beauty of this quantized revision of the classical Poynting Vector is the consistency of the following term: $4\pi G/c^2$. Although Planck units are being used, where $G = c = \hbar = 1$, the numerator $4\pi G$ is the gravitational potential of a neglected point mass. The denominator c^2, the square of the speed of light, can be rewritten as $c \cdot c$; the speed of light is constant in all space and all time.

Continuing to use Planck units, the Poynting Vector entails the negative acceleration, the negative Laplacian of the displacement finction R, $-\nabla^2 R$, which best resembles the classical gravitational field. However, the Poynting Vector also has the R function included. This makes the Poynting vector a non-traditional acceleration field vector, which contradicts the definition of the classical gravitational field.

Although the initial form of the vector provides an approximate proportionality to the classical field, the vector must be rewritten to have a *direct* proportionality to the classical gravitational field.

A. Rewriting the Gravitonic Poynting Vector

Applying Planck units, $\hbar = 1$, and reverting $R = 1/\vec{r}$, the Poynting vector becomes:

$$\frac{4\pi G}{c^2}\frac{-1}{2\gamma^*}\frac{m_s}{\sqrt{6}}\left(\nabla \cdot \left(\nabla \frac{\hat{r}}{r^2}\right)\right)$$

Using Gauss' Law,

$$\frac{4\pi G}{c^2}\frac{-1}{2\gamma^*}\frac{m_s}{\sqrt{6}}\iint \left(\nabla \cdot \frac{\hat{r}}{r^2}\right) dA'$$

dA' is rewritten as $\sigma\alpha dA$. σ is the mass surface density of an oscillating graviton m_Υ/l_P^2 (although in Planck units - thus making it dimensionless), and α is a scaled quantum area $l_P^2\sqrt{6}$ (also dimensionless if squared).

This is coming from the original quantum area $A = 8\pi l_P^2\sqrt{j(j+1)}$[e], with $j = s$ for an azimuthal number $l = n - 1 = 0$[f].

Therefore, $dA' = m_\Upsilon\sqrt{6} \cdot dA$.

Using the mass of an oscillating graviton $m_\Upsilon = 2\pi h/\gamma^* c\rho$[b], the double integral is rewritten:

$$\frac{4\pi G}{c^3}\frac{\pi}{\gamma^2}m_s\iint \left(\nabla \cdot \frac{-\hat{r}}{r^3}\right) dA$$

where ρ (the displacement of gravitational attraction, a.k.a. the maximum displacement of a graviton's oscillation) is the variable r.

Being that the Lorentz Factor γ, previously $\gamma^* = 26.889$, is squared and taken the inverse of, $1/\gamma^2 = 1 - v^2/c^2$. Because gravitons travel at the speed of light c, $1/\gamma^2$ would be a very small number, letting it be $\hbar$.

Therefore,

$$\frac{2\pi G}{c^3} m_s h \iint \left(\nabla \cdot \frac{-\hat{r}}{r^3} \right) dA$$

Using Gauss' Theorem,

$$\frac{2\pi G}{c^3} m_s h \oint -\hat{n} \cdot \frac{\hat{r}}{r^3} ds$$

The Poynting Vector, previously a double area integral, is now a single surface integral. This is considering the gravitational shell of an arbitrary object, which is indeed the range of gravitonic oscillation. For the surface notation is present, which is referred to as the surface area, the notation ds would be the area notation of a gravitational shell $4\pi r dr = 4\pi(r'^3/r^3)\mathbf{r}d\mathbf{r}$.

Using $\mathbf{r} = r - r'$, and letting $r >>> r'$, the surface notation simplifies into $4\pi dr/r^2$.

Therefore, after scaling, the integral identity of the Gravitonic Poynting Vector, with a direct proportionality with the classical gravitational field, is the following:

$$\left|S_h\right\rangle = \frac{8\pi G}{c^3} m_s \hbar \oint -\hat{n} \cdot \frac{\hat{r}}{r^5} dr \tag{3}$$

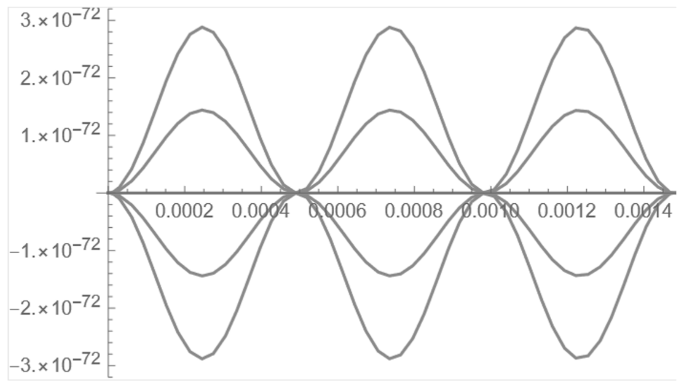
The following identity of the Poynting Vector is a significant derivation in many aspects: 1) the leading coefficient $8\pi G$ is the same coefficient for the energy-momentum-stress tensor in Einstein's field equations, providing a connection with general relativity; 2) the Poynting vector remains spin-dependent and quantized, by nature of quantum particles with spin; and 3) the coefficent $G\hbar/c^3$ is the square of the Planck length, which makes this vector field the quantized equivalent to the classical field.

Most importantly, assuming the graviton's path is irrotational, the integral is evaluated into a field with a proportionality of L^{-2} (L=length), just like the classical field $\vec{g}$.

The purpose for rewriting the differential form of the Poynting vector into an integral form is to show a direct relationship between the particle's field and the classical gravitational field. There is no guarantee that the integrand is a "linear variable," meaning the variable r is indeed radial. Because gravitons can have rotational paths within oscillation, the integrand may even be a tensor.

B. Applying the Vector

Using the initial differential idenity, and having the *Ansatz* (approach) that the displacement function R is the mechanical wave function $\chi(\tau)$, the Laplacian of the function $\nabla^2\chi$ verifies the result of an acceleration function. The graph of the Poynting vector that is plotted (with the values of m_s within the interval [-2,2]) is the following:



The function above displays the graviton's Poynting Vector with the different spin projection values m_s. The functions with the greatest amplitude is where $m_s = \pm 2$.

II. GRAVITONIC STRAND METRIC ANALYSIS

The path strand metric $d\sigma^2$ is an elegant but vicious field equation worthy to tame in order to solve the action of an attracted oscillating graviton.

It was first derived in Part One to form a link between graviton path strands with self-gravitating strings.

For gravitons respond to each other, due to their attraction to energy, they alternate their sense of direction as they oscillate along the displacement ρ. There would be solutions to the metric that would conclude that gravitons are dispersed into deep space, or would lead them to travel into a different dimension. It is within the Hilbert Space will the strand metric be analyzed.

The strand metric is presented as:

$$d\sigma^2 = \partial\chi_q^2 + 2\chi_q \sin(\phi) d\phi \partial\chi_q + \chi_q^2 \sin^2(\phi) d\phi^2 \tag{4}$$

where $\chi(q) = \rho \sin(q\gamma/\rho)$, replacing τv as the general displacement variable q, and $\partial\chi_q = \gamma \cos(q\gamma/\rho)\, dq$.

For it is unprecedented to decipher the change in the graviton's path due to other gravitons, the angle ϕ within the sine functions shall be the magnitude of the phi vector, which is the sum of the angular vectors encompassing all possible dimensions:

$$\vec{\phi} = \vec{\theta} + \vec{\varphi} + \cdots + \vec{\beta} \tag{5}$$

The angular vectors $\vec{\theta}$, $\vec{\varphi}$, $\vec{\beta}$, and so on, are also time-dependent functions that entail the curvature and actual path of a secondary function. It is vital that this secondary function is different from the mechanical wave function χ, which is assumed to be a constant whose derivative is zero.

The "angular equations" are generalized as follows:

$$\vec{\alpha} = \frac{a' \times a''}{(a')^3} a \tag{6}$$

where the secondary function a is a collection of vector functions of each dimension denoted from i to μ, for the nth dimension in the Hilbert space.

For the θ angular equation, its secondary function is $r = (\mathbf{x}_{1\theta}, \mathbf{x}_{2\theta}, \cdots, \mathbf{x}_{n\theta})$, with every third-multiple vector function $(\mathbf{x}_{3N\theta})$ equaling zero.

For the φ angular equation, its secondary function is $s = (\mathbf{x}_{1\varphi}, \mathbf{x}_{2\varphi}, \cdots, \mathbf{x}_{n\varphi})$.

And lastly, for the β angular equation, its secondary function is $\aleph = (\mathbf{x}_{1\beta}, \mathbf{x}_{2\beta}, \cdots, \mathbf{x}_{n\beta})$.

Depending on the number of dimensions, if $\aleph$ is a third-multiple secondary function, it must be a transitionary function to travel into a higher set of three-dimensional space. Furthermore, if $\aleph$ is also a $(1+3n)$th secondary function, every third-multiple vector function $(\mathbf{x}_{3N\beta})$ must equal zero, as like in the r function.

It is important for all $(1+3n)$th secondary functions to have a zero in every third-multiple vector function. It is because in three-dimensional space, θ-angles are restricted to two dimensions. Whereas the other secondary functions have all their vector functions non-zero, for φ-angles encompass all of the dimensions in 3d space.

To prove that the interactions between the even and odd secondary functions do not surpass the barrier of n-dimensions, the cross-product test shall be conducted. Note that this must be done in two- or three-dimensional Hilbert space. In three dimensions, if $r = s$, then the cross product between r and s will result to a product function f in two dimensions. However, if $r \neq s$ in three-dimensional Hilbert space, then $r \times s$ results to a product function f in three-dimensions.

Otherwise, in higher dimensions, the cross-product test can be conducted using the determinant of the n-dimensional "Action Tensor Matrix" $M_{\beta\mu}$. The Action Tensor Matrix includes all possible coordinates from $\hat{\imath}$ to the coordinate of the nth dimension $\hat{\mu}$. The following rows are each of the secondary functions in action, from the established r and s to $\aleph$, the $(n-1)$th-dimensional secondary function dictated by the angle function β.

The Action Tensor shall be seen as follows:

$$M_{\beta\mu} = \begin{bmatrix} i & j & k & \cdots & \mu \\ \mathbf{x}_{1\theta} & \mathbf{x}_{2\theta} & \mathbf{x}_{3\theta} & \cdots & \mathbf{x}_{n\theta} \\ \mathbf{x}_{1\varphi} & \mathbf{x}_{2\varphi} & \mathbf{x}_{3\varphi} & \cdots & \mathbf{x}_{n\varphi} \\ \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ \mathbf{x}_{1\beta} & \mathbf{x}_{2\beta} & \mathbf{x}_{3\beta} & \cdots & \mathbf{x}_{n\beta} \end{bmatrix}$$

As a simplified matrix, the Action Tensor is rewritten as:

$$M_{\beta\mu} = \begin{bmatrix} \varsigma \\ r \\ s \\ \cdot \\ \cdot \\ \cdot \\ \aleph \end{bmatrix}$$

where ς is the collection of coordinates.

The differential notation of the angle $d\phi$ is a function: the "angular gradient":

$$d\phi = \nabla\phi \tag{7}$$

where

$$\nabla\phi = \partial\vec{\theta} + \partial\vec{\varphi} + \cdots + \partial\vec{\beta} \tag{8}$$

Therefore, the simplified graviton path strand metric is:

$$d\sigma = \partial\chi_q + \chi_q \sin\left(||\vec{\phi}||\right) \nabla\phi \tag{9}$$

III. THE ACTION OF AN OSCILLATING GRAVITON PARTICLE

In understanding the behavior of gravitons while in oscillation, knowing their action is crucial to the interpretation of energy-momentum symmetry.

A. The Time-Dependent Energy Action

The time-dependent action integral:

$$J = \int_{\tau_o}^{\tau} \mathcal{L} d\tau \tag{10}$$

demands the use of the Lagrange $\mathcal{L} = T - U$. An oscillating particle has a small amount of potential, described by the Gravitonic Gaussian Potential $V(x)$. Therefore, the

Lagrange will equal the kinetic energy of an oscillating graviton $E_\Upsilon = 2\pi hc/\rho$. However, to illustrate the variation of instantaneous energy of an oscillating particle, the oscillating energy E_Υ shall be revised:

$$E_\Upsilon = \frac{2\pi hc}{\chi(\tau)} \tag{11}$$

where $\chi(\tau)$ is the time-dependent mechanical function $\rho \sin\left(\tau v/\rho\sqrt{1-\frac{v^2}{c^2}}\right)$.

This demonstrates that the particle at any location between 0 and ρ has a varying amount of energy, due to its intensity. However, keeping true to the Heisenberg Uncertainty Principle, as long as the graviton's energy is calculated and it travels at the speed of light c, the amount of time it would take for a graviton to have that amount of energy is uncertain.

Although the time for a graviton to reach ρ would be as simple as $\tau = \rho\gamma/c$, it is truly not simple. For gravitons can travel through dimensions freely and spontaneously, the calculated time would be false. Therefore, the time it would take a graviton to reach ρ is uncertain as long as the graviton's oscillation energy is known and solvable. It also applies to a solvable time interval; the amount of energy for an oscillating graviton must be unknown.

However, since gravitons are relativistic particles as they are quantum, it is calculated in Part One that their relativistic speed, between their linear and vibrational motions, is $0.93c$. Therefore, the Heisenberg Uncertainty shall not be in full effect.

This rewrites the time-dependent action integral to be:

$$J_\tau = \frac{2\pi hc}{\rho}\int_{\tau_o}^{\tau} \csc\left(\frac{\tau v\gamma}{\rho}\right) d\tau \tag{12}$$

The integral is solved, theorizing the Time-Dependent Action of the Graviton Particle:

$$J_\tau = \wp\Big[\ln\left(\csc\left(\frac{\tau v\gamma}{\rho}\right) + \cot\left(\frac{\tau v\gamma}{\rho}\right)\right)\Big] \tag{13}$$

where $\wp$ is the gravitonic Planck action, $\frac{2\pi}{\gamma^*}h$, with values of $1.548 \times 10^{-34}\mathrm{J}\cdot\mathrm{s}$ and $9.665 \times 10^{-16}\mathrm{eV}\cdot\mathrm{s}$.

The time-dependent action is essentially irrotational. In order for the space-time symmetry to be held as true, the time-independent action must be evaluated in the irrotational frame, meaning the path strand metric $d\sigma$ must also be irrotational.

B. The Time-Independent Momentum Action

The time-independent action integral demands the use of the momentum $\mathbf{p}$. Using the energy-momentum relation, the graviton's momentum is derived to be:

$$\mathbf{p}_\Upsilon = \mu\frac{2\pi h}{\rho} \tag{14}$$

where $\mu = 1.006$, which can be overlooked as 1.

As a function, the gravitonic momentum would be revised as:

$$\mathbf{p}(q) = \mu\frac{2\pi h}{\chi(q)} \tag{15}$$

while χ_q is already established.

For the time-independent action integral is an integral with respect to distance, the derivative notion $d\sigma$ is the square-root of the metric of a gravitonic path strand:

$$d\sigma = \partial\chi_q + \chi_q \sin(||\vec{\phi}||)\nabla\phi \tag{16}$$

where $\partial\chi_q$ is already established.

The path strand metric is key to the overall path of a graviton. If a graviton interacts with another, because of their attractive forces, the graviton's straight-line path becomes intertwined as it oscillates from 0 to ρ.

Therefore, the action of the graviton particle is equal to:

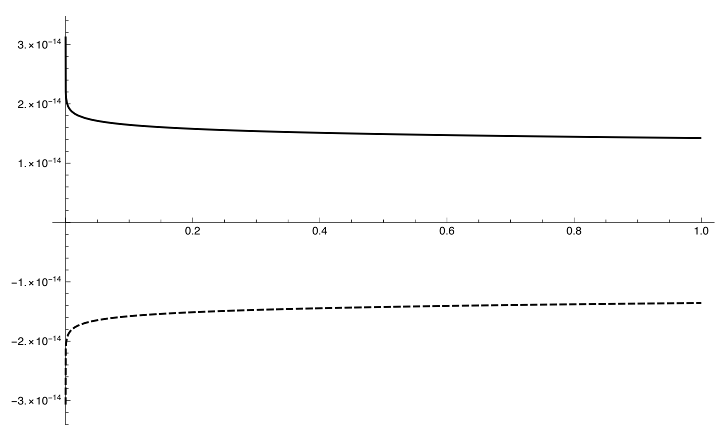
$$J = \int_0^\rho \mathbf{p}(q)\cdot\Big(\partial\chi_q + \chi_q \sin(||\vec{\phi}||)\nabla\phi\Big) \tag{17}$$

C. The Irrotational Action J_i

For an irrotational path strand, no matter the perspective, the angle $\phi = 0$, which simplifies the action intergal into

$$J_i = \frac{2\pi h}{\rho}\int_0^\rho \frac{\partial\chi_q}{\chi(q)} \Rightarrow \wp\Big[\ln\left(\sin\left(\frac{q\gamma}{\rho}\right)\right)\Big] \tag{18}$$

The graph comparing the two gravitonic actions is seen as follows:



where the bolded function is the time-dependent energy action, and the dashed function is the time-independent irrotational momentum action. This graph proves the symmetric nature of energy and momentum, and their respective curves hold true to their quantum operators $\hat{\mathbf{p}} = -i\hbar\nabla$, and $\hat{\mathbf{E}} = i\hbar\nabla$.

D. The Rotational Action J_r

The action of a rotational path strand, where the angle within the metric $\phi > 0$, is unsolvable without further research into the terms of the metric. Nevertheless, the rotational action integral remains as the general time-independent action.

IV. THE EMDEN-SCHROEDINGER-WHEELER (E.S.W.) EQUATION

Quantum foaming is a phenomenon imagined by quantum physicist John Wheeler, which describes that there are quantum interactions within the spacetime fabric in the Planck scale[c]. It was briefly mentioned in Part One that interacting gravitons within oscillation form rotational path strands, which resemble the quantum foaming in regions of gravitation. However, in regions where there is little to no source of gravitation, foaming is still present.

This section will discuss how unbent spacetime has quantum foaming in the Planck scale, yet is still inertial for astronomical objects, by referring to the gravitons in the stable lattice: the network of interacting gravitons in stable energetic equilibrium[d].

Because gravitons in the lattice are connected to each other by bonds of energy fluctuations, it is required of each stable graviton to neutralize an amount of transmitted "neighboring" energy that would compromise the lattice's stable equilibrium. In order for a single reference graviton to neutralize this amount of energy, the particle would need to have a sufficient amount of internal potential energy to somehow "reverse" the transmitted energy.

Despite the theory that stable gravitons have a rest energy of 6.53×10^{-5}eV[g], the rest mass E/c^2 is derived from the energy bond length λ: the average wavelength of an energy wave function of two stable gravitons. For instantaneous energy wave functions that are transmitted from neighboring particles, the rest energy of a reference graviton is based on the amount of internal potential energy that particle has. If the reference graviton sends an energy wave function to either of its neighbors, that particle would have to become reenergized through a neighboring wave function.

Letting these energy transmissions be spontaneous acts, if a neighboring particle sends an energy wave function towards the reference graviton, which has a sufficient amount of potential energy, that particle "reverses" the wave by reflecting the incoming wave into a negative wave function of the same phase and magnitude. This is the basis of quantum foaming and inertial maintenance in unbent spacetime.

This action of energy neutrality - gravitonic stability - requires a net wave function $|\psi\rangle$ (incoming wave constructed with the reflected wave), which must be formulated through a specific differential equation that best describes the physical scenario.

This specific differential equation takes on the roles of three other differential equations. The first of the three is the astrophysical Emden Equation (using the Emden Equation requires the assumption that gravitons are made of plasma, for the equation's use is to determine the radii of plasma stars)

$$\frac{d^2\theta}{d\xi^2} + \frac{2}{\xi}\frac{d\theta}{d\xi} + \theta^n = 0$$

where θ is a function of the dimensionless variable ξ.

The second is the quantum mechanical Schrödinger Equation (by nature of quantum particles)

$$\hat{H}|\psi\rangle = \hat{E}|\psi\rangle,$$

where $\hat{H}$ is the Hamiltonian operator $\frac{-\hbar^2}{2m}\nabla^2 + V$ and $\hat{E}$ is the energy operator $i\hbar\nabla$,

and the third equation is the quantum gravitational Wheeler-DeWitt Equation (stating that spacetime as a whole is stable in the quantum scale)

$$\hat{H}|\psi\rangle = 0.$$

A single bond of stable gravitons in the lattice must have a neutral net energy, and what each graviton must do to obtain equilibrium is to nullify the transmitted energies, so that $E_{in} = -E_{out}$.

Using the three equations, and modifying the grand equation to satisfy the physical scenario, the following equation is derived:

$$\nabla^2|\psi\rangle + |\psi\rangle = -\nabla|\psi\rangle \tag{19}$$

The following equation is the Emden-Schrödinger-Wheeler Equation. The structure of the E.S.W. Equation is similar to that of Emden's Equation, but rewritten as operators the equation is simplified:

$$\hat{\mathbf{H}}|\psi\rangle = -\hat{\mathbf{E}}|\psi\rangle, \tag{20}$$

In the larger view, this supports the postulate that the universe, although riddled with Einsteinian curvatures, is geometrically flat. If all of the curvatures were summed as if they were wave interferences, the entire universe (observed and not) would have no curvature. The energy it would take to (attempt to) curve spacetime, if there is no presence of a mass, will be neutralized to maintain the flat, inertial nature of unbent spacetime.

The only exception to this statement is the formation of Kugelblitz black holes.

A. The Solution to the E.S.W. Equation for Graviton Stability

The solution to the E.S.W. differential equation for gravitonic stability is a wave function, which describes the probability density for a single reference graviton to maintain a zero-net energy.

With an *Ansatz* $\psi = e^{\lambda\xi}$, the function is derived to be:

$$\psi(\xi) = \exp\left[i\frac{m}{\hbar}\xi - \frac{2m}{\hbar}\sqrt{\frac{|V|}{2m}}\xi\right] \tag{21}$$

The Solution is conditionary, depending on the value of the potential energy V, which is resembled as the internal energy of the reference graviton. In this case, the potential energy represents the amount of internal energy a reference graviton has for neutralizing "neighboring energies".

1. V=0

In the case that $V = 0$, the stability wave function $\psi(\xi) = \exp\left[i\frac{m}{\hbar}\xi\right]$. Using Euler's theorem, the function can be rewritten as a generalized function in terms of sines and cosines:

$$\psi(\xi_{V=0}) = A_o \cos\left(\frac{m}{\hbar}\xi\right) + A_1 \sin\left(\frac{m}{\hbar}\xi\right)$$

Only the sine component would be used, for the function is an energy wave with an initial condition at equilibrium. The generalized equation is simplified:

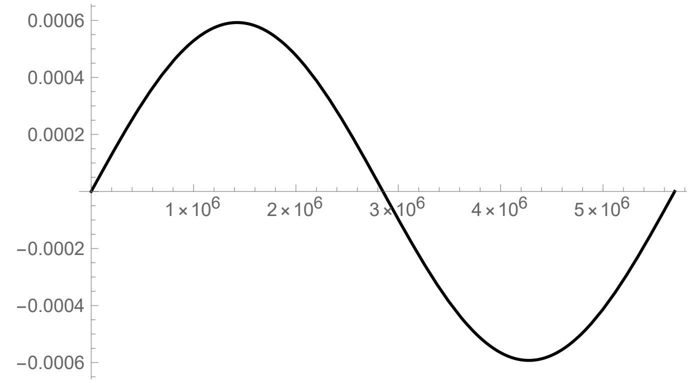
$$= A_1 \sin\left(\frac{2\pi}{\lambda c}\xi\right)$$

where $m = h/(\lambda c)$, and $\lambda = 0.019$m (the length of an energy band between two stable gravitons[5]).

Using normalization to find the leading coefficient, the wave function where the potential energy is zero is equal to:

$$\psi(\xi_{V=0}) = \sqrt{\frac{2}{\lambda c}} \sin\left(\frac{2\pi}{\lambda c}\xi\right) \tag{22}$$

The "No Potential" Particular Solution is proprotional to the wave function of stable gravitions $(\Psi_B(x))$[6], if the function were in Planck Units. That relationship exists because the stable graviton wave function is a function over a period length of λ, the energy bond length. Without a potential barrier, the E.S.W. Solution is a permittable function where transmitted "neighboring" energies are transferred, making fluctuations in the space - quantum foaming.



However, because gravitons contain energy within themselves, any reference graviton must have a potential V. The duration of the condition where $V = 0$ would only be brief, when the reference graviton has emitted energy, leaving it to take in transmitted "neighboring" energies.

2. V>0

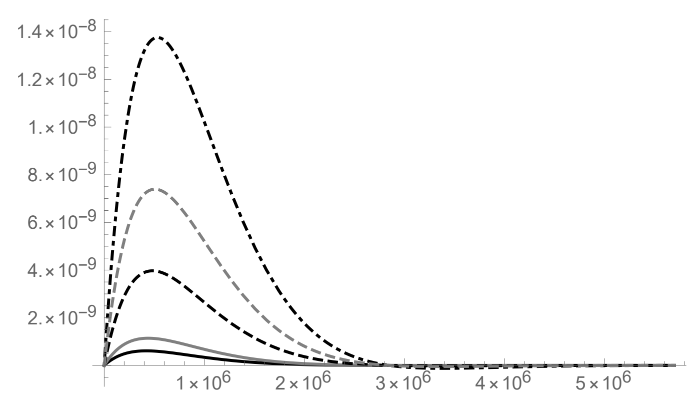
In the case that the potential energy V is greater than zero, $\psi(\xi_{V\to\infty}) = \psi(\xi_{V=0}) \cdot A_o \exp\left[\frac{2m}{\hbar}\sqrt{\frac{V}{2m}}\xi\right]$.

With an increasing value of potential energy V, it would become much easier for a reference graviton to neutralize transmitted "neighboring" energies.

After further derivation, the function with increasing potential resistance, after normalization and scaling, is derived as follows:

$$\psi(\xi_{V\to\infty}) = \frac{4}{3}V\sqrt{\frac{\pi}{\lambda c \cdot e^{8\pi V}}} \sin\left(\frac{2\pi}{\lambda c}\xi\right) \exp\left[\frac{-4\pi}{\lambda c}V\xi\right] \tag{23}$$

and the graph illustrating the solution containing various potential values is shown below:



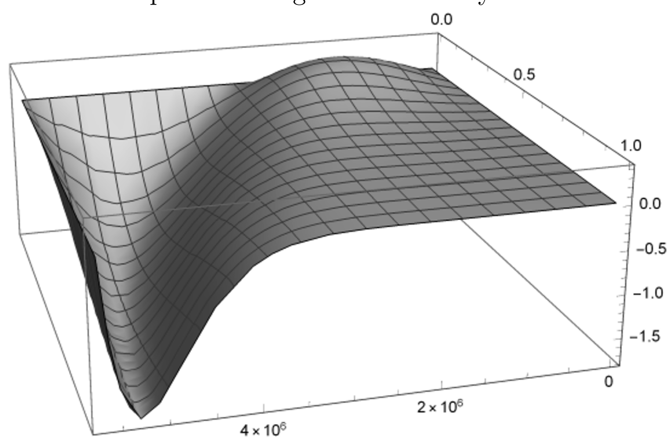
In the graph $V_1 = 0.75$eV is the dot-dashed black curve, $V_2 = 0.8$eV is the dashed gray curve, $V_3 = 0.85$eV is the dashed black curve, $V_4 = 0.95$eV is the solid gray curve, and $V_5 = 1\text{eV}$ is the solid black curve. Those values of internal potential are greater than the reference particle's rest energy.

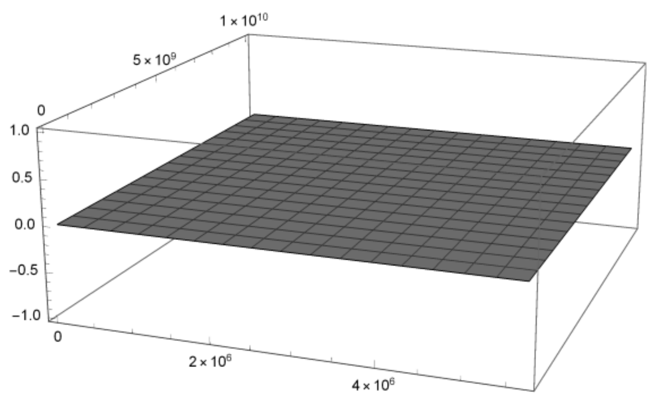
As the value of potential increases, the curve will level off into a zero function.

In the case that $V < 0$, $\psi(\xi)$ remains the same in the case that $V > 0$ due to the absolute value.

In both conditions of the potential V, the periodic domain of the stability wave function is λc. This is to say that the energies from "neighboring" particles that are transmitted to a reference graviton, propogating at $\psi(\xi_{V=0,c=1})$, travel at the speed of light across the energy bond length.

The two three-dimensional models below illustrate the behavior of space due to graviton stability.





The first model is the view of spacetime if the reference graviton had no potential energy in its interior to neutralize the transmitted "neighboring" energy. The second model is spacetime if the reference graviton had the sufficient potential energy to neutralize the transmitted energy.

If the reference graviton had no potential energy to neutralize the "neighboring" energies from the other gravitons in the lattice, spacetime would be elastic rather than inertial. This shows the vital role of the lattice particles to the inertial nature of spacetime.

V. GRAVITONS IN HIGHER DIMENSIONS: INTERACTING WITH THEIR ANTI-PARTICLE

For every particle has an anti-particle, the estranged cousin of the bosonic graviton is the fermionic gravitino. Gravitinos are fermions with a spin $s = 3/2$, and no charge. However, unlike most anti-particles, gravitinos are only present in one space if and only if gravitons are not residing within the same space. Because gravitinos are part of supergravity (SUGRA), select SUGRA principles will be applied to demonstrate graviton-gravitino interactions.

A. The Graviton and Gravitino Spaces

In order to show the interactions between gravitons and gravitinos, the simplistic method of bra-kets will be used. $|\Psi\rangle$ shall be dubbed the "graviton subspace," and $|\Phi\rangle$ the "gravitino subspace." These two fields will be represented generally in N-dimensional Hilbert space, such that:

$$|\Psi\rangle = \sum_{i=1}^{N} |\Phi_i\rangle\langle\Phi_i|\Psi\rangle$$

$$|\Phi\rangle = \sum_{i=1}^{N} |\Psi_i\rangle\langle\Psi_i|\Phi\rangle$$

All possible interacions of the subspaces may be represented using simple linear algebra:

$$\langle\Psi|\Psi\rangle = 1 \tag{24}$$

$$\langle\Phi|\Phi\rangle = 1 \tag{25}$$

$$\langle\Psi|\Phi\rangle = 0 \tag{26}$$

$$\langle\Phi|\Psi\rangle = 0 \tag{27}$$

The interactions above state that identical subspaces interacting with each other have a non-zero energy state of space. However, when different subspaces interact, they nullify each other, rendering a zero total energy state of space. This can lead to the conclusion that, in such a universe where the gravitino and graviton spaces interact, there would be a zero-total energy, and the universe would be classified as geometrically flat.

In other words, such a universe may come from nothing due to its zero total energy nature. This is only if that particular universe has both the graviton and gravitino subspaces interacting simultaneously across the universe. But our universe may be more dynamic than that; when one subspace is needed to interact with an entity, the other subspaces' states are nullified by existing in a higher or lower dimensional space.

However, these two subspaces may fluctuate back and forth from neutral to positive interactions, in order to maintain a neutrality when needed - much like what was shown and theorized in the E.S.W Equation. But when the force of gravity is needed to be applied onto a mass, the two subspaces fluctuate between interactions, such that the graviton's influence is greater than that of the gravitino. That is to say the force of gravity is acting on this mass, and it is not too overbearing for the size of the mass. This verifies that gravitons mediate the gravitational force, and their relativistic motions are what creates the classical gravitational field.

1. The Difference in Gravitonic and Gravitino Interactions to Mass

To describe the link between mass and the subspaces $|\Psi\rangle$ and $|\Phi\rangle$, the following statements can be made where M is the mass of an arbitrary object:

$$M \propto |\Psi\rangle$$

$$M \propto \frac{1}{|\Phi\rangle}$$

The following proportionalities that were derived from the initial statements give a confident, yet indefinitive, relation between the subspaces and the arbitrary mass. They are seen as follows:

$$\nabla^2\Psi \propto 4\pi r\rho \tag{28}$$

$$\nabla^4\Phi \propto \frac{1}{4\pi r^3\rho} \tag{29}$$

$$\nabla^2\Psi \propto \frac{1}{\nabla^4\Phi} \tag{30}$$

where ρ is the density of the massive object.

The last proportionality of the three shows that the gravitino subspace is *much* weaker than the graviton subspace, for the inverse of the fourth order del operator of the gravitino subspace is proportional to the second order del operator of the graviton subspace. In our perspective, the gravitonic influence is dominant over that of gravitinos, and for "anti-mass" the gravitino interaction is dominant over that of gravitons.

As an example of anti- and "normal" matter interacting with each other, we can use the expansion of the universe. The universe is expanding because of anti-matter, letting dark matter have traces of anti-matter. Due to this expansion, "normal" matter is affected due to its displacment from origin. Therefore, anti- and "normal" matter interact with one another indirectly. However, if both forms of matter interact *directly*, they will nullify each other, just like the graviton and gravitino subspaces.

VI. GRAVITONS IN HIGHER DIMENSIONS: DIMENSIONAL TRANSITIONS

As closed strings, gravitons are free to transition into higher or lower sets of dimensions.

According to supergravity in string theory, the largest possible dimension D in which supersymmetric multiplets can exist with spin ≤ 2 is $D = 11$, with a single local symmetry.

A. The Ping Pong Thought Experiment

The concept of this thought experiment is much like two people playing a game of table tennis (or ping pong). The table has two halves: two different yet conjoined sets of dimensions. Bordering the two sides is the net: the barrier between the two sets.

If the server is unable to hit the ball across the net, the ball remains on the serving side. That is to say (in terms of gravitons) that energetic particles could not advance into the higher set, due to an insufficient amount of energy. But if the server is able to hit the ball across the net, the ball advances to the other half. That is to say that energetic particles (with sufficient energy) can transition into higher and lower sets, as if two skilled players are keeping up a round of table tennis.

1. Dimensional Transition Probability

In extension to the Ping Pong Thought Experiment, if the graviton lacks the required amount of energy to make a dimensional transition. The probability to reflect is equal to 1. However, if the graviton has the sufficient amount of energy to make a transition, the probability to transition is equal to 1, such as:

$$E \geq E_t; T = 1, R = 0$$

$$E < E_t; T = 0, R = 1$$

When $T = 1$, there is a complete transition; and when $R = 1$, there is a complete reflection. The transmission coefficients, R and T can be calculated with the following:

$$R = \frac{J_r}{J_i}$$

$$T = \frac{J_t}{J_i}$$

where J_i is the probability current in the wave acting on the barrier, and J_r is the probability current of the reflection. While J_t is the probability current in the wave moving away from the barrier on the other side defined as:

$$J_t = \frac{i\hbar}{2m}\left[\psi_t(x)\frac{d\psi_t^*(x)}{dx} - \psi_t^*(x)\frac{d\psi_t(x)}{dx}\right]$$

This renders a mathematical description of the probability of the graviton moving through dimensions, for gravitons are meant to be in relativistic motion in order to transition into higher dimensions. Although this mathematical description is non-relativistic for simplicity, it may be argued that, at the moment the graviton is transitioning between dimensions, relativistic factors are negligable, and this model may be applied.

B. The Transitionary Energy E_t

It is therorized in Part One that gravitons (as boson particles) are always in the energy state $n = 1$. As gravitons transition into higher sets of dimensions, the particles shall also be in a higher energy state. Gravitons residing in $n = 1$ is depicting that gravitons reside in the first set of four-dimensional spacetime - our set. As gravitons transition through higher sets of 4D spacetime, gravitons are essentially increasing in energy state.

In order for gravitons to transition into higher dimensional sets, the particles need to obtain a specific value of energy, in order to tunnel through the "set barrier".

There are three types of gravitons: dispersed, oscillating, and stable. Hypothesized in Part One, oscillating gravitons are the particles that oscillate within the mass-energy system of either a single astronomical object or a binary. Stable gravitons are the particles that are in constant interaction with other stable gravitons, forming an energetic lattice that composes Einstein's fabric of spacetime.

These two particle types can be classified as "bound gravitons," for they are bound to systems of mass and/or energy. Dispersed gravitons are the "free gravitons" that are dispersed from black holes as gravitational waves, such as during the first expansion of gravitation after the Big Bang.

Each type of graviton is set to a maximum energy to surpass in order to make a dimensional transition.

For dispersed gravitons:

$$E_t = \frac{n^2hc\gamma^*}{16\pi\lambda_C} \tag{31}$$

where λ_C is the dispersed graviton's Compton wavelength 1.6×10^{16}m[7], and $\gamma^* = 26.889$.

For oscillating gravitons:

$$E_t = \frac{n^2hc\gamma^*}{8\rho} \tag{32}$$

using the "oscillator wavelength" $\lambda_o = \rho/2\pi$.

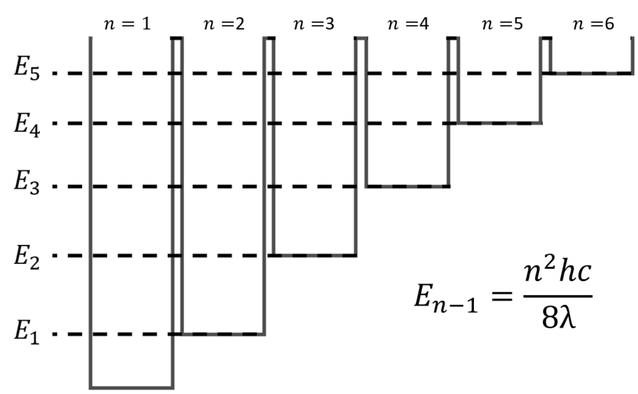
And for stable gravitons:

$$E_t = \frac{n^2hc}{8\lambda_b} + V_b \tag{33}$$

where λ_b is the wavelength of the two-stable-graviton energy bond with a length of 0.019m, and V_b is the potential energy of the energy bond.

It is easier for dispersed gravitons to transition into higher-dimensional sets, for it is a free particle unbound from a system of mass and energy. Oscillating gravitons may be restriced to a mass-energy system, but they have the limited flexibility to transition into higher sets. They only do so at the central node of spacetime curvature, such as a singularity. Lastly, it is nearly impossible for stable gravitons to transition into higher dimensions, for they would have to overcome the potential resistance of the energy bonds V_b. In the presence of an astronomical object, only then could stable gravitons have the possibility to transition into higher sets (as oscillating gravitons).

A particularly elementary model of the universe for transitioning gravitons is seen below:



Each dimensional set is an energy state n, where $E_t = E_{n-1}$.

C. The Transitionary Wave Function $\Psi(q, t)$

In the moment when oscillating gravitons reach the specific transitionary energy, they obtain a secondary wave function: the transitionary wave function $\Psi(q, t)$. It is the wave function that allows oscillating gravitons to transition into higher sets as an act of tunneling.

The generalized format of the wave function is the following:

$$\Psi(q,t) = A_\Psi \psi_0(q)\psi(t) \tag{34}$$

where A_Ψ is a normalized coefficient of the function, which includes the tunneling probability $T = e^{-2\kappa l}$ (where $\kappa = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$ and l is the length of the barrier between the wells of higher dimensional sets). For gravitons are closed strings, and can easily transition into higher sets of dimensions, the tunneling probability T will be equal to one.

$\psi_0(q)$ is the quantum wave function of the oscillating graviton; it is in terms of the generalized variable q, for it encompasses three spacial dimensions. Furthermore, $\psi(t)$ is a normalized wave function for the variations of time perception.

To go about normalizing the whole transitionary wave function, each wave function must be individually normalized. The oscillating function ψ_0 is already normalized, in Part One. This new function $\psi(t)$ must be normalized on its own.

As the oscillating graviton travels from 0 to ρ, it undergoes relativistic mechanics. Therefore, its perception of time is through time dilation. To demonstrate the change of time perception as the graviton oscillates within ρ, the wave function must be a sinusoidal function with no negative values:

$$\psi(t) = A_t|\sin(t)|^2 \tag{35}$$

where $A_t = \rho A$, the normalized coefficient for the wave function.

Upon normalization, the constant A_t was derived to be:

$$A_t = \frac{1}{\rho\sqrt{Z(t)}} \tag{36}$$

where $Z(t)$ is the *Zeitkonzeptionfunktion*, the time-conception function:

$$Z(t) = \frac{-1}{4}\sin^3(t)\cos(t) + \frac{3}{8}t - \frac{3}{16}\sin(2t) \tag{37}$$

Time, in this case, is assumed to be a fourth dimension consisting of "moments". Therefore, time shall be unitless, while space remains in units.

The normalization of the whole transitionary wave function is set as follows:

$$1 = A_\Psi^2 \int_0^\tau \int_0^\sigma \sin^2\left(\frac{n\pi}{\rho}q\right)\frac{1}{Z(t)}\left(|\sin(t)|^2\right)^2 \mathrm{dqdt} \tag{38}$$

A rather unothodox method of normalization, to use a double integral. But for the graviton to transition into a higher set of dimensions, the particle would need to tunnel through space and time to advance into the higher set.

After normalization, the coefficent A_Ψ is derived as:

$$A_\Psi = \rho\sqrt{\frac{\rho}{2\sigma\varsigma}} \tag{39}$$

where σ is the "Spacial Boundary" of the current set of dimensions:

$$\sigma \approx \frac{\sigma}{2} - \frac{\rho}{4\pi n} \tag{40}$$

and ς is the "Periodic Stability" of the transition:

$$\varsigma = \ln(12t - 7) \tag{41}$$

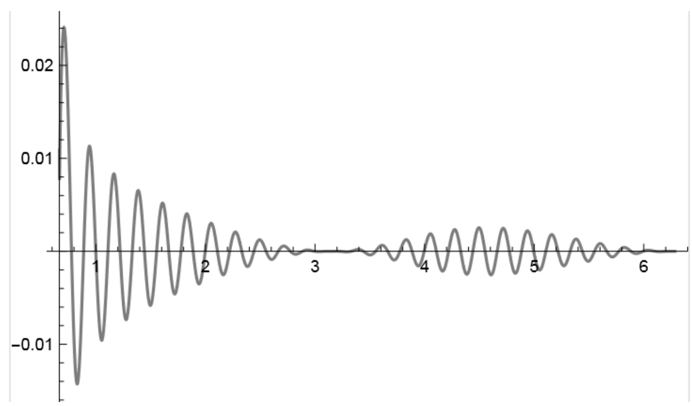
If σ is set to be the current vastness of the observable universe, then σ has a set value.

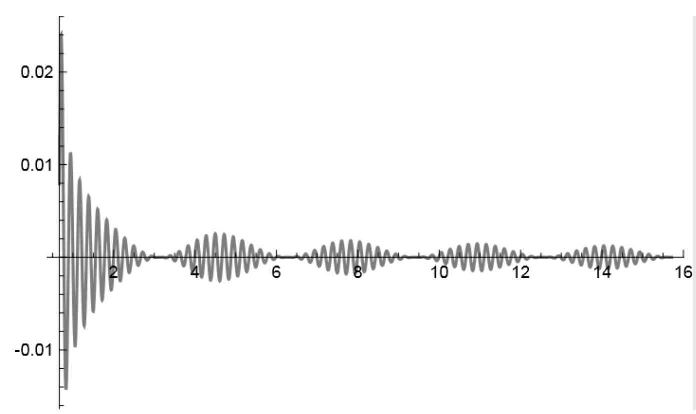
Therefore, the transitionary wave function appears as follows:

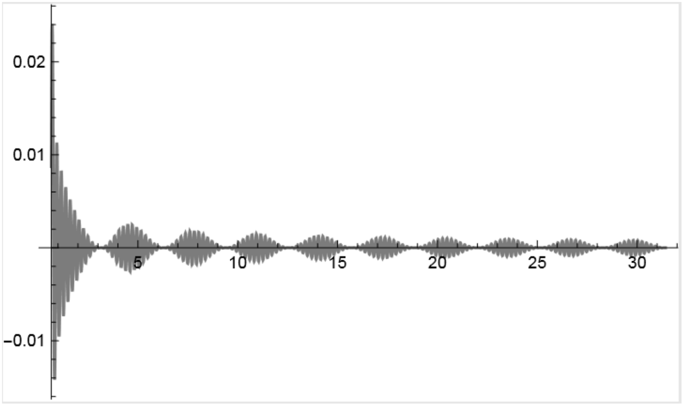
$$\Psi(q,t) = \left(\rho\sqrt{\frac{\rho}{\mathbf{m}}}\right)\sin\left(\frac{n\pi}{\rho}q\right)\frac{1}{\sqrt{\varsigma(t)Z(t)}}|\sin(t)|^2 \tag{42}$$

where $\mathbf{m} = 8.7 \times 10^{26}$m, the Universal Length.

The graph of the transitionary wave function $\Psi(q,t)$, with $q = ct$, in the first set of dimensions $n = 1$, for a value of ρ to be 3.33×10^7m (for the Earth), would look as follows:







These graphs show the transition of a graviton into another dimensional set. They indicate that equilibrium will be achieved over time. When that occurs, the graviton is finally in another set of dimensions. For oscillating gravitons, transitions are meant to be instantaneous at the central node of space-time curvature, at the speed of light c.

D. Probability Density of Gravitonic Influence in Parallel Universes

In string theory, it is supposed that - as gravitons transition into parrallel universes - their influence on energy and matter increase, making the gravitational influence stronger than it is in our universe.

It is also proposed in string theory, that universes that are parallel from one another must obtain uniform physical characteristics, although consisting of certain, finite variables of infinite variations. That is to say, a parallel Earth may rotate at a faster frequency than our Earth, hence making the parallel Earth's gravitational field stronger due to the enforced curving of spacetime by rotating objects.

For the energy state of parallel universes, it is denoted as m, while n is the energy state number for dimensional sets.

In order to provide an analytical model of the probability range of the graviton's influence in each universal state, the multivariable function below will be used as a probability density function:

$$z = \frac{\cos(X^2 + Y^2)}{1 + X^2 + Y^2}$$

where $X = m \cdot x$, and $Y = m \cdot y$. In this case, x and y are the repesentation of all space and all time, respectfully.

It is possible to determine the probability density of a graviton's influence in higher universe states. This will help understand how the gravitational strength increases as gravitons go into parallel universes deemed as higher sets.

To properly apply the graviton into the probability density function, the original function z is scaled into the following:

$$\zeta = \beta z$$

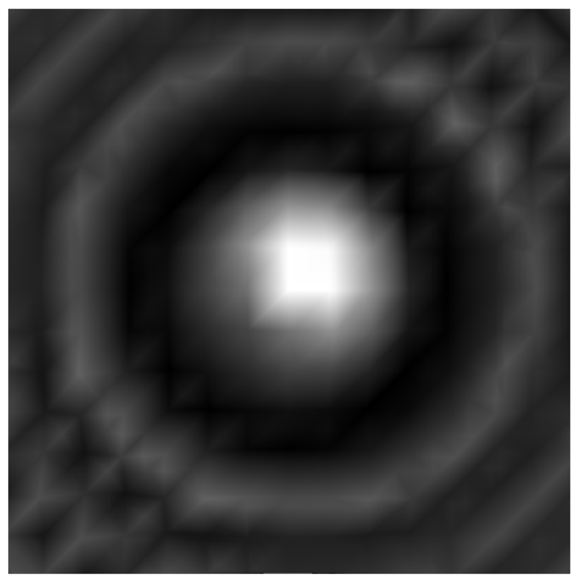
where β is the transitional energy for oscillating gravitons in Planck units $E_{m-1} = m^2\gamma^*/8\rho$.

Therefore, the gravitonic probability density function is depicted as:

$$\zeta = \frac{m^2\gamma^*}{8\rho} \frac{\cos(m^2x^2 + m^2y^2)}{1 + (mx)^2 + (my)^2} \tag{43}$$

The gravitonic probability density function is simulated for universal states $m = 1, 5$ and 10 to show how the probability alters in each state.

These models are based on our reference on our universal state $m = 1$, which is depicted as follows:



Applicable to any heavenly body, whose range of gravitational attraction can be calculated by the following: $\rho = \left(24\pi G m r^2\right)^{1/4}$, the case of $m = 1$ (our universal state) shows that there is a "range of uncertainty" for the graviton's influence - proving the weakness of the gravitational interaction.

Looking at the probability density for $m = 1$, the outer ring best represents the outer boundary of the displacement ρ, where gravitons are less intense. The inner circle region is the displacement interval $[\frac{1}{4}\rho, 0]$, the standard deviation and mean of the Gaussian Potential [8], which is where the very-intense gravitons can be found. The shaded region between the outer ring and inner circle is the region of uncertainty for gravitons.

Keeping true to the Uncertainty Principle of oscillating gravitons, the location of the graviton is $\rho[0 \leq x \leq 1]$[9], which the model describes beautifully. It also maps the locations of the two extreme forms of intensity: low intensity along the outer ring, and high intensity within the central region.

The following models are for $m = 5$, and $m = 10$ respectfully:

It is shown that, as the universal state increases, the graviton's "range of uncertainty" becomes narrower and more certain. However, there is still a range of uncertainty, even in the higher universal states. This is because of Heisenberg's Uncertainty Principle. Because the transitional energy and velocity of the particle are certain, its time and position remain uncertain - hence the consistency of the "range of uncertainty."

For observers in higher universal states, their reference on their own state would be identical to our reference on our own state. Therefore, the strength of gravity is relative to our perspective, thus supporting that gravity is a manifestation of relativity.

CONCLUSION

In this attempt to understand gravitation and their quanta in the quantum framework, gravitons shall obey the laws of General Relativity as they obey the laws of quantum mechanics.

The paper proposed three main considerations that were theoretically addressed and identified with a mathematical theorem.

1) The graviton's particular Poynting Vector is the quantized equivalent of the classical gravitational field vector. The vector has a differential definition and an integral definition, providing an approximate and a direct proportionality to the classical gravitational field. Applying the Mechanical Oscillation Function $\chi(\tau)$ into the differential definition, the second order del operator provides a function based on acceleration. Its significance is derived by the classical understanding of the gravitational field being an acceleration vector.

2) Rotational path strand metrics contribute to the quantum foaming of spacetime, while the potential stability function contributes to the inertial nature of spacetime. The path strand metric is dependent on the instantaneous motions of a reference graviton in oscillation, and its interactions with other oscillating gravitons. However, in the stable graviton lattice, transmitted "neighboring" energies are forcefully nullified by a reference graviton that has a sufficient amount of internal potential energy V. The nullification of "neighboring" energies prevents making spacetime elastic, maintaining the classical nature of inertial space.

3) With sufficent energy, gravitons can freely move into higher sets of dimensions - even universes. Upon reaching the sufficient amount of energy, an oscillating graviton obtains a third wave function: the transitionary wave function. It has been analytically proven that gravitonic influence in higher universal states is stronger than the gravitonic influnce in our universe, therefore supporting the hypothesis that gravity is stronger in parallel universes deemed as a higher set.

The set-backs of the paper are the following: there is still no complete quantum field theory for gravitational interactions and the graviton, and General Relativity remains as not renormalized. There will be a third extension to the graviton theory entity, discussing the usage of statistical mechanics and thermodynamics towards interacting gravitons in the present time and in the Big Bang itself.

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