Spontaneous Symmetry Breaking in Einstein-Cartan Gravity coupled to the SU(2)xU(1) Electroweak Theory

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Abstract

We develop an alternative to the Higgs mechanism for spontaneously breaking the local SU(2)xU(1) gauge symmetry in the Electroweak Theory by coupling to Einstein-Cartan gravity in curved spacetime. This approach exhibits a local scale invariance in the unbroken phase, while the gravitational sector does not propagate in the conventional quantum field theory definition. The mass terms for the quantum fields are then generated by fixing a local reference mass scale. The reference mass scale remains fixed up to a local U(1) transformation plus a corresponding local scale transformation. This results in a spontaneously broken local SU(2) symmetry, while the Higgs boson remains a complex scalar field. We outline subtle changes in the resulting field interactions, while maintaining renormalizability of the quantum field theory.

1 Einstein-Cartan Gravity coupled to a Dirac Spinor

Here we outline the basic formulation of General Relativity[1][2] coupled to a Dirac spinor in curved spacetime[4][5][6]. In this formalism, we do exclude the conventional restriction on torsion and follow the approach introduced by Cartan[3]. The analysis does not result in a propagating theory of quantum gravity, and the lack of renormalizability in the traditional sense[7][8] does not pose any inconsistency. Furthermore, this approach does not lead to a symmetric energy-momentum tensor as in the Belinfante-Rosenfeld procedure[9][10].

For group and matrix indices we choose the lower case Roman letters (a, b, c, d), for flat spacetime indices we choose the lower case Roman letters (m, n, p, q), and for curved spacetime indices we choose the lower case Greek letters (μ, ν, ρ, σ).

We adopt the following conventions for the metric tensor and vierbein connection.

$$\begin{aligned} \eta_{mn} &= (-+++) \\ g_{\mu\nu} &= \eta_{mn} e^m_\mu e^n_\nu \end{aligned} \tag{1}$$

We choose the Clifford algebra for γ matrices with this metric signature as follows.

$$\{\gamma_m,\, \gamma_n\} = -2\eta_{mn}$$
$$\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 \tag{2}$$

The theory for a Dirac spinor in curved spacetime is symmetric with both general covariant and local Lorentz transformations. The covariant derivative acting on the vierbein, Dirac spinor, and γ matrices is defined as follows. We use the convention that $\Gamma^{\sigma}_{\mu\nu}$ is the general covariant connection and the ${\omega_\mu}^{mn}$ is the local Lorentz spin connection. We find it important to note that since η_{mn} is invariant under local Lorentz transformations, the vierbein transforms only as a general covariant vector.

$$\nabla_\mu\, e^n_\nu = \partial_\mu\, e^n_\nu + {\Gamma_{\mu\nu}}^{\rho}\, e^n_\rho$$
$$\nabla_\mu \psi = \partial_\mu \psi + \frac{1}{2}\,{\omega_\mu}^{mn}\, S_{mn}\psi$$
$$\nabla_\mu \gamma_m = {\omega_{\mu m}}^{n}\, \gamma_n \tag{3}$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$
$$\nabla_\mu \bar{\psi} = \partial_\mu \bar{\psi} - \frac{1}{2}\,\bar{\psi}\,{\omega_\mu}^{mn} S_{mn} \tag{4}$$

In the spinor representation, we choose the following conventions for the local Lorentz group.

$$\psi'(x) = \Lambda(x)\psi(x)$$
$$\Lambda(x) = \exp\left\{-\frac{1}{2}\theta^{mn}(x)S_{mn}\right\}$$

$$\bar{\psi}'(x) = \bar{\psi}(x)\Lambda^{-1}(x)$$
$$\Lambda^{-1}(x) = \exp\left\{+\frac{1}{2}\theta^{mn}(x)S_{mn}\right\}$$

$$S_{mn} = -\frac{1}{4}[\gamma_m, \gamma_n] \tag{5}$$

This representation satisfies the SO$(1,3)$ Lie algebra of the generators S_{mn} for the local Lorentz group.

$$[S_{mn}, S_{pq}] = -\,\eta_{mp}S_{nq} - \eta_{nq}S_{mp} + \eta_{mq}S_{np} + \eta_{np}S_{mq} \tag{6}$$

We now adopt the following fundamental definition, which allows for both a unique covariant derivative and Riemann curvature tensor. The general covariant connection $\Gamma^{\sigma}_{\mu\nu}$ can then be eliminated from the theory in favor of the vierbein and spin connection. Due to the presence of torsion, the veirbein and local Lorentz spin connections remain as independent fields.

$$\omega_{\mu}{}^{mn} := e^{\nu m}\nabla_{\mu}e^{n}_{\nu} \tag{7}$$

This definition leads directly to the desired results. After integrating by parts to determine the scalar curvature R, we find it does not contain any derivates on the connections (e, ω).

$$\begin{aligned}
\nabla_{\mu}\gamma_{\nu} &= 0 \\
\nabla_{\mu}(\bar{\psi}\gamma_{\nu}\psi) &= (\bar{\psi}\partial_{\mu}\gamma_{\nu}\psi) + \Gamma_{\mu\nu}{}^{\rho}(\bar{\psi}\gamma_{\rho}\psi) \\
R_{\mu\nu}{}^{mn} &= \nabla_{[\mu}\omega_{\nu]}{}^{mn} + \omega_{[\mu}{}^{mp}\omega_{\nu]p}{}^{n} \\
R &= \eta_{pq}(\omega_{\mu}{}^{\nu p}\omega_{\nu}{}^{q\mu} + \omega_{\mu}{}^{\mu p}\omega_{\nu}{}^{\nu q})
\end{aligned} \tag{8}$$

We now write the Lagrangian density for Einstein-Cartan Gravity coupled to a Dirac spinor in curved spacetime and the corresponding gravitational field equations.

$$\begin{aligned}
eL &= \frac{1}{2\kappa}R + \frac{1}{2}\bar{\psi}e^{\mu}_{m}\gamma^{m}i\nabla_{\mu}\psi - \frac{1}{2}(i\nabla_{\mu}\bar{\psi})e^{\mu}_{m}\gamma^{m}\psi + m\bar{\psi}\psi \\
&= \frac{1}{2\kappa}R + \frac{1}{2}\bar{\psi}e^{\mu}_{m}\gamma^{m}i\partial_{\mu}\psi - \frac{1}{2}(i\partial_{\mu}\bar{\psi})e^{\mu}_{m}\gamma^{m}\psi + \frac{i}{4}\omega_{\mu}{}^{pq}e^{\mu}_{m}S^{m}{}_{pq} + m\bar{\psi}\psi
\end{aligned}$$

$$\begin{aligned}
e\kappa\frac{\delta L}{\delta e^{\mu}_{m}} &= R_{\mu}{}^{m} - \frac{1}{2}e^{m}_{\mu}R + \frac{\kappa}{2}\bar{\psi}\gamma^{m}i\partial_{\mu}\psi - \frac{\kappa}{2}(i\partial_{\mu}\bar{\psi})\gamma^{m}\psi + \frac{i\kappa}{4}\omega_{\mu}{}^{pq}S^{m}{}_{pq} = 0 \\
e\kappa\frac{\delta L}{\delta\omega^{\mu}{}_{pq}} &= \omega_{\mu}{}^{pq} + \frac{i\kappa}{4}e^{m}_{\mu}S_{m}{}^{pq} = 0
\end{aligned} \tag{9}$$

where $e = det(e^{\mu}_{m})$, $R_{\mu}{}^{m} = e^{\nu}_{n}R_{\mu\nu}{}^{mn} = e^{\nu}_{n}R_{\nu\mu}{}^{nm}$, $R = e^{\mu}_{m}R_{\mu}{}^{m}$, and $S^{m}{}_{pq} = \bar{\psi}\{\gamma^{m}, S_{pq}\}\psi$ is the totally anti-symmetric spin field. Here, we vary only with respect to the totally anti-symmetric spin connection, since the other components can be eliminated by their equation of motion.

2 Einstein-Cartan Gravity coupled to the Electroweak Theory

We present a theory of Einstein-Cartan Gravity coupled to SU(2)xU(1) Electroweak Theory[11][12][13] in curved spacetime and discuss the details of a local scale invariance in the unbroken phase[17][18]. However, we do not follow the formalism of the Higgs mechanism for spontaneous symmetry breaking[14][15][16]. We show how the electroweak symmetry can be spontaneously broken by fixing a reference mass scale.

We choose a unitary gauge only for the local SU(2) symmetry, and the resulting Higgs boson remains a complex field in this approach. While the local SU(2) symmetry is spontaneously broken, the reference mass scale remains fixed up to a local U(1) transformation plus a corresponding local scale transformation. Therefore, we do not remove all the physical degrees of freedom for the Higgs boson as in[19], since the real and imaginary components cannot both be fixed to a constant by a local scale transformation.

We choose the complex scalar SU(2) doublet with weak hypercharge $Y = +1$. We follow the convention $Y/2 = Q - J_3$, where Q is the electric charge and J_3 is the corresponding SU(2) generator. For the scalar SU(2) doublet, we use the standard representation.

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

$$J_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{10}$$

We also define the following local SU(2) transformation $T(x)$ and the SU(2) matrix m(x).

$$T = \begin{pmatrix} \phi_0\,/(|\phi_0| + i\,|\phi_+|) & -\phi_+/(|\phi_0| + i\,|\phi_+|) \\ \bar{\phi}_+/(|\phi_0| - i\,|\phi_+|) & \bar{\phi}_0\,/(|\phi_0| - i\,|\phi_+|) \end{pmatrix}$$

$$T\phi = \begin{pmatrix} 0 \\ |\phi_0| + i\,|\phi_+| \end{pmatrix}$$

$$TmT^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{11}$$

We choose an SU(2) doublet for the (neutrino, electron) fields with $Y = -1$, and an SU(2) doublet for the (proton, neutron) fields with $Y = +1$. We follow the standard representation for chiral spinors. $(\nu, e, p, n)_L = [(1 - \gamma_5)/2](\nu, e, p, n)$ and $(\nu, e, p, n)_R = [(1 + \gamma_5)/2](\nu, e, p, n)$.

$$T\psi = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$T\chi = \begin{pmatrix} p_L \\ n_L \end{pmatrix} \tag{12}$$

We choose SU(2) singlets for the (neutrino, neutron)$_R$ fields with $Y = 0$, an SU(2) singlet for the electron field e_R with $Y = -2$, and an SU(2) singlet for the proton field p_R with $Y = +2$.

The complete Lagrangian density follows, where τ_a are the Pauli matrices.

$$
\begin{aligned}
(\not\nabla,\, \not W^a,\, \not B) &= e^\mu_m \gamma^m (\nabla_\mu,\, W^a_\mu,\, B_\mu) \\
W^a_{\mu\rho} &= \nabla_{[\mu} W_{\rho]}{}^a + g\epsilon^a{}_{bc} W_\mu{}^b W_\rho{}^c \\
B_{\mu\rho} &= \nabla_{[\mu} B_{\rho]}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
eL = {} & \frac{1}{6}(\phi^\dagger\phi)R + g^{\mu\rho}(i\,\partial_\mu\phi + \frac{1}{2}g\tau_a W^a_\mu\phi + \frac{1}{2}g' B_\mu\phi)^\dagger(i\,\partial_\rho\phi + \frac{1}{2}g\tau_a W^a_\rho\phi + \frac{1}{2}g' B_\rho\phi) + \frac{1}{2}\lambda^2(\phi^\dagger\phi)^2 \\
& + \bar\psi(i\not\nabla + \frac{1}{2}g\tau_a \not W^a - \frac{1}{2}g'\not B)\psi + \bar\nu_R i\not\nabla\nu_R + \bar e_R(i\not\nabla - g'\not B)e_R + G_\nu(\bar\psi m\phi\nu_R + \bar\nu_R\phi^\dagger m^\dagger\psi) + G_e(\bar\psi\phi\, e_R + \bar e_R\phi^\dagger\psi) \\
& + \bar\chi(i\not\nabla + \frac{1}{2}g\tau_a \not W^a + \frac{1}{2}g'\not B)\chi + \bar p_R(i\not\nabla + g'\not B)p_R + \bar n_R i\not\nabla n_R + G_p(\bar\chi m\phi p_R + \bar p_R\phi^\dagger m^\dagger\chi) + G_n(\bar\chi\phi\, n_R + \bar n_R\phi^\dagger\chi) \\
& + \frac{1}{4}W^a_{\mu\rho}W_a^{\mu\rho} + \frac{1}{4}B_{\mu\rho}B^{\mu\rho}
\end{aligned} \tag{14}
$$

There exists a local scale invariance $\Omega(x)$ of the action $S = \int d^4xL$.
We present the field transformations in four spacetime dimensions.

$$
\begin{aligned}
g^{\mu\rho} &\to \Omega^2(x) g^{\mu\rho} \\
e^\mu_m &\to \Omega(x) e^\mu_m \\
e &\to \Omega^4(x) e \\
\omega^{mn}_\mu &\to \omega^{mn}_\mu - g^{\rho\sigma} e^{[m}_\mu e^{n]}_\rho \Omega^{-1}(x)\partial_\sigma\Omega(x) \\
\phi &\to \Omega(x)\phi \\
\psi &\to \Omega^{3/2}(x)\psi \\
\chi &\to \Omega^{3/2}(x)\chi \\
(\nu, e, p, n)_R &\to \Omega^{3/2}(x)(\nu, e, p, n)_R \\
W^a_\mu &\to W^a_\mu \\
B_\mu &\to B_\mu
\end{aligned} \tag{15}
$$

We now fix a reference mass scale in the unitary gauge for SU(2) with the transformation $T(x)$.

$$
\begin{aligned}
T\phi &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ s + iH(x) \end{pmatrix} \\
\phi^\dagger\phi &= \frac{1}{2}(s^2 + H^2)
\end{aligned} \tag{16}
$$

In this approach, $H(x)$ is the Higgs boson and $\kappa s^2 = 6$.

The SU(2) symmetry has been (spontaneously broken) by fixing the local scale invariance and choosing a reference mass scale defined by the gravitational coupling constant. We can now show the resulting Lagrangian density representing the physical degrees of freedom for the field theory.

3 Field Definitions in the Spontaneously Broken Theory

Finally, we examine the physical content of the spontaneously broken theory. Following convention, we define the Weinberg angle in terms of the coupling constants.

$$\sin\theta = \frac{g}{\sqrt{g^2 + g'^2}}$$
$$\cos\theta = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(W^1_{\mu} \mp i\, W^2_{\mu})$$

$$Z_{\mu} = -W^3_{\mu}\sin\theta + B_{\mu}\cos\theta$$
$$A_{\mu} = +W^3_{\mu}\cos\theta + B_{\mu}\sin\theta$$

$$W^1_{\mu} = \frac{1}{\sqrt{2}}(W^+_{\mu} + W^-_{\mu})$$
$$W^2_{\mu} = \frac{i}{\sqrt{2}}(W^+_{\mu} - W^-_{\mu})$$

$$W^3_{\mu} = A_{\mu}\cos\theta - Z_{\mu}\sin\theta$$
$$B_{\mu} = A_{\mu}\sin\theta + Z_{\mu}\cos\theta$$

$$\begin{aligned} M_H &= s\lambda \\ M_W &= \frac{sg}{2} \\ M_Z &= \frac{s}{2}\sqrt{g^2 + g'^2} \\ M_A &= 0 \end{aligned} \tag{17}$$

All the fermion masses are given by $sG_f/\sqrt{2}$ with the relevant fermion subscript for G_f. The fermion interaction term takes the following form, where the weak neutral current J^{μ}_{Z} can be easily found using the above definitions.

$$L_{int} = \frac{g}{2\sqrt{2}}(W^+_{\mu}\bar{\nu}\gamma^{\mu}(1-\gamma_5)e + W^+_{\mu}\bar{p}\gamma^{\mu}(1-\gamma_5)n + H.C.) + Z_{\mu}\, J^{\mu}_{Z} + \frac{gg'}{\sqrt{g^2+g'^2}}A_{\mu}(\bar{p}\gamma^{\mu}p - \bar{e}\gamma^{\mu}e) \tag{18}$$

The fermion interaction term is the same as in the standard model. However, there is a chiral change to the Yukawa couplings between the Higgs boson and the fermions in form of $iHG_f\bar{f}\gamma_5 f/\sqrt{2}$. This brings into question the actual parity of the physical Higgs boson.

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