# Conformal Symmetry Breaking in Einstein-Cartan Gravity coupled to the Electroweak Theory

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## Abstract

We develop an alternative to the Higgs mechanism for spontaneously breaking the local SU(2)xU(1) gauge invariance of the Electroweak Theory by coupling to Einstein-Cartan gravity in curved spacetime. The theory exhibits a local scale invariance in the unbroken phase, while the gravitational sector does not propagate according to the conventional quantum field theory definition. We define a unitary gauge for the local SU(2) invariance which results in a complex Higgs scalar field. This approach fixes the local SU(2) gauge without directly breaking the local U(1). We show how the electroweak symmetry can be spontaneously broken by choosing a reference mass scale to fix the local scale invariance. The mass terms for the quantum fields are then generated without adding any additional symmetry breaking terms to the theory. We point out subtle differences of the quantum field interactions in the broken phase.

## 1 Einstein-Cartan Gravity coupled to a Dirac Spinor

Here we outline the basic formulation of General Relativity[1][2] coupled to a Dirac spinor in curved spacetime[4][5][6]. In this formalism, we exclude the conventional restriction on torsion and follow the approach introduced by Cartan[3]. The analysis does not result in a propagating theory of quantum gravity, and the lack of renormalizability in the traditional sense[7][8] does not pose any inconsistency. Furthermore, this approach does not necessarily lead to a symmetric canonical energy-momentum tensor as in the Belinfante-Rosenfeld procedure[9][10].

For group and matrix indices we choose the lower case Roman letters (a, b, c, d), for flat spacetime indices we choose the lower case Roman letters (m, n, p, q), and for curved spacetime indices we choose the lower case Greek letters ($\mu$, $\nu$, $\rho$, $\sigma$).

We adopt the following conventions for the metric tensor and vierbein connection.

$$\begin{aligned} \eta^{mn} &= (-+++) \\ g^{\mu\nu} &= \eta^{mn} e^{\mu}_{m} e^{\nu}_{n} \end{aligned} \tag{1}$$

We choose the Clifford algebra for $\gamma$ matrices with this metric signature as follows.

$$\begin{aligned} \{\gamma^m, \gamma^n\} &= -2\eta^{mn} \\ \gamma_5 &= i\gamma^0\gamma^1\gamma^2\gamma^3 \end{aligned} \tag{2}$$

In the spinor representation, we adopt the following conventions for the local Lorentz group.

$$\begin{aligned} \psi'(x) &= \Lambda(x)\psi(x) \\ \Lambda(x) &= \exp\left(-\frac{1}{2}\theta^{mn}(x)S_{mn}\right) \\ \\ \bar{\psi} &= \psi^\dagger\gamma^0 \\ \bar{\psi}'(x) &= \bar{\psi}(x)\Lambda^{-1}(x) \\ \Lambda^{-1}(x) &= \exp\left(+\frac{1}{2}\theta^{mn}(x)S_{mn}\right) \\ \\ S_{mn} &= -\frac{1}{4}[\gamma_m, \gamma_n] \end{aligned} \tag{3}$$

This representation satisfies the $\mathrm{SO}(1,3)$ Lie algebra of the generators $S_{mn}$ for the local Lorentz group.

$$[S_{mn}, S_{pq}] = -\,\eta_{mp}S_{nq} - \eta_{nq}S_{mp} + \eta_{mq}S_{np} + \eta_{np}S_{mq} \tag{4}$$

The theory for a Dirac spinor in curved spacetime is symmetric with respect to both general covariant and local Lorentz transformations. We use the convention that $\Gamma_{\mu\nu}{}^{\rho}$ is the general covariant connection and $\omega_\mu{}^{mn}$ is the local Lorentz spin connection. In our approach, the index order of the connections are set deliberately. We also find it important to note that since $\eta^{mn}$ and the Clifford algebra are invariant under local Lorentz transformations, the vierbein transforms only as a general covariant vector. The covariant derivative acting on the vierbein and Dirac spinor is defined as follows.

$$\begin{aligned} \nabla_\mu\, e^n_\nu &= \partial_\mu\, e^n_\nu + \Gamma_{\mu\nu}{}^{\rho}\, e^n_\rho \\ \nabla_\mu\psi &= \partial_\mu\psi + \frac{1}{2}\,\omega_\mu{}^{mn}\, S_{mn}\psi \\ \nabla_\mu\bar{\psi} &= \partial_\mu\bar{\psi} - \frac{1}{2}\,\bar{\psi}\, S_{mn}\omega_\mu{}^{mn} \end{aligned} \tag{5}$$

We make use of the following fundamental definitions, which allow for both a unique covariant derivative and Riemann curvature tensor. The general covariant connection can then be eliminated from the theory in favor of the vierbein and spin connection which remain as independent fields.

$$
\begin{aligned}
\omega_\mu{}^{mn} &:= e^{\nu m}\nabla_\mu e^n_\nu \\
R_{\mu\nu}{}^{mn} &:= e^{\sigma m}[\nabla_\mu, \nabla_\nu]\, e^n_\sigma
\end{aligned} \tag{6}
$$

These definitions lead directly to the desired results with $\omega_\mu{}^{mn} + \omega_\mu{}^{nm} = 0$ giving $\nabla_\mu g^{\nu\rho} = 0$.

$$
\begin{aligned}
\nabla_\mu \gamma_\nu &= 0 \\
\nabla_\mu \gamma_m &= -\omega_{\mu m}{}^n\, \gamma_n \\
\nabla_\mu(\bar{\psi}\gamma_\nu\psi) &= \partial_\mu(\bar{\psi}\gamma_\nu\psi) + \Gamma_{\mu\nu}{}^\rho(\bar{\psi}\gamma_\rho\psi) \\
\\
R_{\mu\nu}{}^{mn} &= \nabla_{[\mu}\omega_{\nu]}{}^{mn} + \omega_{[\mu}{}^{mp}\omega_{\nu]p}{}^n \\
R &= e^\mu_m e^\nu_n \omega_{[\nu}{}^{mp}\omega_{\mu]p}{}^n
\end{aligned} \tag{7}
$$

where $\gamma_\nu = e^m_\nu\gamma_m$ and $R = e^\mu_m e^\nu_n R_{\mu\nu}{}^{mn}$. After integrating by parts to evaluate the scalar curvature $R$, we note that it no longer contains any derivates on the connections $(e, \omega)$. We now write the Lagrangian density and gravitational field equations for Einstein-Cartan Gravity coupled to a Dirac spinor.

$$
\begin{aligned}
eL &= \frac{1}{2\kappa}R + \frac{1}{2}\bar{\psi}e^\mu_m\gamma^m i\nabla_\mu\psi - \frac{1}{2}(i\nabla_\mu\bar{\psi})e^\mu_m\gamma^m\psi + m\bar{\psi}\psi \\
&= \frac{1}{2\kappa}R + \frac{1}{2}\bar{\psi}e^\mu_m\gamma^m i\partial_\mu\psi - \frac{1}{2}(i\partial_\mu\bar{\psi})e^\mu_m\gamma^m\psi + \frac{i}{4}\omega_\mu{}^{pq}e^\mu_m S^m{}_{pq} + m\bar{\psi}\psi
\end{aligned}
$$

$$
\begin{aligned}
e\kappa\frac{\delta L}{\delta e^\mu_m} &= R_\mu{}^m - \frac{1}{2}e^m_\mu R + \frac{\kappa}{2}\bar{\psi}\gamma^m i\partial_\mu\psi - \frac{\kappa}{2}(i\partial_\mu\bar{\psi})\gamma^m\psi + \frac{i\kappa}{4}\omega_\mu{}^{pq}S^m{}_{pq} = 0 \\
e\kappa\frac{\delta L}{\delta\omega^\mu{}_{pq}} &= \omega_\mu{}^{pq} + \frac{i\kappa}{4}e^m_\mu S_m{}^{pq} = 0
\end{aligned} \tag{8}
$$

where $e = det(e^\mu_m)$ and $S^m{}_{pq} = \bar{\psi}\{\gamma^m, S_{pq}\}\psi$ is the totally anti-symmetric spin field. The spin connection is now totally anti-symmetric since the other components may be eliminated by their equations of motion.

## 2 Einstein-Cartan Gravity coupled to the Electroweak Theory

We develop the formalism for Einstein-Cartan Gravity coupled to the Electroweak Theory[11][12][13] in curved spacetime. This approach exhibits a local scale invariance in the unbroken phase[17][18]. However, we do not follow the procedure of the Higgs mechanism for spontaneously breaking the local SU(2)xU(1) gauge invariance of the Electroweak Theory[14][15][16]. Instead, we show how the electroweak symmetry can be spontaneously broken by choosing a reference mass scale to fix the local scale invariance.

We define a unitary gauge for the local SU(2) invariance that results in a complex Higgs scalar field. This approach fixes the local SU(2) gauge without directly breaking the local U(1). Technically, we fix the local SU(2) unitary gauge then break the remaining local U(1) by choosing a reference mass scale. Therefore, the reference mass is arbitrary up to a local U(1) transformation and a corresponding local scale transformation. We do not reduce the Higgs scalar to a constant as in[19], since the real and imaginary components cannot both be fixed to a constant by a single local scale transformation.

We introduce the complex scalar SU(2) doublet with weak hypercharge $Y = +1/2$. We follow the convention $Y = Q - J_3$, where Q is the electromagnetic charge and $J_3$ is the corresponding SU(2) generator.

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

$$J_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{9}$$

We also define the following local SU(2) transformation $T(x)$ and the reversal matrix m(x).

$$T = \begin{pmatrix} \phi_0 \,/(|\phi_0| + i\,|\phi_+|) & -\phi_+/(|\phi_0| + i\,|\phi_+|) \\ \bar{\phi}_+/(|\phi_0| - i\,|\phi_+|) & \bar{\phi}_0 \,/(|\phi_0| - i\,|\phi_+|) \end{pmatrix}$$

$$T\phi = \begin{pmatrix} 0 \\ |\phi_0| + i\,|\phi_+| \end{pmatrix}$$

$$TmT^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{10}$$

We introduce a chiral SU(2) doublet for the $(neutrino, electron)_L$ fields with $Y = -1/2$, and a chiral SU(2) doublet for the $(proton, neutron)_L$ fields with $Y = +1/2$. We follow the standard representation for chiral spinors. $(\nu, e, p, n)_L = [(1 - \gamma_5)/2](\nu, e, p, n)$ and $(\nu, e, p, n)_R = [(1 + \gamma_5)/2](\nu, e, p, n)$.

$$T\psi = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$T\chi = \begin{pmatrix} p_L \\ n_L \end{pmatrix} \tag{11}$$

By convention, we choose SU(2) singlets for the $(\nu, n)_R$ fields with $Y = 0$, an SU(2) singlet for the electron field $e_R$ with $Y = -1$, and an SU(2) singlet for the proton field $p_R$ with $Y = +1$.

The Lagrangian density follows, where $\tau_a$ are the Pauli matrices and $\epsilon^a{}_{bc}$ are the SU(2) structure constants.

$$
\begin{aligned}
(\nabla\!\!\!/,\, W\!\!\!/^a,\, B\!\!\!/) &= e^\mu_m \gamma^m (\nabla_\mu,\, W^a_\mu,\, B_\mu) \\
W^a_{\mu\rho} &= \nabla_{[\mu} W_{\rho]}{}^a + g\epsilon^a{}_{bc} W_\mu{}^b W_\rho{}^c \\
B_{\mu\rho} &= \nabla_{[\mu} B_{\rho]}
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
eL = {} & \frac{1}{6}(\phi^\dagger\phi)R + g^{\mu\rho}(i\,\partial_\mu\phi + \frac{1}{2}g\tau_a W^a_\mu\phi + \frac{1}{2}g' B_\mu\phi)^\dagger(i\,\partial_\rho\phi + \frac{1}{2}g\tau_a W^a_\rho\phi + \frac{1}{2}g' B_\rho\phi) + \lambda^2(\phi^\dagger\phi)^2 \\
& + \bar{\psi}(i\nabla\!\!\!/ + \frac{1}{2}g\tau_a W\!\!\!/^a - \frac{1}{2}g' B\!\!\!/)\psi + \bar{\nu}_R i\nabla\!\!\!/\nu_R + \bar{e}_R(i\nabla\!\!\!/ - g' B\!\!\!/)e_R + G_\nu(\bar{\psi}m\phi\nu_R + \bar{\nu}_R\phi^\dagger m^\dagger\psi) + G_e(\bar{\psi}\phi\, e_R + \bar{e}_R\phi^\dagger\psi) \\
& + \bar{\chi}(i\nabla\!\!\!/ + \frac{1}{2}g\tau_a W\!\!\!/^a + \frac{1}{2}g' B\!\!\!/)\chi + \bar{p}_R(i\nabla\!\!\!/ + g' B\!\!\!/)p_R + \bar{n}_R i\nabla\!\!\!/ n_R + G_p(\bar{\chi}m\phi p_R + \bar{p}_R\phi^\dagger m^\dagger\chi) + G_n(\bar{\chi}\phi\, n_R + \bar{n}_R\phi^\dagger\chi) \\
& + \frac{1}{4}W^a_{\mu\rho}W^{\mu\rho}_a + \frac{1}{4}B_{\mu\rho}B^{\mu\rho}
\end{aligned} \tag{13}
$$

There exists a local scale invariance $\Omega(x)$ of the action $S = \int d^4xL$. We present the corresponding field transformations that generate the invariance in four spacetime dimensions. We also find it important to note that the embedded torsion tensor does not transform.

$$
\begin{aligned}
g^{\mu\rho} &\to \Omega^2(x) g^{\mu\rho} \\
e^\mu_m &\to \Omega(x) e^\mu_m \\
e &\to \Omega^4(x) e \\
\omega^{mn}_\mu &\to \omega^{mn}_\mu - g^{\rho\sigma} e^{[m}_\mu e^{n]}_\rho \Omega^{-1}(x)\partial_\sigma\Omega(x) \\
\phi &\to \Omega(x)\phi \\
\psi &\to \Omega^{3/2}(x)\psi \\
\chi &\to \Omega^{3/2}(x)\chi \\
(\nu, e, p, n)_R &\to \Omega^{3/2}(x)(\nu, e, p, n)_R \\
W^a_\mu &\to W^a_\mu \\
B_\mu &\to B_\mu
\end{aligned} \tag{14}
$$

We now fix the local scale invariance by choosing a reference mass scale in the unitary gauge for SU(2).

$$
T\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ s + iH(x) \end{pmatrix}
$$

$$
\phi^\dagger\phi = \frac{1}{2}(s^2 + H^2) \tag{15}
$$

We identify $H(x)$ as the Higgs boson and $\kappa s^2 = 6$.

The Electroweak symmetry has been spontaneously broken by choosing a reference mass scale to fix the local scale invariance in relation to the gravitational coupling constant. The classical gravity sector connections $(e, \omega)$ do not exhibit quantum fluctuations and now may be set to their vacuum expectation values $(\delta, 0)$. They may be thought of as auxiliary fields in the quantum theory to complete the local scale invariance. All masses for the quantum fields are generated by the reference mass scale, and the Higgs boson does not develop a vacuum expectation value by adding any additional terms to the Lagrangian density.

## 3 Field Transformations in the Spontaneously Broken Theory

Here we examine the physical content of the spontaneously broken quantum field theory. We adopt the Weinberg angle in terms of the coupling constants to make the field transformations more transparent.

$$\cos\theta = \frac{g}{\sqrt{g^2+g'^2}}$$
$$\sin\theta = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$W_\mu = \frac{1}{\sqrt{2}}(W^1_\mu - i\,W^2_\mu)$$

$$Z_\mu = -W^3_\mu \cos\theta + B_\mu \sin\theta$$
$$A_\mu = +W^3_\mu \sin\theta + B_\mu \cos\theta$$

$$W^1_\mu = \frac{1}{\sqrt{2}}(W_\mu + \overline{W}_\mu)$$
$$W^2_\mu = \frac{i}{\sqrt{2}}(W_\mu - \overline{W}_\mu)$$

$$W^3_\mu = A_\mu \sin\theta - Z_\mu \cos\theta$$
$$B_\mu = A_\mu \cos\theta + Z_\mu \sin\theta$$

$$M_H = s\lambda$$
$$M_W = \frac{sg}{2}$$
$$M_Z = \frac{sg}{2\cos\theta}$$

$$M_A = 0 \tag{16}$$

All the fermion masses are given by $sG_f/\sqrt{2}$. The fermion and vector boson interaction terms take the same form as in the Standard Model, where the weak neutral current $J^\mu_Z$ can be found using the above field definitions. For simplicity, we do not show the interaction terms containing only vector bosons.

$$L_f = \frac{g}{2\sqrt{2}}\{[\,\bar{\nu}\,\not{W}(1-\gamma_5)\,e\,] + [\,\bar{e}\,\overline{\not{W}}(1-\gamma_5)\,\nu\,] + [\,\bar{p}\,\not{W}(1-\gamma_5)\,n\,] + [\,\bar{n}\,\overline{\not{W}}(1-\gamma_5)\,p\,]\}$$

$$+\; Z_\mu\, J^\mu_Z \;-\; g\sin\theta\,(\bar{e}\,\not{A}\,e) \;+\; g\sin\theta\,(\bar{p}\,\not{A}\,p) \tag{17}$$

The primary differences in this approach are the absence of a single Higgs coupled to two vector bosons and a chiral change to the Yukawa coupling of the Higgs to fermions in form of $iHG_f\bar{f}\gamma_5 f/\sqrt{2}$. These results bring into question the Higgs decay channels to vector bosons and its expected positive parity.

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