Exponential-Grating Monochromator

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Abstract

A monochromator optical design is described, which comprises a grazing-incidence reflection grating and two grazing-incidence mirrors, one on each side of the grating. The grating operates in conical diffraction mode and has an exponential phase function, so that the wavelength can be scanned by translating the grating surface in the phase-gradient direction. The wavelength can be continuously tuned over a wide spectral range while maintaining zero-aberration performance over the full range, and perfect blazing is also achieved over the full illumination aperture at all wavelengths.

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Aspnes [1, 2] developed a monochromator design that uses an exponential diffraction grating to achieve wavelength tunability via translational displacement of the grating. As a result of the exponential phase distribution on the grating surface, a surface translation along the exponential gradient direction is equivalent to a uniform dimensional scaling of the local grating period, so the monochromator's selected wavelength is similarly scaled. However, Aspnes' design has limited utility for grazing-incidence EUV and X-ray gratings because the grating would have to be excessively long in the translation direction. (The illumination area on the grating is greatly elongated due to the shallow grazing angle, and the grating aperture would need to be even longer to accommodate the scan range.)

Hettrick [3] developed an alternative exponential grating design in which the scan direction is transverse, not parallel, to the illumination spot's long dimension, so scanning can be achieved with a grating of practical size. But Hettrick's design requires curved grating lines, which cannot be easily manufactured for EUV/X-ray applications.

An alternative monochromator grating design that overcomes the limitations of Aspnes' and Hettrick's designs is illustrated in Figures 1A (plan view) and 1B (side view). The grating is planar and will be described with reference to x_1, x_2, x_3 Cartesian coordinates with the x_1 and x_2 axes parallel to the grating plane. The grating substrate is in the plane $x_3 = 0$. The grating lines are straight and orthogonal to the x_1 axis, and wavelength tuning is achieved by translating the grating along a scan direction parallel to the x_1 axis. The grating operates in conical diffraction mode, with the incidence plane transverse to the scan direction. The incident beam and diffracted beam cover an illumination spot that is greatly elongated in the x_2 direction due to the shallow grazing incidence.

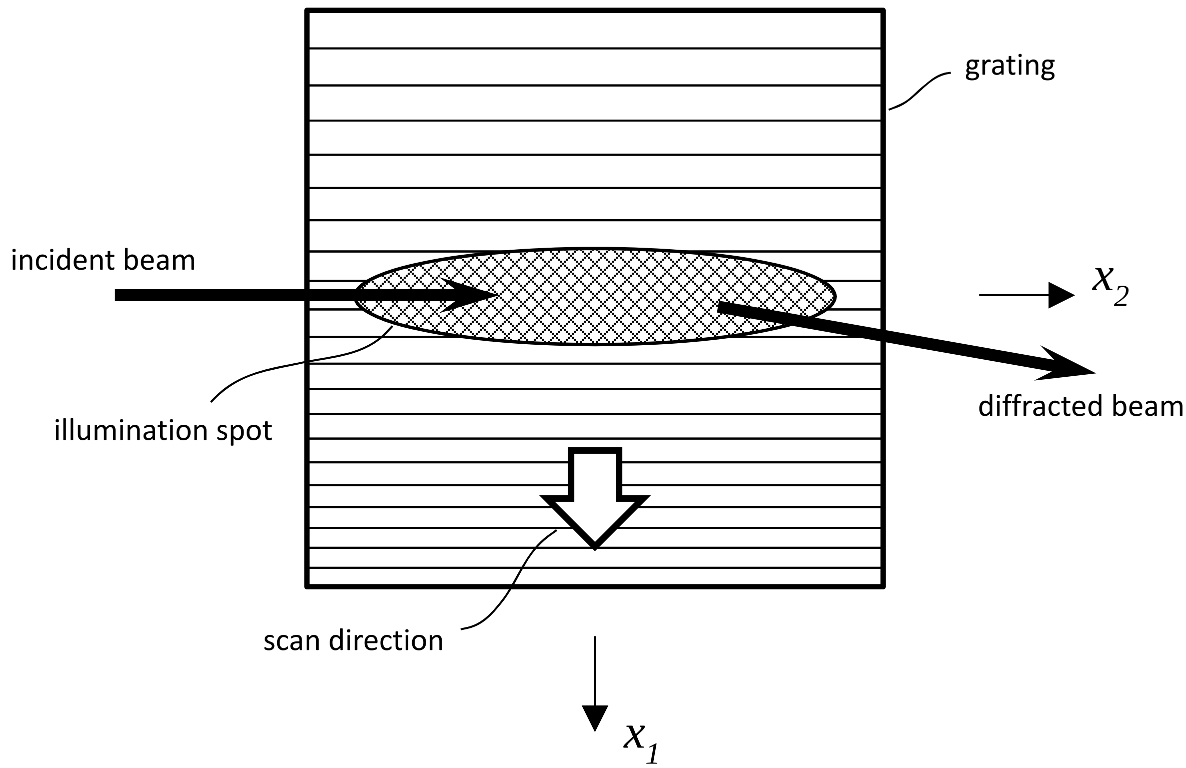

Figure 1A

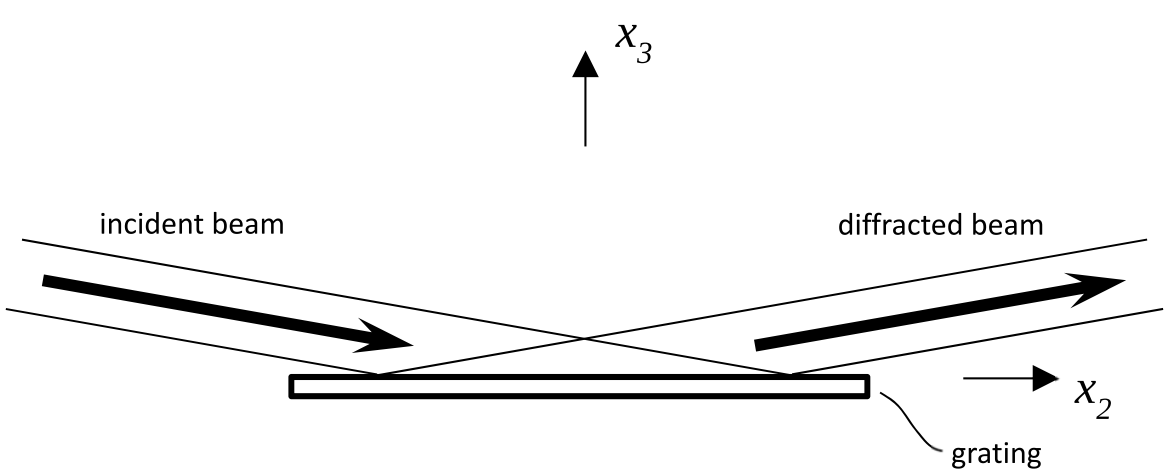

Figure 1B

The grating profile has a blazed, sawtooth form, as illustrated schematically in Figure 2. The grating "lines" are boundaries between grating facets. The grating structure is defined by a phase function $\Omega[x_1, x_2]$, a continuous function of x_1 and x_2 that takes on integer values on the grating lines. (In this description, function arguments are delimited by square braces "[…]" while round braces "(…)" are reserved for grouping. Phase quantities are defined in cycle units; 1 cycle = 2π radian.) The grating phase is an exponential function of x_1,

$$\Omega = (g_0 / c)(\exp[c\, x_1] - 1) \tag{1}$$

The grating line density (phase gradient) is

$$\left(\frac{\partial \Omega}{\partial x_1}, \quad \frac{\partial \Omega}{\partial x_2} \right) = \left(g, \quad 0 \right), \quad g = g_0 \exp[c\, x_1] \tag{2}$$

(The local grating period is $1/g$.)

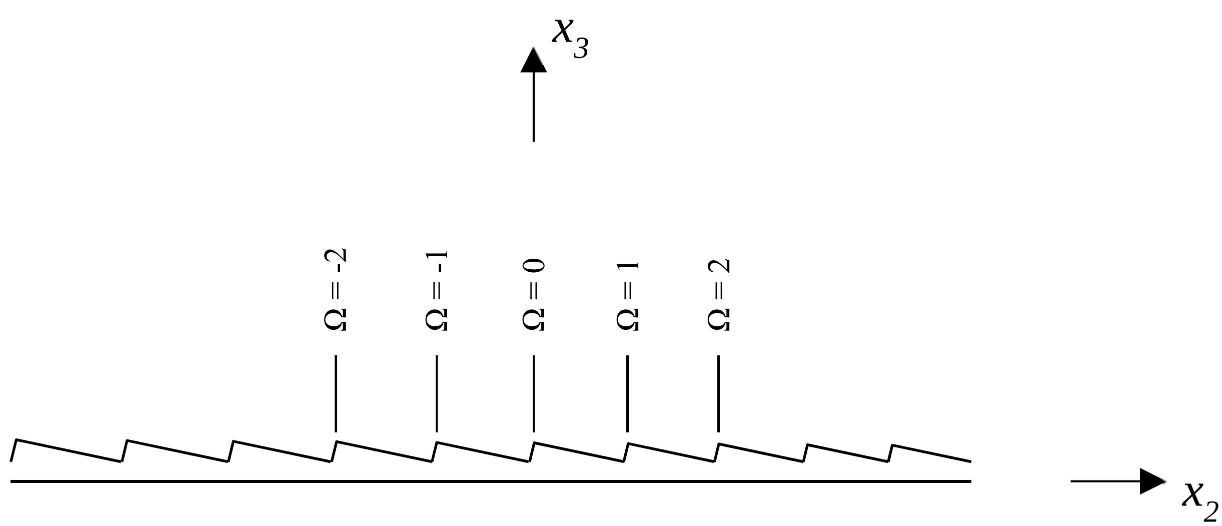

Figure 2

The grating design will be described for a particular wavelength λ, which is selected by the monochromator slit at a particular grating position. As a result of the grating's exponential phase, a lateral translation of the grating in the x_1 direction is equivalent to a uniform scale change of the local grating period, so the selected wavelength will be scaled by the same factor.

The incident beam has an optical phase function $\Phi[x_1, x_2, x_3]$, for wavelength λ originating from a particular source point. The incident electromagnetic field amplitude is proportional to $\exp[i\, 2\pi\, \Phi]$. The field's spatial frequency vector $\mathbf{f}$ (aka. "wave vector") is the phase gradient,

$$\mathbf{f} = \left(f_1, \quad f_2, \quad f_3 \right) = \left(\frac{\partial \Phi}{\partial x_1}, \quad \frac{\partial \Phi}{\partial x_2}, \quad \frac{\partial \Phi}{\partial x_3} \right) \tag{3}$$

The field satisfies the "eikonal equation" (which is the basis of geometric optics),

$$f = |\mathbf{f}| = \sqrt{f_1^{\,2} + f_2^{\,2} + f_3^{\,2}} = 1/\lambda \tag{4}$$

The diffracted field is similarly characterized by its phase Φ' and phase gradient $\mathbf{f}'$,

$$\mathbf{f}' = \left(f_1', \quad f_2', \quad f_3' \right) = \left(\frac{\partial \Phi'}{\partial x_1}, \quad \frac{\partial \Phi'}{\partial x_2}, \quad \frac{\partial \Phi'}{\partial x_3} \right) \tag{5}$$

$$\sqrt{f_1'^2 + f_2'^2 + f_3'^2} = f \tag{6}$$

The diffracted beam's phase function is defined by the relation $\Phi' = \Phi + \Omega$ in the grating plane ($x_3 = 0$),

$$\Phi'[x_1, x_2, 0] = \Phi[x_1, x_2, 0] + \Omega[x_1, x_2] \tag{7}$$

The additive relation applies to the phase gradient (x_1 and x_2 derivatives of Eq. (7)),

$$\left(f_1', \quad f_2' \right) = \left(f_1 + g, \quad f_2 \right) \tag{8}$$

(Eq's. (7) and (8) are for order-1 diffraction; for order-m diffraction replace Ω and g by $m\Omega$ and $m\,g$.) f_3 and f_3' are determined from Eq's. (4) and (6) (with square root signs consistent with the diffraction geometry illustrated in Figure 1B),

$$f_3 = -\sqrt{f^2 - f_1^2 - f_2^2} \tag{9}$$

$$f_3' = \sqrt{f^2 - f_1'^2 - f_2'^2} \tag{10}$$

The output beam can be focused to a point in the monochromator's exit slit by means of a grazing-incidence mirror following the grating. The optics need only be designed to provide zero-aberration focusing for a single wavelength; it will automatically also exhibit the same zero-aberration performance for all other wavelengths.

The incident wave geometry can be defined to achieve perfect blazing over the full beam aperture at the design wavelength, and based on the wavelength-scaling property of the exponential grating perfect blazing will also be achieved at all other wavelengths.

Figure 3 shows an optical schematic of the monochromator. Broadband radiation is spatially filtered by an entrance slit, is reflected by a curved mirror, diffracted by the grating, and focused by a second curved mirror onto the exit slit. The two mirrors provide sufficient degrees of freedom to achieve optimal, full-aperture blazing and zero-aberration point imaging at any selected wavelength.

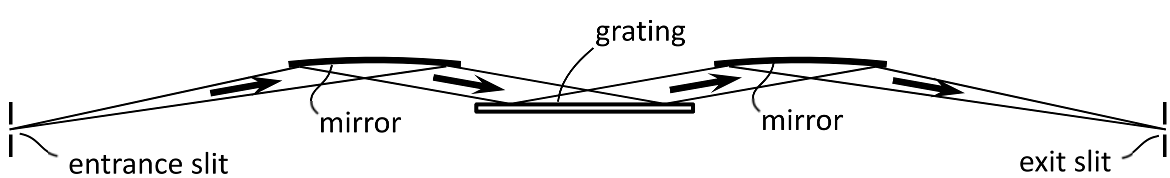

Figure 3

The blaze condition, which generally maximizes diffraction efficiency, is satisfied when the diffracted wave vector $\mathbf{f}'$ determined from Eq's. (8) and (10) coincides with the ray direction defined by geometric optics for reflection at the grating facets. The facet surface normal vector

$\mathbf{n}$ at any particular grating point is determined from the incident and reflected wave vectors $\mathbf{f}$ and $\mathbf{f}'$ according to geometric optics,

$$\mathbf{n} = \frac{\mathbf{f}' - \mathbf{f}}{|\mathbf{f}' - \mathbf{f}|} = \frac{\left(g \quad 0 \quad \sqrt{f^2 - (f_1 + g)^2 - f_2^2} + \sqrt{f^2 - f_1^2 - f_2^2}\right)}{|\ldots|} = \left(\sin\beta \quad 0 \quad \cos\beta\right) \qquad (11)$$

where β is the blaze angle, which is constant across the full grating. Eq. (11) implies the following relation, from which β can be determined,

$$\sqrt{f^2 - (f_1 + g)^2 - f_2^2} + \sqrt{f^2 - f_1^2 - f_2^2} = g \cot\beta \qquad (12)$$

With β predefined and g specified by Eq. (2), we will use Eq. (12) to determine f_1 and f_2.

Eq. (12) is solved for f_1

$$f_1 = -\tfrac{1}{2} g \pm \sqrt{f^2 - f_2^2 - (\tfrac{1}{2} g / \sin\beta)^2}\ \cos\beta \qquad (13)$$

(The square root sign is a design choice.) f_1 and f_2 are derivatives of Φ (Eq. (3)), so Eq. (13) amounts to a partial differential equation for Φ. A solution can be obtained by defining Φ so that f_2 is a constant,

$$\Phi[x_1, x_2, 0] = \Phi[x_1, 0, 0] + f_2\, x_2 \quad (\text{constant } f_2\ ) \qquad (14)$$

This makes the right side of Eq. (13) a function of only x_1 (not x_2), which can be directly integrated to obtain Φ.

f_2 is defined in terms of the conic angle γ (for conical diffraction),

$$f_2 = f \cos\gamma \qquad (15)$$

Eq's. (2), (3), (14), and (15) are substituted into Eq. (13) to obtain the differential equation

$$\frac{d}{dx_1}\Phi[x_1, 0, 0] = -\tfrac{1}{2} g_0 \exp[c\, x_1] \pm \sqrt{f^2 \sin^2\gamma - (\tfrac{1}{2} g_0 \exp[c\, x_1] / \sin\beta)^2}\ \cos\beta \qquad (16)$$

Eq. (16) is integrated to obtain

$$\Phi[x_1, 0, 0] = \frac{1}{c}\left(\begin{array}{l} -\tfrac{1}{2} g_0 \exp[c\, x_1] \\ \pm f \sin\gamma \cos\beta \left(\sqrt{1 - \left(\dfrac{\tfrac{1}{2} g_0 \exp[c\, x_1]}{f \sin\gamma \sin\beta}\right)^2} - \tanh^{-1}\sqrt{1 - \left(\dfrac{\tfrac{1}{2} g_0 \exp[c\, x_1]}{f \sin\gamma \sin\beta}\right)^2} \right) \end{array} \right) + const \qquad (17)$$

Eq. (17) is combined with Eq. (14) to obtain $\Phi[x_1, x_2, 0]$, and then with Eq. (7) to obtain $\Phi'[x_1, x_2, 0]$. The full phase functions $\Phi[x_1, x_2, x_3]$ and $\Phi'[x_1, x_2, x_3]$ can then be obtained by integrating the phase from the grating surface along optical rays, and the curved mirror shapes **3.2** and **3.3** can be determined by phase matching between incident and reflected beams.

The above design outline represents just one of a broad class of possible exponential grating designs. For a flat grating with scanning via translation in the x_1 direction, the phase function $\Omega[x_1, x_2]$ can be an exponential function of x_1 times an arbitrary function of x_2,

$$\Omega[x_1, x_2] = \exp[c\, x_1]\, u[x_2] \qquad (18)$$

The grating need not be flat; it can have any curved substrate shape that has translational symmetry in the x_1 direction, as illustrated in Figure 4. The substrate surface comprises (x_1, x_2, x_3) coordinate points defined by a surface height function, h, which is a function only of x_2,

$$x_3 = h[x_2] \tag{19}$$

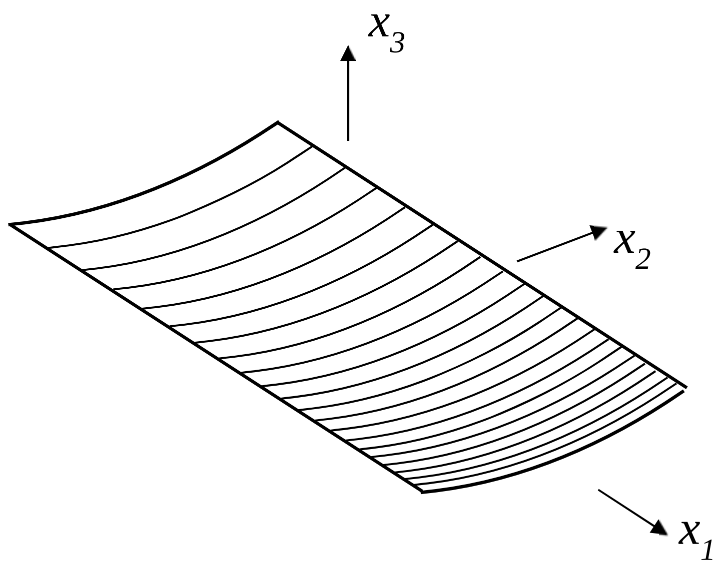

Figure 4

The grating can alternatively be scanned via rotation around a fixed axis, provided that the substrate surface has rotational symmetry around the axis. For example, Figure 5 illustrates a cylindrical grating with grating lines parallel to the cylinder axis. Other possible shapes include toroidal, conical, or planar (with the plane orthogonal to the rotation axis). The surface can be parameterized in terms of two non-Cartesian coordinates θ and p, where θ is the rotational angle around the symmetry axis. (Surface point coordinates (x_1, x_2, x_3) are functions of θ and p.) The constant- p contours are orthogonal to the axis so that as the surface is rotated around the axis, each surface point's θ coordinate changes but p does not change. The grating phase function is separable in θ and p and has an exponential θ dependence,

$$\Omega[\theta, p] = \exp[c\,\theta]u[p] \tag{20}$$

With this type of phase function, axial rotation of the grating has the effect of applying a uniform scale change to Ω.

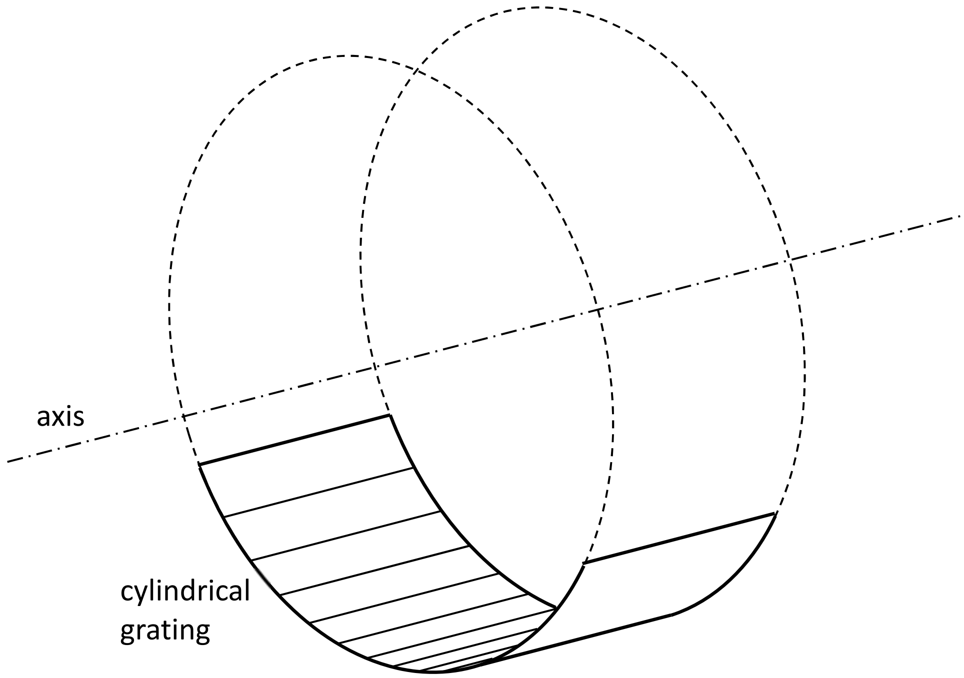

Figure 5

All of these grating types can be used with two mirrors, as illustrated in Figure 3, to provide aberration-free point imaging and optimize full-aperture blazing. The grating shape and curvature can be selected to simplify the mirror design, or to possibly eliminate the need for one or both mirrors.

References

[1] Aspnes, David E. (1985). U. S. Patent No. 4,492,466.

[2] Aspnes, D. E. (1982). High-efficiency concave-grating monochromator with wavelength-independent focusing characteristics. Journal of the Optical Society of America, 72(8), 1056. https://doi.org/10.1364/josa.72.001056

[3] Hettrick, M. C. (2016). Divergent groove gratings: wavelength scanning in fixed geometry spectrometers. Optics Express, 24(23), 26646. https://doi.org/10.1364/oe.24.026646