Question 479 : Some Integrals

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Abstract. This note presents some definite integrals.

Resumen. Esta nota presenta algunas integrales definidas.

El número pi se define por: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.14159265\ldots$, la constante de Catalan se define por: $G = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2} = 0.91596559\ldots$, el número $\Gamma(1/4) = 3.62560990\ldots$ se define por: $\Gamma(1/4) = \int_0^{\infty} e^{-t}t^{-3/4}dt = 4\prod_{n=1}^{\infty}\sqrt[4]{1+\frac{1}{n}}\left(1+\frac{1}{4n}\right)^{-1}$, el número $\ln 2$ se define por: $\ln 2 = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n} = 0.69314718\ldots$. En esta nota mostramos algunas integrales definidas que involucran a los números $\pi, G, \Gamma(1/4), \ln 2$.

Integrales

Entry 1.

$$\frac{\pi}{6} = \int_0^1 \sin^{-1}\left(\frac{4\sqrt{1+12x-9x^2}+1-9x}{5+15x}\right)dx \qquad (1)$$

$$\frac{\pi}{3} = \int_0^1 \sin^{-1}\left(\frac{2\sqrt{1+12x-9x^2}+18x-2}{5+15x}\right)dx \qquad (2)$$

Entry 2.

$$\frac{\Gamma(1/4)^2}{2\sqrt{2\pi}} + \frac{2\pi\sqrt{2\pi}}{\Gamma(1/4)^2} - 1 - \sqrt{2} = \int_{1+\sqrt{2}}^{\infty}\left(1-\sqrt{1-\frac{6}{x^2}+\frac{1}{x^4}}\right)dx \qquad (3)$$

Entry 3.

$$\frac{\pi^3}{16}+G=\int_0^{\infty}\tan^{-1}\left\{\exp\left(\frac{1-\sqrt{1+4x}}{2}\right)\right\}dx \tag{4}$$

Entry 4.

$$\begin{aligned}\frac{\pi^3}{16}-G=&\int_0^{\infty}\tan^{-1}\left\{\exp\left(\frac{-1-\sqrt{1+4x}}{2}\right)\right\}dx+\\&+\int_{-1/4}^{0}\left(\tan^{-1}\left\{\exp\left(\frac{-1-\sqrt{1+4x}}{2}\right)\right\}-\tan^{-1}\left\{\exp\left(\frac{-1+\sqrt{1+4x}}{2}\right)\right\}\right)dx\end{aligned} \tag{5}$$

Entry 5.

$$2G-\frac{\pi}{4}\ln 2=\int_{\ln 2}^{\infty}\tan^{-1}\left(\sqrt[3]{\sqrt{\frac{1}{27}+4e^{-2x}}+2e^{-x}}-\sqrt[3]{\sqrt{\frac{1}{27}+4e^{-2x}}-2e^{-x}}\right)dx \tag{6}$$

Entry 6.

$$3-\frac{\pi}{2}=\int_{-\ln 2}^{\infty}\left(\sqrt[3]{\sqrt{\frac{1}{27}+\frac{e^{-2x}}{4}}+\frac{e^{-x}}{2}}-\sqrt[3]{\sqrt{\frac{1}{27}+\frac{e^{-2x}}{4}}-\frac{e^{-x}}{2}}\right)dx \tag{7}$$

Entry 7.

$$\frac{G}{2}+\frac{3\pi\ln 2}{8}=\int_{-\ln 2\sqrt{2}}^{\infty}\cos^{-1}\left(-\frac{2}{\sqrt{3}}\cos\left(\frac{2\pi}{3}+\frac{1}{3}\cos^{-1}\left(\frac{3\sqrt{3}e^{-x}}{16}\right)\right)\right)dx \tag{8}$$

Entry 8.

$$\pi=\frac{14}{5}+\frac{32}{5}\int_0^1\sqrt{\frac{2(1-x)}{5\sqrt{1089+120x+16x^2}+52x+123}}\,dx \tag{9}$$

$$\pi=\frac{14}{5}+\frac{32}{5}\int_0^1\sqrt{\frac{2x}{5\sqrt{1225-152x+16x^2}-52x+175}}\,dx \tag{10}$$

Entry 9.

$$\frac{\sqrt{\pi}}{8}\left(2+\sqrt{2}\right)=\int_0^1\sqrt{\ln\left(\frac{1+\sqrt{1+8x}}{4x}\right)}\,dx \tag{11}$$

Entry 10.

$$\frac{\pi}{8}=\int_0^1 \cosh^{-1}\left(\frac{1}{2}\sqrt{f(x)-8}+\frac{1}{2}\sqrt{-16-f(x)+2\sqrt{64+8f(x)+(f(x))^2}}\right)dx \qquad (12)$$

$$f(x)=\left(512+x^{-2}\right)^{1/3} \qquad (13)$$

Entry 11.

$$\pi=\frac{256}{3\sqrt{3}}\int_0^\infty\left(1-\frac{2}{\sqrt{3}}\sqrt[4]{\sinh(3x)}\sqrt{\sqrt{\sqrt{3+4\sinh^2 x}-\sinh x}-\sqrt{\sinh x}}\right)\cosh(3x)\,dx \qquad (14)$$

Entry 12.

$$\frac{3\pi}{16}=\int_0^\infty\left(1-\frac{1}{\sqrt{2}}\sqrt{\sqrt{2x\sqrt{\frac{4}{3}+u^{2/3}+v^{2/3}}-x\left(u^{1/3}+v^{1/3}\right)}-\sqrt{x\left(u^{1/3}+v^{1/3}\right)}}\right)dx \qquad (15)$$

$$u:=u(x)=\sqrt{\frac{64}{27}+\frac{x^2}{4}}+\frac{x}{2} \quad , \quad v:=v(x)=\sqrt{\frac{64}{27}+\frac{x^2}{4}}-\frac{x}{2} \qquad (16)$$

Referencias

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