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Proposal of solution of the Riemann hypothesis 2

Abstract. In this paper we will resume and conclude the last work.

1 The previous work [1]

This paragraph sums up the key points of the first work:

1.1) The property of the non-trivial zeros to be symmetric about the critical line and about the real line.

1.2) The equation $\zeta(s)=\zeta(1-s)$ satisfied by all non-trivial zeros, which implicates their symmetry about the point P($\frac{1}{2}$, 0).

1.3) The passage from non-trivial zeros z_n to couples of non-trivial zeros $(z_{n(1)}, z_{n(2)})$ (with imaginary part $\pm it$, with equal t), due to the presence of the zeta function in both sides of its functional equation:

$$\zeta(s) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s)\ \zeta(1-s)$$

1.4) The presence of couples of non-trivial zeros asymmetric about the point P($\frac{1}{2}$, 0) (thus $\zeta(s) \neq \zeta(1-s)$ for these couples), if the Riemann hypothesis is false. The absence of these kind of couples, if the Riemann hypothesis is true.

2 The reason in favour of the Riemann hypothesis

The main reason for which we can say that the Riemann hypothesis is true, is the following statement:

● *" The Riemann hypothesis is the only case in which both conditions **1.1** and **1.2**are true for all couples of non-trivial zeros. This shows that the Riemann hypothesis provides the most coherent distribution of the non-trivial zeros in the critical strip, since we won't have a contrast between the properties(**1.1** and **1.2**)of these zeros".*

Expressing this concept in formulas, we obtain that this system is valid only for the Riemann hypothesis:

$$\blacklozenge \begin{cases} \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-s) \end{cases} & simmetry \quad about \quad P(\frac{1}{2};0) \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(\overline{s}) \end{cases} & simmetry \quad about \quad the \quad real \quad line \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-\overline{s}) \end{cases} & simmetry \quad about \quad the \quad critical \quad line \end{cases}$$

for every couples of non-trivial zeros (z_j, z_k) with imaginary part $\pm it$, with equal t.

Proof of the statement ●:

We can easily observe that the system ♦ is possible only if we have at least one limit case, that is when $z_j \equiv z_k$. In fact, if the Riemann hypothesis is false, we will have six couples of non-trivial zeros so that two are symmetric only about the point $P(\frac{1}{2};0)$, two only about the real line and two only about the critical line, as we have shown in the previous paper **[1]**.
Instead, if it's true, we will have the limit case $\zeta(z_j) \equiv \zeta(s) \equiv \zeta(z_k) \equiv \zeta(1-\overline{s})$.

At this point we study all the cases.
First of all, we exclude the impossible ones:

$$\textbf{A)}\ \begin{cases} \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(1-s)\end{cases}\\ \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(\overline{s})\end{cases}\\ \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(1-\overline{s})\end{cases}\end{cases}$$

$$\textbf{B)}\ \begin{cases} \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(1-s)\end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(1-s)\\ \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(\overline{s})\end{cases} & \\ \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(1-\overline{s})\end{cases} & \end{cases}$$

$$\textbf{C)}\ \begin{cases} \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(1-s)\end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(1-s)\\ \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(\overline{s})\end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(\overline{s})\\ \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(1-\overline{s})\end{cases} & \end{cases}$$

$$\textbf{D)}\ \begin{cases} \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(1-s)\end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(1-s)\\ \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(\overline{s})\end{cases} & \\ \begin{cases}\zeta(z_j)=\zeta(s)\\ \zeta(z_k)=\zeta(1-\overline{s})\end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(1-\overline{s})\end{cases}$$

$$\textbf{E)}\quad \begin{cases} \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-s) \end{cases} & \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(\overline{s}) \end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(\overline{s}) \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-\overline{s}) \end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(1-\overline{s}) \end{cases}$$

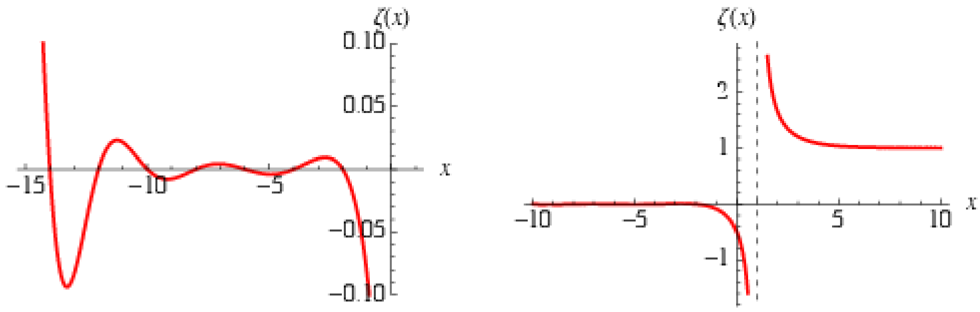
Now we study the possible cases:

$$\textbf{F)}\quad \begin{cases} \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-s) \end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(1-s) \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(\overline{s}) \end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(\overline{s}) \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-\overline{s}) \end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(1-\overline{s}) \end{cases}$$

The only solution of the case F is $z_j \equiv z_k \equiv \frac{1}{2}$. However, since $\zeta(\frac{1}{2}) = -1{,}46...$ and so $\neq 0$ **[2],** we can exclude it, because we will have no non-trivial zeros.

$$\textbf{G)}\quad \begin{cases} \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-s) \end{cases} & \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(\overline{s}) \end{cases} & \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(\overline{s}) \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-\overline{s}) \end{cases} & \end{cases}$$

Also this case can be excluded, since the zeta function has no zeros on the real line in the critical strip.
This graph of $\zeta(s)$, where s is a real number, shows it **[3]** :



H) $$\begin{cases} \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-s) \end{cases} \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(\overline{s}) \end{cases} \\ \begin{cases} \zeta(z_j)=\zeta(s) \\ \zeta(z_k)=\zeta(1-\overline{s}) \end{cases} \end{cases} \quad \wedge\, \zeta(z_j)\equiv\zeta(s)\equiv\zeta(z_k)\equiv\zeta(1-\overline{s})$$

The case H represents the Riemann hypothesis and, since the previous cases are impossible or excluded, the Riemann hypothesis is the only case in which the system ♦ has solutions; and this confirms our initial statement ●.

References:

[1] *Nicolò Rigamonti "Proposal of solution of the Riemann hypothesis": http://vixra.org/abs/1808.0193*
[2] *Wikipedia, Riemann zeta function: https://en.wikipedia.org/wiki/Riemann_zeta_function*
[3] *Wolfram MathWorld, Riemann zeta function: http://mathworld.wolfram.com/RiemannZetaFunction.html*