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THE ISOTROPIC CONSTANT CONJECTURE IS TRUE

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ABSTRACT. In this preprint we will prove the isotropic constant conjecture.

1. INTRODUCTION

All the problems that we investigate in this preprint have their roots in classical mechanics. However, our tools are from the asymptotic convex geometry. We will prove the following theorem.

Theorem 1. *Let the domain* $D \subset \mathbf{R}^n$ *be isotropic. Then*

$$\int_D ||x||^2 dx < nC,$$

where $C > 0$, *where* C *does not depend on* n.

2. DEFINITIONS AND KNOWN RESULTS

A domain $|D| = 1$ is isotropic if the barycenter is at the origin and

$$\int_D < y, x >^2 \, dx = \alpha^2 ||y||^2,$$

for some constant $\alpha > 0$ and all $y \in \mathbb{R}^n$. The following integral can be calculated in spherical coordinates.

$$\int_{B_n} ||x||^2 dx = \frac{n}{n+2} |B_n|, \tag{1}$$

where B_n is the unit ball.

3. THE PROOF OF THE MAIN THEOREM

Let D be an isotropic domain. Because centralized balls are isotropic then via scaling there exists a ball L with the same isotropic constant α. So the real question is the volume of that ball L. Now, we integrate the following in spherical coordinates and obtain

$$\begin{aligned} \frac{1}{|L|^{(n+2)/n}} \int_D ||x||^2 dx = \frac{1}{|L|^{(n+2)/n}} \int_L ||x||^2 dx = \frac{1}{|L|^{(n+2)/n}} \frac{nR^{n+2}}{n+2} |B_n| \\ = \frac{n}{n+2} |B|^{-2/n}, \end{aligned} \tag{2}$$

where B_n is the n dimensional unit ball and we used scaling in order to rewrite the expression. Rewriting the above gives

$$\frac{1}{|L|} \int_L ||x||^2 dx = \frac{n}{n+2} |B_n|^{-2/n} |L|^{2/n} < c_n |B_n|^{-2/n}, \tag{3}$$

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where $c_n > 0$ and

$$\lim_{n\to\infty} c_n = 1. \tag{4}$$

The bound follows when we consider the bound

$$|L| \leq c\sqrt{n}. \tag{5}$$

For the bound (5) see for example [2]. Now, it follows from (3) that

$$\sqrt{\frac{n}{n+2}}R < \sqrt{c_n}|B_n|^{-1/n} = \sqrt{c_n}r,$$

where r is the radius of the ball with the unit volume. Let $\epsilon > 0$ then there exists $n \geq 1$ s.t

$$R < r + \epsilon$$

Thus,

$$|L|^{1/n} = |B_n|^{1/n}R = r^{-1} * R < \frac{(r+\epsilon)}{r}$$

So it follows that

$$|L| < (\frac{(r+\epsilon)}{r})^n < (\frac{(r+1)}{r})^n$$

Thus,

$$|L| < C_1, \tag{6}$$

and we have theorem 1.

References

[1] K.Ball, *An Elementary Introduction to Modern Convex Geometry*, Flavors of Geometry, MSRI Publications, Volume 31, 1997.

[2] A.Giannopoulos, *Notes on isotropic convex bodies*, Warsaw, October 2003.

[3] Daniel Hug, Wolfgang Weil, *A Course on Convex Geometry*, available at https://www.fmf.uni-lj.si/ lavric/hug&weil.pdf

[4] B.Klartag *On Isotropic Constants and Mahler Volumes* Alailable at http://www.math.tau.ac.il/ klartagb/papers/1710.08084.pdf

[5] V.D.Milman and A. Pajor *Isotropic Position and Inertia Ellipsoids and Zonoids of the Unit Nall of a Normed n-Dimensional Space*, GAFA-Seminar 87-88, Springer Lecture Notes in Math., v.1376, 64-104, (1989).

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