The Seiberg-Witten equations for vector fields

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Abstract

By analogy with the Seiberg-Witten equations, we define equations for a spinor and a vector field.

1 Recalls of differential geometry

The $\text{Spin}^\mathbb{C}$-structures are reductions of a $SO(n),S^1$-fiber bundle to the group $\text{Spin}^\mathbb{C}(n) = \text{Spin}(n) \times \{1, -1\} S^1$. For a four-manifold it exists always a $\text{Spin}^\mathbb{C}$-structure for the tangent fiber bundle [F].

The Dirac operator is defined over the $\text{Spin}^\mathbb{C}$-structure with help of a connection $A$ for the associated line bundle.

$$\mathcal{D}_A = \sum_i \epsilon_i \nabla^A_{e_i}$$

with $\nabla^A$ the connection defined by the Levi-Civita connection and the connection $A$ of the determinant fiber bundle of the $\text{Spin}^\mathbb{C}$-structure.

The self-dual part of the curvature (which is a 2-form) of the connection $A$ is considered:

$$\Omega_A^+$$

A self-dual 2-form with imaginary values, bound to a spinor $\psi \in S^+$ is also defined by [F]:

$$\omega(\psi)(X, Y) = \langle X.Y, \psi \rangle + \langle X, Y \rangle |\psi|^2$$

2 Recalls of the Seiberg-Witten equations

The Seiberg-Witten equations are the following ones [F] [M]:

1) $$\mathcal{D}_A(\psi) = 0$$

2) $$\Omega_A^+ = -(1/4)\omega(\psi)$$
3 The SW equations for vector fields

By analogy with the usual Seiberg-Witten equations, we are tempted to define equations for a spinor $\psi$ and a vector field $X$:

1) $D_X(\psi) = (D + iX)(\psi) = 0$

2) $id(X^*)^+ = -(1/4)\omega(\psi)$

with $X^*$ the dual form of $X$, $d$ is the differential for the forms. The first equation makes use of the Clifford multiplication.

References


