# From Maxwell's equations to Electro-Magnetic Waves

**Sjaak Uitterdijk**
sjaakenlutske@hetnet.nl

*Abstract – This article shows how Electro-Magnetic source and wave are related and why the propagation velocity of light in vacuum is c, exclusively relative to its source.*

## 1. Introduction

Starting with the explanation of the most basic concepts: static electric and magnetic fields, the Maxwell equations for the dynamic fields are shown to be hypotheses instead of laws. These hypotheses are used to show mathematically how an EM wave must look like.
For this study reference [1] has been used as guidebook.

## 2. The static electric field

### 2.1 From force to voltage

About two and a half centuries ago Coulomb discovered the repulsive force between like electrical charges objects and the attractive force between unlike charged objects. Just like Newton discovered the attractive force between masses.
The last mentioned one has been mathematically expressed as $F_G = GMm/r^2$, with [G] is $Nm^2kg^{-2}$. The distance between the centres of the objects is defined as r.

Coulomb's force has been mathematically expressed in basically exactly the same way by means of $F_C = CQq/r^2$, with $C = 1/4\pi\varepsilon$ and ε the so-called dielectric permittivity of the medium in which the objects are located. As a result [C] is $Nm^2C^{-2}$. In order to avoid confusion between both C's, the first mentioned one will from now on be presented as $C_\varepsilon$, so $[C_\varepsilon]$ is $Nm^2C^{-2}$. The dimension of ε thus is: $N^{-1}m^{-2}C^2$. Let for the ease of the considerations Q be the main object and q, *being much smaller*, the object in the sphere of influence of Q. The charge q is supposed to be small enough to have negligible influence on the sphere of influence of Q.

Let us describe that sphere of influence of Q by means of the words 'electric field' of Q. Such a field has, according to Coulomb's experiences through his experiments, the possibility to attract or repulse an object with an electric charge of q. Let us call the strength of this field $E_r$ at the distance r from Q and the related force $F_{Cr}$. So $F_{Cr} = qE_r$.

Given the relation $F_{Cr} = C_\varepsilon Qq/r^2$, the electric field strength $E_r$ has to be presented as $E_r = C_\varepsilon Q/r^2$. Given the dimension of the variables on the right side of the equation, the dimension of $E_r$ is N/C.

Remark: N/C can also be written as: Nm/Cm = VAs/Cm = VC/Cm =V/m, because Nm and VAs are both expressions for energy, so the dimension of $E_r$ is also V/m, as normally used.

In order to move, in the electric field of Q, the object q from $P_1$ to $P_2$, the integral $\int_s qE_s ds$ represents the work that has to be carried out in order to do so. The quantity $qE_s$ represents at any place on that path the force in the direction of the movement. The total mentioned work thus equals the difference in *potential energy* in the two chosen points.
If the path is a closed curve, the result must be zero, so $\oint_s qE_s ds = 0$ and thus $\oint_s E_s ds = 0$. *
The quantity $E_s ds$ represents a voltage/potential, so $\int_s E_s ds$ from $P_1$ to $P_2$ results in $V_{P2} – V_{P1}$, the difference in voltage between the points $P_1$ and $P_2$.

* *The symbols $\oint_s$, resp. $\int_s$ are used to express a line integral along a closed, resp. open curve s.*

### 2.2 The static electric flux

The static electric flux $\Phi_E$ is also a measure of the electric field strength at any distance from the charge Q, in the sense of the amount of electric field *through a certain surface*.
The mathematical presentation is $\Phi_E$ = §§$_S$ $E_r dS$ *, with $E_r$ perpendicular to surface element dS.
In case of an electric field equally spread over a sphere around object Q, the field strength $E_r$ is the same at each element dS on that sphere.
So the surface integral at distance r from Q is $4\pi r^2 E_r = 4\pi r^2 . C_\varepsilon Q/r^2 = Q/\varepsilon$ [Vm].

The result effectively shows that, whichever closed surface is chosen around Q, $\Phi_E = Q/\varepsilon$.

### 2.3 The static dielectric flux

The expression in 2.2 for the static electric flux can also be written as: Q = §§$_S$ $\varepsilon E_r\, dS$ = §§$_S$ $D_r\, dS$
The variable D is normally used as symbol for dielectric displacement.
This name will be commented in chapter 3.
The equation shows that the dimension of $D_r$ (from now on written as D) equals the dimension of $\varepsilon E_r$ ($E_r$ from now on written as E) and Q/S, being $Cm^{-2}$.
Differentiating to time, the equation D=Q/S results in: $dD/dt = dQ/dt/S = I_D/S$, with $I_D$ used as symbol for dielectric current. But dQ/dt in a *static* situation is zero.
Therefor the situation has to be transferred to a dynamic one.

### 2.4 The dynamic dielectric flux

In order to obtain a meaningful concept of an electric charge that changes with time, without applying solid conductors in which electrons operate as moving electric charges, we can imagine a moving charge Q in empty space or tangible medium. In empty space the electric permittivity has to be chosen as $\varepsilon_0$, in a tangible medium as $\varepsilon_0\varepsilon_r$, shortly written as ε, like up to now.
Imagine a charge Q in a reference system relative to which Q moves with constant velocity v, say along the x-axis. During time t until t+Δt, Q moves from x to x+vΔt =x+Δr. At time t the electric field strength at distance r from x can be expressed as in 2.1: $E(t) = C_\varepsilon Q/r^2$. At time t+Δt the field strength, at this same position relative to x, is $E(t+\Delta t) = C_\varepsilon Q/(r-\Delta r)^2$. So $E(t+\Delta t)-E(t) = 2\Delta r\, C_\varepsilon Q/r^3$.

Multiplying both sides with ε, results in: $\Delta D/\Delta t = \varepsilon 2\Delta r\, C_\varepsilon Q/r^3/\Delta t = vQ/r^3/2\pi$  ($\varepsilon C_\varepsilon = 1/4\pi$).

Multiplying both sides with an arbitrary small surface element dS results in $I_D = dSvQ/2\pi r^3$ [A].

This is not a surprising result, because what is the fundamental difference between moving charges in an empty space/tangible medium, respectively in a conductor?
This concept has been used in [2]! In the model applied there, orbiting electrons around an atomic nucleus are considered as circular shaped electric currents, generating magnetic fields through the, by these orbits, enclosed surfaces.
In order to look for another approach too, a changing Q as function of time will be considered from the point of view that Q does not change its position, but is value.
Suppose at time t the charge is Q and at time t+Δt it is Q+ΔQ. At distance r from this changing electric charge, D(t) is $\varepsilon C_\varepsilon Q/r^2 = Q/4\pi r^2$ while $D(t+\Delta t) = (Q+\Delta Q)/4\pi r^2$.

So $\Delta D/\Delta t = (\Delta Q/\Delta t)/4\pi r^2$ [$Cm^{-2}s^{-1} = Am^{-2}$].

Multiplying both sides with an arbitrary small surface element dS shows: $I_D = dS\Delta Q/\Delta t/4\pi r^2$ [A].

This is the moment to start the investigation of magnetic fields, because electric as well as dielectric currents create magnetic fields.
From now on a current can be either an electric or a dielectric current.

* *The symbols §§$_S$, resp. ∫∫$_S$ represent the surface integral over a closed, resp. open surface S.*

# 3. The magnetic field

### 3.1 The static magnetic field

Two types of static magnetic fields will be considered: the one created by a current through an infinite long straight conductor, the other by a circular shaped current.

A straight line current creates a circular shaped magnetic field with this current as centre and in a plane perpendicular to this line. Its strength H at distance r from this current is $I/2\pi r$ [A/m].

A circular shaped current creates a magnetic field through the surface enclosed by this current, perpendicular to this surface. Its strength H in the centre of this circle is equal to $I/2r$ [A/m].

Remark about the similarity between electric and magnetic fields:

> A fundamental difference between a static electric field and a static magnetic field is that the electric field is an open one, leading to the results: $\S_s E\, ds=0$ and $\S\S_S E\, dS = Q/\varepsilon = \Phi_E$, while the magnetic field is a closed one, leading to the results: $\S_s H\, ds=I$ and $\S\S_S H\, dS = 0$.
>
> The last mentioned expression is normally written as $\Phi_B = \S\S_S \mu H\, dS = \S\S_S B\, dS = 0$, with $\Phi_B$ the so-called magnetic flux [Vs] and B the magnetic flux density [$Vs/m^2$].
>
> From the point of view of similarity with the magnetic field it is strange that the electric flux has not been defined as $\Phi_D = \S\S_S \varepsilon E\, dS = \S\S_S D\, dS = Q$. The dimension of $\Phi_D$ is As (or C). And the dimension of D, as an electric flux density, still As $m^{-2}$, or $Cm^{-2}$.
>
> Such a convention would lead to the following table for static fields:

| | **Electric field** | | **Magnetic field** | |
|---|---|---|---|---|
| Field strength | E | V/m | H | A/m |
| Medium properties | $\varepsilon$ | As/Vm | $\mu$ | Vs/Am |
| Flux density | $D=\varepsilon E$ | $As/m^2$ | $B=\mu H$ | $Vs/m^2$ |
| Flux | $\Phi_D$ | As | $\Phi_B$ | Vs |
| | | | | |
| Physical laws | | | | |
| Closed curve integrals | $\S_s E\, ds = 0$ | | $\S_s H\, ds = I$ | |
| Closed surface integrals | $\S\S_S D\, dS = Q$ | | $\S\S_S B\, dS = 0$ | |

### 3.2 The dynamic magnetic field

The just mentioned equation: $\S_s H\, ds = I$ can simply be proven in case of a straight line current, Because $H= I/2\pi r$ at each point around the current at distance r, and H and ds do have the same direction in each point too, the closed integral along this circle is $I/2\pi r * 2\pi r = I$.
Reference [1] at this place declares at page 256: "Investigation of other magnetic fields finally leads to the conclusion that $\S_s H_s ds = I$, with I the sum of all currents enclosed in s, is a general property of the magnetic field."
Seemingly a theoretical substantiation of this general property is too complex, maybe even impossible. The expression thus should have been qualified as hypothesis.
Finally the closed curve integral is presented as: $\S_s H\, ds = d/dt \int\int_S D\, dS + \int\int_S \gamma E\, dS$,
with $\int\int_S \gamma E\, dS$ defined as conduction current.
However the expression is not helpful trying to understand how an EM wave is generated, because an EM wave is normally not generated in fields including conduction currents.
The complete equation is officially called: "Ampère's circuital law (with Maxwell's addition)", or shortly Maxwell's nth equation, with 'n' now-a-days undefined!

In this article only $\S_s H\, ds = d/dt \int\int_S D\, dS$ will be used and called Maxwell-Ampère equation.

Remark:

Before we continue with this equation we have to take care of the fact that the equation $\Phi_D = \S\S_S D\, dS$ concerns a closed surface integral over the charge Q that creates the electric flux density D in a static situation, while in $\S_s H\, ds = d/dt \iint_S D\, dS$ the electric flux density is supposed to go through an open surface enclosed by the curve 's' meant in the left side of the equation. We thus have to accept even more penetrating that the expression $\S_s H\, ds = d/dt \iint_S D\, dS$ is not a law but a hypothesis: valid as long as it has not been proven to be invalid.

The second difference between $\S\S_S D\, dS$ and $\iint_S D\, dS$ is that he first mentioned one equals Q in the static situation, while the second one is supposed to be applied in a dynamic situation, given the fact that the differentiation, applied to it, is supposed to be meaningful. The same kind of remark is applicable to the following consideration.

It is generally accepted that a voltage can be generated in a closed wire by changing a magnetic flux through the *open* surface enclosed by the wire, mathematically expressed by $V = d\Phi_B/dt$. This $\Phi_B$ thus is not the same magnetic flux as in $\Phi_B = \S\S_S B\, dS$. So just like in the electric situation the relation $\int_s E\, ds = d/dt \iint_S B\, dS$ should have been qualified as a hypothesis too. Summarized:

Maxwell-Ampère equation $\S_s H\, ds = d/dt \iint_S D\, dS$
Maxwell-Faraday equation $\int_s E\, ds = d/dt \iint_S B\, dS$

## 4. The Electro Magnetic wave

The Maxwell-Ampère equation can also be presented as $\S_s H\, ds = d/dt \iint_S \varepsilon E\, dS$,
like the Maxwell-Faraday equation can be presented as $\int_s E\, ds = d/dt \iint_S \mu H\, dS$.
In this way they clearly show that the generation of an EM wave is based on a feedback loop, unavoidably resulting in an oscillating phenomenon that has lead to the generally accepted model with sinusoidal shaped E and H fields, in planes perpendicular to each other.
The propagation of these fields is supposed to be like drawn in figure 1, copied from [1] at page 318. An interesting question is whether the mutual phase between these fields is indeed zero, or possibly $90^0$, shown in figure 2, like the voltage and current in a LC-oscillator.

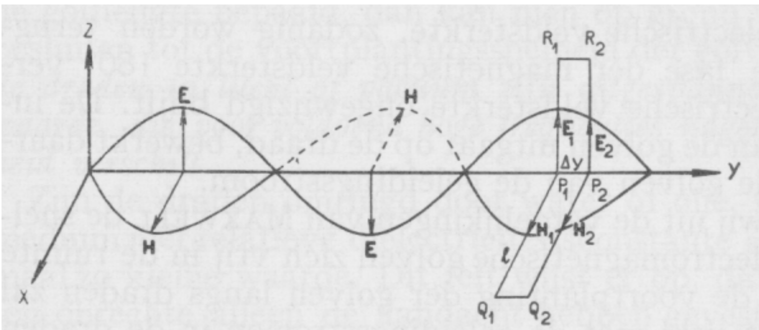


*Figure 1 showing zero phase between E and H fields*

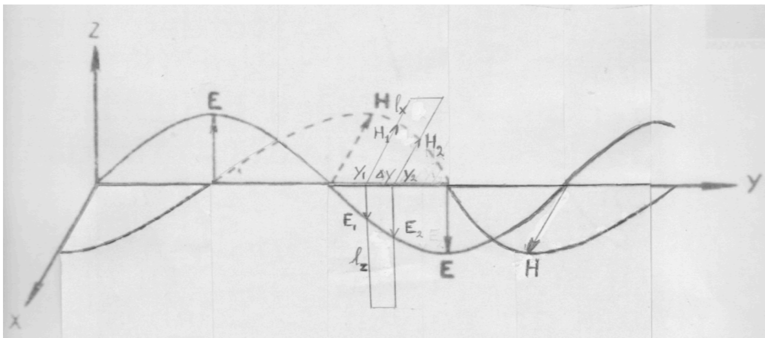


*Figure 2 showing $90^0$ phase between E and H fields*

Reference [1] shows a simple derivation of the speed of light based on figure 1. Such a derivation can also be applied to figure 2. The translation of that derivation, a bit adapted here, is shown. It doesn't matter whether figure 1 or 2 is used.

The propagation direction of the EM wave is assumed to be along the y-axis. The electric resp. magnetic fields at position $y_1$ resp. $y_2$ are: $E_1$ and $H_1$ resp. $E_2$ and $H_2$. Between these moments the EM wave is supposed to travel the distance Δy during the time Δt with speed v=Δy/Δt.

The Maxwell-Ampère respectively Maxwell-Faraday equation applied, leads to:
$§_s H\, ds = (H_2–H_1)l_x = d/dt∫∫_S D\, dS = dD/dt.l_zΔy = εdE/dt.l_zΔy = ε(E_2–E_1)/Δt.l_zΔy = εv(E_2–E_1)l_z$
resp.,
$§_s E\, ds = (E_2–E_1)l_z = d/dt∫∫_S B\, dS = dB/dt.l_xΔy = μdH/dt.l_xΔy = μ(H_2–H_1)/Δt.l_xΔy = μv(H_2–H_1)l_x$

N.B. The electric and magnetic fields in the y-direction are zero!

So $(H_2–H_1)l_x = εv(E_2–E_1)l_z$ respectively $(E_2–E_1)l_z = μv(H_2–H_1)l_x$

At this point the difference between figure 1 and 2 shows up!
Suppose we take in figure 2 the places along the y-axis around a maximum H, where $H_1= H_2$ and thus around E = 0 where $E_1 = –E_2$. At such places $H_2–H_1=0$, so the 2 equations are clearly invalid. Taking figure 1, there are no such places, so this phenomenon is already a sufficient reason to reject the possibility of whatever phase shift between the E and H field, other than $0^0$ or $180^0$.

Applying the first equation in the second equation shows:
$(E_2–E_1)l_z = μv\{εv(E_2–E_1)l_z\} = εμv^2(E_2–E_1)l_z$, so $εμv^2=1$, resulting in $v=1/\sqrt{εμ}$.

Reference [1] uses from the beginning the assumption $l_x = l_z$. Not doing so in the next step and defining ΔE as $E_2–E_1$ resp. ΔH as $H_2–H_1$, leads to: $ε(ΔE\, l_z)^2 = μ(ΔH\, l_x)^2$.
A basically correct approach would be to chose $l_x$ as well as $l_z$ infinite, resulting in $ΔE = ΔH\sqrt{μ/ε}$, with $\sqrt{μ/ε}$ the so-called characteristic impedance of the medium under consideration.

Such an outcome also forces to conclude that the E and H fields must be synchronized *in phase*.

A much more important conclusion of this consideration is the confirmation of the remark made already at the beginning:
"Because an electric charge is a quantity that can exist independent of other quantities, the start of the generation of an EM wave must be found in a source that eventually produces a dQ/dt."
That means that the very first dQ/dt is generated in the source of the EM wave, with the conclusion that thus the propagation speed of an EM wave is $1/\sqrt{εμ}$ *relative to its source only*.
Secondly: the larger this dQ/dt, given the same dQ, the higher the frequency of the EM wave will be, also confirming that the EM wave and its source are inextricably linked to each other at the moment of emission. In [2] it has already been argued about 3 years ago that, given a certain ΔQ, the smaller Δt is, the higher the frequency of the emitted photon.

With this conclusion the hypothesis in the Special Theory of Relativity: the speed of light is c relative to *any* reference, has to be rejected, and thus this theory! See also [3].

## 5. The integral and differential presentations of the Maxwell equations

### 5.1 The integral presentation (IP).

In the previous chapter the IP of the Maxwell equations have been used.
An EM wave is a 3-dimensional phenomenon, so the two variables E and H and their related variables, are effectively vectors. In figure 1 these variables are presented as 1-dimensional: E is only defined in the z-direction, like H only in the x-direction.

In order to try to understand what happens 3-dimensionally this caricature has been chosen. Thus the vectors **E** and **H** and their related variables have been presented and used as E resp. H. Consequently these variables will not leave the x-z plane, thus do not show any propagation of the EM wave out of the source.

Centuries old experiences have learned that these fields do, for some not yet explained reason, not stay in that one plane. The most likely reason is that an E/H field doesn't cause a H/E field as one line exactly perpendicular to E/H, as suggested up to now. For example, a circular shaped electric current creates H fields curving around this current as closed loops.
So indeed figure 1 is a caricature of reality.

### 5.2 The differential presentation (DP)

This presentation is as follows:

| | | | |
|---|---|---|---|
| $\nabla \cdot \mathbf{E} = \rho/\varepsilon$ | in stead of | $\S\S_S\, \mathbf{E}\, dS = Q/\varepsilon$ | $(\rho = Q/m^3)$ |
| $\nabla \cdot \mathbf{H} = 0$ | in stead of | $\S\S_S\, \mathbf{H}\, dS = 0$ | |
| $\nabla \times \mathbf{E} = -\mu\partial\mathbf{H}/\partial t$ | in stead of | $\int_s \mathbf{E}\; ds = d/dt \iint_S \mu\mathbf{H}\, dS$ | |
| $\nabla \times \mathbf{H} = -\varepsilon\partial\mathbf{E}/\partial t$ | in stead of | $\S_s\, \mathbf{H}\, ds = d/dt \iint_S \varepsilon\mathbf{E}\; dS$ | |

The outcome of $\nabla.\mathbf{F}$ is the quantity: $\partial F_x/\partial x + \partial F_y/\partial y + \partial F_z/\partial z$.
The outcome of $\nabla \times \mathbf{F}$ is the vector: $(\partial F_y/\partial z - \partial F_z/\partial y, \partial F_z/\partial x - \partial F_x/\partial z, \partial F_x/\partial y - \partial F_y/\partial x)$.

The fundamental problem with these presentations is, at least to the opinion of the author, that in the first instance every feeling with reality is lost.
But $\nabla \times \mathbf{F}$ can be simplified by looking at it as only a derivative of **F** to 'place'.
Doing so the dimension of **E** changes from V/m to V/m$^2$ resp. the one of **H** from A/m to A/m$^2$.

In the IP situation on both sides of the equation the dimension is V, resp. A. So, from that point of view there is no principle difference between the two types of presentation.

The DP shows, after applying esoteric $\nabla$ operations: $\partial^2E/\partial t^2 = c^2\nabla^2E$ and $\partial^2H/\partial t^2 = c^2\nabla^2H$.
Reference [4] presents the following information about such a type of equation:

> "Solutions of this equation describe propagation of disturbances out from the region at a fixed speed in one or in all spatial directions, as do physical waves from plane or localized sources; the constant c is identified with the propagation speed of the wave."

*Seemingly the speed c with respect to its "localized source" is meant!*

The IP thus is much more clear and much more related to reality in proving that $c = \sqrt{1/\varepsilon_0\mu_0}$.
The DP does neither show any indication about the mutual phase between the E and H fields!
It shows that if $\partial\mathbf{H}/\partial t = 0$ then $\nabla \times \mathbf{E} = 0$. But $\nabla \times \mathbf{E}$ is in the most simple imagination dE/d'place', not being able to specify 'place' better than an unknown direction at an unknown position.

## Conclusion

The article shows in the simplest way how the EM wave can mathematically be deduced from the Maxwell-Ampère and Maxwell-Faraday equations, with, as spin of, the evidence that the reference for the propagation speed of an EM wave can only be its source.
Such a conclusion forces to reject the Special Theory of Relativity.

## References


[1] Leerboek der natuurkunde, Prof. Dr. R. Kronig, Delft, 1962 (in Dutch)
[2] Why a photon is not a particle http://vixra.org/abs/1505.0225
[3] Einstein's and Galilei's Principle of Relativity http://vixra.org/abs/1611.0111
[4] https://en.wikipedia.org/wiki/Wave_equation