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THE ISOTROPIC CONSTANT CONJECTURE IS TRUE

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ABSTRACT. In this preprint we will prove the isotropic constant conjecture.

1. INTRODUCTION

All the problems that we investigate in this preprint have their roots in classical mechanics. However, our tools are from the asymptotic convex geometry. We will prove the following theorem.

Theorem 1. *Let the convex body $D \subset \mathbf{R}^n$ be isotropic. Then*

$$\int_D ||x||^2 dx < nC,$$

where $C > 0$ and C does not depend on n.

Above result implies the hyperplane conjecture [1][3]. Thus, any convex body of volume 1 has a $n-1$ dimensional hyperplane section that has volume greater than some universal constant [2].

2. DEFINITIONS AND KNOWN RESULTS

A convex body $|D| = 1$ is isotropic if the barycenter is at the origin and

$$\int_D < y, x >^2 dx = \alpha^2 ||y||^2,$$

for some constant $\alpha > 0$ and all $y \in \mathbb{R}^n$. The following integral can be calculated in spherical coordinates.

$$\int_{B_n} ||x||^2 dx = \frac{n}{n+2} |B_n|, \tag{1}$$

where B_n is the unit ball.

3. THE PROOF OF THE MAIN THEOREM

Let D be an isotropic convex body. Because centralized balls are isotropic then via scaling there exists a ball L with the same isotropic constant α. So the real question is the volume of that ball L. Now, we integrate the following in spherical coordinates and obtain

$$\begin{aligned}\frac{1}{|L|^{(n+2)/n}} \int_D ||x||^2 dx = \frac{1}{|L|^{(n+2)/n}} \int_L ||x||^2 dx = \frac{1}{|L|^{(n+2)/n}} \frac{nR^{n+2}}{n+2} |B_n| \\ = \frac{n}{n+2} |B|^{-2/n},\end{aligned} \tag{2}$$

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where B_n is the n dimensional unit ball. Rewriting the above gives

$$\frac{1}{|L|}\int_L ||x||^2 dx = \frac{n}{n+2}|B_n|^{-2/n}|L|^{2/n} = c_n|B_n|^{-2/n}, \tag{3}$$

where $c_n > 0$ and

$$\lim_{n\to\infty} c_n = 1. \tag{4}$$

The bound follows when we consider the bound

$$|L| < Cn. \tag{5}$$

For the bound (5) see for example [3] and [2]. Now, it follows from (3) that

$$\sqrt{\frac{n}{n+2}}R = \sqrt{c_n}|B_n|^{-1/n} = \sqrt{c_n}r,$$

where r is the radius of the ball with the unit volume. Let $1 > \epsilon > 0$ then we can assume that there exists m s.t for $n \geq m$

$$R < r + \epsilon.$$

Otherwise for some $1 > \alpha > 0$, it holds that

$$r + 2\alpha < R = C_n r,$$

so that for large n

$$C_n\alpha < R - r$$

Thus,

$$C_n(\alpha - r) < -r$$

so that

$$C_n < \frac{r}{r-\alpha}.$$

Because above holds for all $0 < \alpha < 1$,

$$(C_n) \leq 1,$$

for large n, which is a contradiction. Thus,

$$|L| < \frac{|B_n|}{|B_n|}(\frac{(r+\epsilon)}{r})^n < C$$

and asymptotically

$$|L| \sim 1.$$

Then we have the theorem 1 via equation (3) and a standard estimate

$$|B_n|^{-2/n} \leq Cn.$$

References

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