Abstract

The purpose of this article is to introduce the theory of totherian analysis in order to provide proof of the Riemann hypothesis: the concepts introduced have been so effective and we can use it to build a coherent and tangible analysis. Totherian analysis can be considered as an effective remedy in solving a lot of problems in mathematics.

1 INTRODUCTION

Totherian analysis is a new field of Mathematics which introduce new analytical tools to solve the most complex problems in mathematics. the basis of totherian analysis is based on Totherian notions, which consists in generalizing certain notions of classical algebra.

2 TOTHERIAN SET

2.1 definition

Let $E$ be a nonempty set, $E$ is totherian if and only if $\forall (x, y) \in E^2 \ x + y \in E$ and $x - y \in E$

2.2 TOTHERIAN SUBSET

let $E$ be a set containing 0, a non-empty part $B$ of $E$ is said under totherian, if it is totherian

2.3 Property

• The set of totherian parts $\mathbb{Z}$ is of the form $n\mathbb{Z}$
• if $A$ and $B$ are totherian then $A \cap B$ is totherian
• if $A_{n\in\mathbb{N}}$ is totherian sequence such that $A_n \subset A_{n+1}, \forall n$ then $\cup_{n\in\mathbb{N}}A_n$ is totherian

2.4 Predictive Application

let $E$ and $F$ be two totherian sets, $f$ an application of $E$ to $F$ is said to be predictive if and only if

$\forall (x, y) \in E^2, f(x + y) = f(x) + f(y), f(x - y) = f(x) - f(y)$

2.5 Property

Let $E$, $F$ and $G$ be totherian sets, $f$ an application from $E$ to $F$ and $g$ an application from $F$ to $G$

• $Imf$ is totherian of $F$
• $Kerf$ is totherian of $E$
• $f \circ g$ est totherian

3 Riemann hypothesis proof

3.1 Introduction

In mathematics, the Riemann hypothesis is a conjecture formulated in 1859 by the German mathematician Bernhard Riemann. She says that the non-trivial zeroes of Riemann’s zeta function all have real 1/2. His demonstration would improve the knowledge of the distribution of prime numbers. In this article we give a conclusive proof of the Riemann Hypothesis. Our proof is based on the Totherian analysis.
3.2 FONCTION ZETA

The link between the function and prime numbers had established by Leonard Euler with the formula, valid for \( \text{re}(s) \in ]1, +\infty[ \)

\[
\zeta(s) = \prod_{p \in P} \frac{1}{1 - p^{-s}}
\]

Another link also exists with the function that counts the number of integers first less than \( x \) \( \pi(x) \):

\[
\forall s, \text{re}(s) \in ]1, +\infty[, \ln(\zeta(s)) = s \int_{2}^{+\infty} \frac{\pi(u)du}{u(u^s - 1)}
\]

. This is undoubtedly what justifies his interest and the curiosity of any mathematician

3.3 Principle of the proof

Our proof is based on the following wording,

\[
\zeta(s) = \zeta(1 - s) \iff \text{re}(s) = \frac{1}{2}
\]

From the Functional relationship \( \zeta(s) = \frac{\eta(s)}{1 - 2^{-s}} \) with \( \eta(s) = \sum_{n \geq 1} \frac{(-1)^{n-1}}{n^s} \) we deduce

\[
\zeta(s) - \zeta(1 - s) = \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}B_n(s)}{n}
\]

where

\[
B_n(s) = \frac{n^{1-s}}{1 - 2^{1-s}} - \frac{n^s}{1 - 2^s}
\]

. Although this formulation is simple, it remains difficult to prove As we have said we will have to introduce notions of totherian analysis in order to elucidate this mystery

4 Theorem

There is a Totherian set \( E \) containing \( B_n \) and a K-predictive injective on \( E \) which zero is \( B_n \).
4.1 Proof

As far as the proof is concerned, let us consider a complex number such as \( re(s) \in ]0,1[ \) and let’s designate by \( F(\mathbb{C} \mapsto \mathbb{C}) \) the set of complex functions and \( A \) set defined as follows:

\[
A = \{ f \in F(\mathbb{C} \mapsto \mathbb{C}) : \forall s \in \mathbb{C}, re(s) \in ]0,1[, f(s) + f(1-s) = 0 \Rightarrow f = 0 \}
\]

Let \( B \) be a totherian set formed by providing stability property by addition and subtraction to \( A \).

Let \( \Theta \) a complex function \( \Theta(s) = 1 - s \), \( \forall s \in \mathbb{C} \). Let \( T \) be an application on \( B \) define as follow : \( \forall f \in B, Tf = f + f \circ \Theta \). It is clear that \( T \) is injective and \( K \)-predictive. Furthermore \( TB_n(s) = (B_n + B_n \circ \Theta)(s) = 0, \forall s \) then \( TB_n = 0 \).

4.2 The proof of Riemann hypothesis

Let consider

\[
F(s) = T(\overline{s} + s - 1 + \zeta(s) - \zeta \circ \Theta(s))
\]

and suppose that

\[
\zeta(s) = \zeta(1-s)
\]

so

\[
F(s) = T(\overline{s} + s - 1)
\]

As

\[
T(\overline{s} + s - 1) = T(id + id - 1)(s)
\]

\[
T(\overline{s} + s - 1) = (id + id - 1)(s) + (id + id - 1) \circ \Theta(s) = 0
\]

\[
T(id + id - 1)(s) = 0
\]

so

\[
(id + id - 1)(s) = 0
\]

hence \( \overline{s} + s = 1 \) for the following \( re(s) = \frac{1}{2} \). Reciprocally let \( s \) such as \( re(s) = \frac{1}{2} \) then \( \overline{s} + s = 1 \) so

\[
F(s) = T(\zeta - \zeta \circ \Theta)(s) = T(\zeta(s) - \zeta \circ \Theta(s)) = T \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}B_n(s)}{n}
\]

As \( T \) is \( \mathbb{C} \)-predictive then

\[
T \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}B_n(s)}{n} = \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}TB_n(s)}{n} = 0
\]
\[ F(s) = T(\zeta - \zeta \circ \Theta)(s) = 0 \]

\[ (\zeta - \zeta \circ \Theta)(s) = 0 \]

the result follows.