Disproof of the Riemann Hypothesis

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Occupy a Nice Place in the Future

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We present a disproof by direct contradiction. We use an elementary representation of the Riemann ζ function to show that there are infinitely many non-trivial zeros of ζ off the critical line. All of these zeros are in the neighborhood of infinity and we define that neighborhood.

Let \( \mathbb{R} \) be separated between real numbers in the neighborhood of the origin \( \hat{0} \) and real numbers in the neighborhood of infinity \( \pm \hat{\infty} \). Call real numbers in the neighborhood of zero \( \mathbb{R}_0 \) and call real numbers in the neighborhood of infinity \( \hat{\mathbb{R}} \) so that for \( \mathbb{R} \sim \mathbb{R}_0 \) we have

\[
\forall x \in \mathbb{R}_0 \ \exists \ b \in \mathbb{R} : x = \hat{0} + b ,
\]

and

\[
\forall x \in \hat{\mathbb{R}} \ \exists \ b \in \mathbb{R} , b > 0 : x = \pm (\hat{\infty} - b) .
\]

The symbol \( \hat{\infty} \) inherits all canonically non-standard analytical properties of \( \infty \) except for additive absorption [1]. Therefore, every number in the neighborhood of infinity is distinct and there are as many real numbers in the neighborhood of infinity as there are non-zero real numbers in the neighborhood of the origin. Every \( \hat{\mathbb{R}} \) number is incomparably large in absolute value compared to every \( \mathbb{R}_0 \) number so

\[
x \in \mathbb{R}_0 , \ y \in \hat{\mathbb{R}} \quad \implies \quad \frac{x}{y} = 0 .
\]

Riemann has continued the domain of the Dirichlet function to \( \mathbb{C} \) so that we have the Euler product

\[
\zeta(z) = \prod_{p \in \text{primes}} \frac{1}{1 - p^{-z}} , \quad \text{for} \quad z \in \mathbb{C} .
\]

We will consider the further analytic continuation to the exterior neighborhood of infinity around \( \mathbb{C} \). Complex numbers including numbers in the neighborhood of infinity have the form

\[
z = x + iy , \quad \text{where} \quad x, y \in \mathbb{R}_0 \cup \hat{\mathbb{R}} .
\]

To disprove Riemann’s hypothesis, let \( z_0 \) be a number in the neighborhood of negative real infinity in the form

\[
z_0 = -(\hat{\infty} - b) + iy_0 , \quad \text{with} \quad b, y_0 \in \mathbb{R} , b > 0 .
\]

Therefore,

\[
\zeta(z_0) = \prod_{p \in \text{primes}} \frac{1}{1 - p^{-\hat{\infty} - b} p^{-iy_0}} .
\]

Using the formula

\[
a^{b+ic} = a^b \left[ \cos(c \ln a) + i \sin(c \ln a) \right] ,
\]

and choosing \((y_0 \ln p') = 2n\pi\) for some prime \(p'\) we get

\[
\zeta(z_0) = \frac{1}{1 - \hat{\infty}} \left( \prod_{p \in \text{primes}} \frac{1}{1 - p^{-z_0}} \right) .
\]

By equation (3) we get

\[
\zeta(z_0) = 0 .
\]

Since \(-\hat{\infty} + b \neq 1/2\), we have disproven Riemann’s hypothesis. The Riemann ζ function has a zero at every point in the neighborhood of minus real infinity which has the form of \(z_0\) with appropriate \(y_0\). In other work [2], we have shown that ζ will have zeros off the critical line in the neighborhood of infinity.