

# Origin of the inertial mass (II): Vector gravitational theory

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**Abstract.** We deduce the inductive forces of a vector gravitational theory and we will study whether these forces can be identified with the forces of inertia that act on a body when it is accelerated.

## 1. Introduction

In a previous work we investigate the induction phenomena derived from a scalar gravitational theory<sup>1</sup>. We find that there are forces that have their origin in the movement of the field source, but they do not satisfactorily explain the origin of inertia

Now we propose a vector gravitational theory and we will also investigate its inductive effects, with the idea of explaining the phenomenon of inertia. An investigation of this type was made in a famous work by Sciama<sup>2,+</sup>

When studying the effect of the Universe as a whole, we have to choose a cosmic model. As we are interested in the qualitative aspects, we consider the simplest cosmic model: static, with homogeneous density, large enough and of finite age. In later works we will examine the problem with more realistic cosmological models.

We advise the previous reading of the first part of this research: «Origin of the inertial mass (I): Scalar gravitational theory»<sup>1</sup>.

## 2 Vector gravitational theory

To obtain a vector gravitational theory we have to adapt the electromagnetic theory to the gravity<sup>3</sup>. The gravitational potential is a tetravector that we define by

$$\phi^k = (\phi, c\mathbf{A})$$

$\phi$  is the scalar potential and  $\mathbf{A}$  the vector potential. The source is the current tetradsity

$$j^k = \rho u^k = (j^o, \mathbf{j}) \quad (1)$$

$\rho$  is the proper density of matter and  $u^k$  is the tetravelocity defined by

$$u^k = \frac{dx^k}{d\tau} \quad (2)$$

$d\tau$  is the proper time of the source particle.

The field equation is

$$\nabla^2 \phi^k - \frac{1}{c^2} \frac{\partial^2 \phi^k}{\partial t^2} = \frac{4\pi G}{c} j^k .$$

## 3 The proper time

We assume that movement is relative. In other words, it is equivalent to say that a body moves with respect to the Universe, that to say that it is the Universe that moves with respect to the body. Both situations are equivalent kinematically and dynamically. In our simplified cosmic model all the constituent bodies of the Universe are at rest among themselves, with the exception of the C body whose movement we study.

When we assume that the Universe as a whole is in motion, the proper time of its constituents coincides with the coordinate time  $t$  of the reference system with respect to which the Universe is at rest, which is naturally an inertial reference system, where all clocks that are at rest can be synchronized. Therefore, the tetravelocity of any fixed body to the whole of the Universe in any reference system is

$$u^k = \frac{dx^k}{d\tau} = \frac{dx^k}{dt},$$

therefore

$$\mathbf{u} = (u^\alpha) = \left( \frac{dx^\alpha}{dt} \right); \quad u^0 = c; \quad j^0 = \rho c; \quad \mathbf{j} = \rho \mathbf{u}.$$

Then the field equation is divided into a scalar field and a vector field

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 4\pi G \rho; \quad \nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = \frac{4\pi G}{c^2} \mathbf{j}. \quad (3)$$

#### 4 Retarded potentials

Equations (3) are solved with the retarded potentials technique

$$\phi(\mathbf{r}, t) = -G \int \frac{[\rho]}{r'} dV'; \quad \mathbf{A}(\mathbf{r}, t) = -\frac{G}{c^2} \int \frac{[\mathbf{j}]}{r'} dV' \quad (4)$$

$\mathbf{r}$  is the position vector of the field point,  $\mathbf{r}'$  is the position of the field point with respect to the source at the moment of generating the force (or position retarded) and the brackets are values at the moment of emission or retarded values, that is in the instant  $t' = t - r'/c$ . The vector  $\boldsymbol{\sigma}$  is the position of a source point, therefore  $\mathbf{r}' = \mathbf{r} - \boldsymbol{\sigma}$ . From (4) we note that vector potential is of second order with respect to  $c$ .

If the source moves with a velocity  $\mathbf{u}$  then the potentials of Liénard-Wiechert have to be applied

$$d\phi = -G \frac{[dm]}{r' - \frac{\mathbf{u} \cdot \mathbf{r}'}{c}} = -G \frac{[dm]}{s}; \quad d\mathbf{A} = -\frac{G}{c^2} \frac{[dm] \mathbf{u}}{s}. \quad (5)$$

For the calculation of (5) we must use relationships <sup>4</sup>

$$\begin{aligned} \nabla \left( \frac{1}{s} \right) &= -\frac{\mathbf{r}'}{s^2 r'} + \frac{\mathbf{u}}{cs^2} - \frac{\mathbf{r}'(\mathbf{r}' \cdot \mathbf{u})}{cs^3 r'} - \frac{\mathbf{r}'(\mathbf{r}' \cdot \mathbf{a})}{c^2 s^3} + \frac{u^2 \mathbf{r}'}{c^2 s^3} \\ \frac{\partial}{\partial t} \left( \frac{\mathbf{u}}{s} \right) &= \frac{r'}{s} \left[ \frac{\mathbf{a}}{s} + \frac{\mathbf{u}(\mathbf{r}' \cdot \mathbf{u})}{s^2 r'} + \frac{\mathbf{u}(\mathbf{r}' \cdot \mathbf{a})}{cs^2} - \frac{\mathbf{u}u^2}{cs^2} \right]. \end{aligned} \quad (6)$$

$\mathbf{u}$  is the retarded velocity of the source and  $\mathbf{a}$  the acceleration, therefore

$$\mathbf{u} = \frac{d\boldsymbol{\sigma}}{dt'} = -\frac{d\mathbf{r}'}{dt'}.$$

The gravitoelectric field  $\mathbf{E}$  and the gravitomagnetic field  $\mathbf{B}$  are derived from the potentials  $\phi$  and  $\mathbf{A}$  in the same way as in electromagnetism

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}; \quad \mathbf{B} = \nabla \wedge \mathbf{A}, \quad (7)$$

the force acting on a particle of gravitational mass  $m_g$  that has a velocity  $\mathbf{w}$  is

$$\mathbf{F} = m_g \mathbf{E} + m_g \mathbf{w} \wedge \mathbf{B}. \quad (8)$$

#### 5 Induction of forces on a body that has a rectilinear acceleration

Suppose a body C that moves with accelerated and rectilinear movement with respect to the whole of the Universe. The situation is equivalent to saying that the Universe moves with respect to the body C.  $\mathbf{u}$  and  $\mathbf{a}$  are the velocity and acceleration of the Universe with respect to the body C. Therefore the body C has velocity  $-\mathbf{u}$  and acceleration  $-\mathbf{a}$  in relation to the Universe.

To integrate (5) we assume the Universe divided into concentric spherical shells of thickness

$d\sigma$ .  $dm$  is an infinitesimal portion of mass of one of those shells, then the force exerted on body C is deduced from (5) and (8)

$$-m_g \nabla d\phi - \frac{\partial(d\mathbf{A})}{\partial t} = G[dm]m_g \nabla \left( \frac{1}{s} \right) + \frac{G}{c^2}[dm]m_g \frac{\partial}{\partial t} \left( \frac{\mathbf{u}}{s} \right).$$

Comparing (4) and (5) and assuming non-relativistic velocities

$$[dm] = \left( 1 - \frac{\mathbf{r}' \cdot \mathbf{u}}{r'c} \right) [\rho] dV' \approx [\rho] dV',$$

therefore

$$-m_g \nabla d\phi - \frac{\partial(d\mathbf{A})}{\partial t} = G[\rho]m_g \nabla \left( \frac{1}{s} \right) dV' + \frac{G}{c^2}[\rho]m_g \frac{\partial}{\partial t} \left( \frac{\mathbf{u}}{s} \right) dV'.$$

From (6) we find that the only inductive terms (which are those that depend on the movement of the source) are

$$\nabla \left( \frac{1}{s} \right) = -\frac{\mathbf{r}'(\mathbf{r}' \cdot \mathbf{a})}{c^2 s^3} \approx -\frac{\boldsymbol{\sigma}(\boldsymbol{\sigma} \cdot \mathbf{a})}{c^2 s^3}; \quad \frac{\partial}{\partial t} \left( \frac{\mathbf{u}}{s} \right) = \frac{\mathbf{a}}{s} + \frac{\mathbf{u}(\mathbf{r}' \cdot \mathbf{a})}{cs^2} \approx \frac{\mathbf{a}}{s} - \frac{\mathbf{u}(\boldsymbol{\sigma} \cdot \mathbf{a})}{cs^2}$$

we have put  $r' \approx s \approx \sigma$  and we neglect the terms that depend on  $1/\sigma^2$  because they are negligible compared to those that depend on  $1/\sigma$ .

For simplicity we assume that the acceleration  $-\mathbf{a}$  of the Universe is parallel to the  $x$  axis  $\mathbf{a} = a\mathbf{i}$  and that in polar coordinates the vector  $\boldsymbol{\sigma}$  is

$$\boldsymbol{\sigma} = \sigma \sin \theta \cos \varphi \mathbf{i} + \sigma \sin \theta \sin \varphi \mathbf{j} + \sigma \cos \theta \mathbf{k} \quad (9)$$

using the results found in the reference 1 we arrived after the integration to

$$-m_g \nabla \phi - \frac{\partial \mathbf{A}}{\partial t} = -\frac{2\pi}{3} \frac{G}{c^2} \rho m_g \sigma^2 \mathbf{a} + 2\pi \frac{G}{c^2} \rho m_g \sigma^2 \mathbf{a} = \frac{4\pi}{3} G \rho t^2 m_g \mathbf{a} \quad (10)$$

the limit of spatial integration is the furthest point from the Universe that has a causal relationship with the body C, that is  $\sigma = ct$ , where  $t$  is the age of the Universe.

The force (10) acts on the body C when it has the rectilinear acceleration  $\mathbf{a}$ , and is caused by the action of the whole of the Universe. We identify this force with the force of inertia, therefore

$$-m_i(-\mathbf{a}) = \frac{4\pi}{3} G \rho t^2 m_g \mathbf{a} \Rightarrow m_i = \frac{4\pi}{3} G \rho t^2 m_g, \quad (11)$$

$m_i$  is the inertial mass. As in the present moment the inertial mass is assumed equal to the gravitational mass, then by (11)

$$\frac{4\pi}{3} G \rho t_0^2 = 1 \Rightarrow \phi_0 = c^2 \quad (12)$$

$t_0$  is the current age of the Universe and  $\phi_0$  is the current gravitational potential in the position occupied by the body C. Naturally, these results are valid in the framework of the cosmic model considered.

(11) means that the inertial mass is the result of the action of the whole of the Universe on the body, it is not an innate magnitude of the matter, but it is acquired by the cosmic action. The numerical value (12) is approximately correct. Therefore we conclude, within the limitations of our cosmic model, that the Mach's principle is fulfilled, which states that the inertia of a body is produced by the inductive forces of the Universe.

## 6 Induction force on a body in uniform motion

Suppose a body C that has a uniform and rectilinear movement with respect to the Universe.  $\mathbf{u}$  is the velocity of the Universe with respect to C. Next we calculate the induction force that is exerted on C.

The only terms of (6) that we have to use in this problem is the second and third of the first equation, the other terms not null can be neglected for non-relativistic velocities, then

$$\nabla \left( \frac{1}{s} \right) = \frac{\mathbf{u}}{cs^2} - \frac{\mathbf{r}'(\mathbf{r}' \cdot \mathbf{u})}{cs^3 r'}$$

and the induction force produced by an element of the spherical shell is

$$-m_g \nabla d\phi - \frac{\partial(d\mathbf{A})}{\partial t} = G\rho m_g \left[ \frac{\mathbf{u}}{cs^2} - \frac{\mathbf{r}'(\mathbf{r}' \cdot \mathbf{u})}{cs^3 r'} \right] \sigma^2 \sin\theta d\theta d\phi d\sigma.$$

Integrating for the whole of the Universe and neglecting terms, the induction force acting on the body C is

$$\mathbf{F} = \frac{8\pi}{3} G\rho t m_g \mathbf{u}, \quad (13)$$

by the principle of dynamic equilibrium the sum of all the forces acting on the body is zero. We assume that on the body acts in addition to the force (13), the force of inertia, which for a body in rectilinear motion is the equation (10), then when we apply the principle of dynamic equilibrium is found

$$\mathbf{F} + \mathbf{F}_i = 0 \Rightarrow \frac{8\pi}{3} G\rho t m_g \mathbf{u} = \frac{4\pi}{3} G\rho t^2 m_g \mathbf{a} \Rightarrow \mathbf{a} = \frac{2\mathbf{u}}{t} \quad (14)$$

$t$  is the age of the Universe and  $\mathbf{a}$  is the acceleration of the body C. The equation (15) tells us that when a body is in uniform motion with respect to the Universe, an induction force appears that produces an acceleration given by (14), of magnitude so extraordinarily small that we can consider it null. Therefore there is no force of inertia on a body in uniform motion and therefore remains with equal velocity, a statement that corresponds to the principle of inertia.

## 7 Participation of the Universe in the formation of the inertial mass

According to Mach's principle, the inertial mass of a body is generated by the action of the whole Universe. On a body C act forces that are produced in all parts of the observable Universe. Some of these forces originate in very distant objects, that is, they were produced a long time ago; while the forces exerted by nearby objects were produced a short time ago.

Now we study the participation in the generation of the inertial mass of the different epochs of the Universe. We consider the Universe formed by  $N$  spherical shells in whose center is the body C. To the furthest shell we give the numeration  $n = 0$  and in it are the objects that produce the force at the beginning of the Universe, whose effects reach the body C in the present moment, when the age of the Universe is  $t_0$ .

The thickness of the spherical shells is  $c\tau$  where  $\tau = t_0/N$ . The inertial mass produced by shell  $n$  is by (11)

$$\delta m_i = \frac{8\pi}{3} G\rho m_g t\tau \quad (15)$$

$t$  is the time takes the gravitational interaction to travel from the spherical shell  $n$  to the body C, therefore

$$t = t_0 - n\tau$$

of (11) and (15) we get

$$\frac{\delta m_i}{m_i} = \frac{2}{N} \left( 1 - \frac{t'}{t_0} \right) \quad (16)$$

$t' = n\tau$  is the time retarded.

We must remember that (16) is applicable to the cosmic model that we are considering which has a constant and uniform density of matter in the Universe. But independently of the cosmic model, (16) shows us that each epoch of the Universe contributes differently to the formation of the inertial mass of a body. (16) shows that the first moments of the Universe (when  $t' = 0$ ) have a greater contribution to the inertial mass; while the present Universe ( $t' = t_0$ ) the contribution is minimal. This result can be more pronounced in a realistic cosmic model.

## 8 Mach's principle and the Big Bang

Now we will use a more realistic cosmic model with the exclusive idea of studying the influence that the Big Bang has on the formation of the inertial mass, and considering mainly the qualitative aspects.

Now we suppose, for example, a Universe where the density depends on the cosmic age

according to the law

$$\rho(t') = \rho_0 \frac{t_0}{t'} \quad (17)$$

$\rho_0$  is the density at the present moment and  $t'$  is the age of the Universe at the time of generating the force or retarded time. By (17) the density of the Universe would be very high when its age is small, a situation similar to what happens in the Big Bang, therefore (17) will inform us about how the inertial mass is generated in a Universe that has a high density at its beginning.

By (16) the contribution to the inertial mass of a spherical shell located at a distance  $ct$  and thickness  $cdt$  is

$$dm_i = \frac{8\pi}{3} G \rho_0 \frac{t_0}{t'} m_g t dt = \frac{8\pi}{3} G \rho_0 t_0 m_g \frac{t}{t_0 - t} dt$$

when we integrate for the whole Universe we find a singularity in  $t = t_0$ . To avoid this singularity we calculate the action of the Universe from a time  $t_c$  (instead of  $t_0$ )

$$\frac{m_i}{m_g} = -\frac{8\pi}{3} G \rho_0 t_0^2 \left[ \frac{t_c}{t_0} + \ln \left( 1 - \frac{t_c}{t_0} \right) \right], \quad (18)$$

with the current values of  $\rho_0$  and  $t_0$  is found

$$\frac{8\pi}{3} G \rho_0 t_0^2 \approx 0,92.$$

we verified that the inertial mass observed in the current epoch would be generated by the forces produced by the Universe from  $t_c = 0,856t_0$ . But taking into account ancient cosmic actions, the relationship (18) increases, which means that the inertial mass becomes much greater than the gravitational mass.

For example, if we take the limit at  $t_c = 0,999t_0$ , which corresponds to an age of the Universe of about fourteen million years, the ratio between the inertial mass and the gravitational mass is approximately 5,4.

The conclusion is that if the density in the past was greater than at present (as it must happen in a cosmic model with Big Bang), the participation in the inertial mass of the primitive Universe increases, until being determinant. That is to say, that the inertial mass would be generated mainly by the first stages of the Universe, until being very superior to the mass observed.

We conclude that if the Universe had a Big Bang, the Mach's principle would not be fulfilled and vice versa.

## 9 Force induced by cosmic expansion

Next, we calculate the induction force produced by a movement of cosmic expansion. We assume that the observer is at the center of the Universe, therefore  $r' = \sigma$ . And that the expansion velocity is radial

$$\mathbf{u} = u \frac{\boldsymbol{\sigma}}{\sigma} = u \sin \theta \cos \varphi \mathbf{i} + u \sin \theta \sin \varphi \mathbf{j} + u \cos \theta \mathbf{k}$$

where  $u$  is any function of  $\sigma$ .

We follow the same procedure as in the previous sections. So when calculating the angular integrals that are derived from the two equations (6) we find that the result is zero. That is, a cosmic expansion does not produce induction force, which is logical for reasons of symmetry.

## 10 Conclusions

We have used a vector gravitational theory, from which we deduce induction forces. The results show that it is possible to identify the induction force produced by the Universe with the force of inertia acting on an accelerated body.

We have verified that there are no forces of induction on a body that has a uniform movement, as is required by the principle of inertia. We verify that the cosmic expansion does not produce induction forces.

We have found incompatibility between Mach's principle and a cosmic model with Big Bang.

In this model the inertial mass of a body is mainly produced by the first moments of the Universe and the generated mass significantly exceeds the observed inertial mass.

The conclusion is that although the vector gravitational theory is an approximation to the correct theory, and the cosmic model that we have used is very simple, we find that the induction forces produced by the whole Universe are comparable in sense and magnitude to the forces of inertia that are observed in accelerated bodies, which is a strong support for Mach's principle.

### **Bibliography**

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