The Theories Of The Graviton
Part One: The Classical Framework of the Particle’s Nature and Mechanics

Noah M. MacKay\textsuperscript{1, a)} and Aaron M. Bain\textsuperscript{1, b)}

Physics Department, East Carolina University, Greenville, NC, USA

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Gravitons are the quantum particles of gravity that, if proven to exist, would potentially connect quantum mechanics with gravitation. This particular paper, however, mostly describes the particle’s nature and mechanics in the framework of classical physics: the same framework in which General Relativity was formulated. Although a classically-dominant perspective of gravitonic nature and mechanics, both quantum and classical physics are used to propose theorems, and to suppose two main considerations of the graviton: 1) The roles of the quantum graviton are vital in the overall nature of classical gravitation, 2) the particles contribute to the formation and potential energy of gravitational waves.

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INTRODUCTION

The graviton is a theoretical particle of the gravitational field. It was theorized by Dmitrii Blokhintsev and F. M. Gal’perin in 1934 as a way to state that a particle can harness the gravitational force - an attempt to solve gravitational quantum field theory\textsuperscript{[1]}. However, despite such theories that mathematically support the existence and creation of gravitons\textsuperscript{[2]}, there were (and still are) no experiments successful enough to discover the supposed massless, 2-spin tensor boson\textsuperscript{[3]}. Such a discovery would truly verify quantum gravity\textsuperscript{[3]}, thus constructing a potential bridge between gravitational and quantum mechanics. The discovery of gravitational waves in 2016 raised a considerable question: Could gravity have properties of wave-particle duality?

The main questions this entire paper is attempting to answer are: If the research in the near-future does indeed confirm the existence of the graviton, in what ways are gravitons important to the astro- and quantum physical phenomena of our universe, and what sort of mechanics are involved with this particle of gravity?

The reason why gravitons are of great importance is because quantum gravity will construct a new way of seeing the universe, where the force that holds the entire universe together is all because of these quantum particles, and that there would eventually be a theory of everything.

Beginning the paper is the classical description of the conceptual nature of graviton particles, proposing what they are, and how they influence the existence of classical gravitation. The section describes the fundamental hypothesis of gravitonic mechanics called gravitonic oscillation (how relativistic gravitonic motions create the gravitational field), while referring to the Theory of General Relativity and attempting to form a connection with loop quantum gravity. Following gravitonic oscillation is the description of particle flow and intensity, which is influenced by its oscillation.

The oscillator function of the graviton is later derived, which maps out the continuous motions of gravitons while in oscillation, along with the corresponding quantum wave function of the particle. After the wave function derivation follows the application of the Heisenberg uncertainty principle to gravitons. The graviton’s 2-spin mechanics is thereafter addressed; because this paper is classically-dominated, it is visualized that the full spin mechanics is the curl of the graviton’s Poynting Vector. It is possible to formulate the classical theorem in the quantum framework.

After analyzing the graviton as it behaves in a single-mass system, gravitonic interactions within a binary, either collapsing and stable/orbital, are reviewed, accounting for the classical interpretations of two revolving astronomical bodies. The classical conservations of angular momentum and energy are manipulated to satisfy the mechanics of all interacting gravitons between the masses, forming “gravitonic field energy,” which serves as the potential energy of the resulting gravitational waves.

I. THE NATURE OF GRAVITONS

In the previous theories that attempt to understand gravitation by using quantum mechanics, it is acknowledged that the Theory of General Relativity is formulated within the framework of classical physics.
Gravitons, as fundamental particles of gravitation, shall obey and be dictated by the laws of General Relativity as they (by nature of particles) obey and are dictated by the laws of quantum mechanics.

It is idealized in many other theories of the graviton that the particles have no mass or charge (to say that gravity itself is massless and chargeless), have a spin number of 2 (because the Stress-Energy Tensor of Einstein’s General Relativity Field Equation is a second-order tensor), and, having the nickname of the Blue Photon, that its natural speed is the speed of light \( c \sim 3 \times 10^8 \text{ m/s} \).

In this theory of the graviton, it is presumed that gravitons have a non-zero mass, although it is significantly small for it to be considered as zero. It is thought that gravitons indeed have 2-spin properties, have no charge, and that they travel at \( c \), for the speed of gravitation is also the speed of light\(^4\). Importantly, gravitons are attracted to the highest quantity and most proximate sources of mass and energy (which, according to Einstein’s Theory of General Relativity, are sources of classical gravity)\(^5\).

It was stated that the graviton shall be dictated by the Theories of Relativity. This speculative that its composition, importantly the particle mass and volume, is dependent on its natural speed and its motions influenced by a specific astronomical mass. This supports that the amount of gravitational strength is variant for every astronomical object, that the most massive mass has the highest strength of gravitation.

It is also thought that the graviton maintains its state throughout its lifespan. For gravitons are always interacting with astronomical masses, with other gravitons, and with other particles such as photons \(^5\), gravitons never decay. This is due to the acknowledgement that gravity can reach into places and depths light cannot exist; that gravity plays a role in the classical and quantum mechanics of all objects of volume, matter, and charge; and that fields of gravitation from one source can intervene with gravity fields from another.

In addition, because of interactions between gravitons and photons, gravitational and electrostatic mechanics are equal. Interactions with other gravitons is due to the hypothesis that, for gravitons react to energy, they self gravitate.

II. THE HYPOTHESIS OF GRAVITONIC OSCILLATION

The gravitonic oscillation hypothesis can be viewed as the emission of gravitons between two masses, particles, atoms, etc. However, this hypothesis refutes the consideration that gravitons are emitted from the objects themselves, just as electrons emit photons when they decrease in energy levels. The hypothesis proposes that gravitons are not formed by the quantum DNA of particles that are emitted to illustrate gravity, but rather pre-existing and influenced by mass and energy.

In the observable universe, gravitation exists, either in larger quantities or in lesser ones. Since gravitons are indeed the mediators of the gravitational force, then the fabric of space is supposed to be made of infinite amounts of graviton particles. These particles respond to energy, therefore stating they respond to themselves. Therefore, in the regions of the universe where there is little gravitation, the network of interacting gravitons would be resembled as (in 2-dimensional space - a single layer of three-dimensional space) a lattice web, resembling that the interacting gravitons are in stable equilibrium, thus becoming non-relativistic, and having no net energy. These interacting gravitons form bonds based on energy, and this can be resembled as rubber bands; the stable graviton bonds can be disfigured by any exterior source of mass and/or energy, such as passing-by gravitational waves causing the bands to warp like ocean waves.

The regions where there are larger gravitational quantities reside within a mass-energy system. As a macroscopic object with mass curves spacetime, the gravitons in the lattice, that are involved, endure a pseudo-pressure given off by the mass, causing the gravitons to become excited and build up a net energy. By doing so, these gravitons in the stable bonds are more attracted to the object with mass than they are to the other gravitons. If visualizing these energy bonds as rubber bands, the more tension applied to the band, the more likely it is for the bond to break. When the gravitons build up a maximal amount of potential energy, their energy bonds break, and the particles escape into space at the speed of light (thus becoming relativistic), due to the conservation of momentum.

However, each of the excited gravitons that escaped the planet’s pressure will have to interact with the stable gravitons in unbent spacetime. Therefore, the energy from the excited graviton will be transferred into the stable gravitons. This would slow down the excited graviton to a halt. Afterwards, the once-excited graviton has its energy transferred and is displaced from the planet curving spacetime. Because energy is essential for the existence of all matter, to say the displaced graviton has spent all of its energy, it would seize to exist. The graviton would need to be reenergized in order to remain a graviton.

The stable gravitons, initially having no net energy and having received transferred energy, now have a net energy. For the stable gravitons, the net energy must
be neutralized. Therefore, the stable gravitons give back the energy to the displaced graviton, making it exponentially energetic as it travels back to the mass curving spacetime. The motions repeat as the graviton regains potential energy from the stable gravitons and the mass curving spacetime. Therefore, the excited gravitons behave in a harmonic oscillator.

The hypothesis of gravitonic oscillation, however, implies that: 1) the spacial vacuum (a frictionless medium) is elastic, and that 2) the gravitons in oscillation will never reform a stable bond.

In regards to implication 1), in the Planck scale, the fabric of space (the lattice of stable gravitons) is elastic, but in the scale of astronomical masses, space remains inertial. Stable gravitons with no net energies are transparent to uniform-moving masses of much larger quantities, for these masses also bend spacetime (distort the graviton lattice). But relativistic gravitons have increased mass, stating the mass-graviton interactions bring rise to the gravitational force. Therefore, gravitonic elasticity does not affect astronomical masses, but relativistic gravitons influence the masses to endure gravitation.

To implication 2), if a rubber band ruptures by stretching it too far/hard, the band remains ruptured, and will never be rebound, unless tended to. In the case of oscillating gravitons, they not only interact with the stable gravitons in unbent spacetime, but also with other oscillating gravitons. This may explain quantum foam, that oscillating gravitons are interacting with each other. Therefore, if the oscillating gravitons continue to respond to each other, they will eventually reform the stable energy bonds they once had before being disturbed by the astronomical mass.

This results to two classifications for graviton particles: stable gravitons and oscillating gravitons. Stable gravitons are the non-relativistic particles in stable bonds with other gravitons in unbent spacetime. Oscillating gravitons are the relativistic particles that are unbound from stable equilibrium - yet bound to the influence of the astronomical mass. Their relativistic motions form the classical gravitational field.

The image above displays a computed simulation of the graviton oscillation hypothesis. The center is a singularity, on which the mass and energy is focused. The particles are the oscillating gravitons moving either away or toward the center. The white circle traces around the region where there are more excited gravitons. The distance from the singularity to any point of the circle’s edge marks the distance of maximal particle displacement. The simulation only recognizes the oscillating gravitons; the stable gravitons are neglected from the simulation, although some can be seen outside the circle.

III. OSCILLATING GRAVITONS AS “PATH STRANDS”

In the Planck scale, oscillating gravitons are described earlier as, namely, the particles oscillating within the gravitonic lattice. However, as time elapses, the observations of the graviton’s oscillation would instead be seen as a “path strand” from the mass-energy node in spacetime curvature, for the motions of the oscillating graviton are instantaneous.

The path strands do not refer in any way to the strings in string theory; they refer to the visualization of instantaneous gravitonic motions as hair-length strands. Since hair strands are mentioned and referred to, there are hair strands that are straight (irrotational) and there are hair strands that are curly (looped).

For gravitonic oscillation in a single-mass system, these strands would most likely be straight and irrotational, if not interacting with other oscillating gravitons, and that they only interact with the stable gravitons. This ensures that irrotational, relativistic gravitonic motions form the classical gravitational field, which is
also irrotational. Irrotational path strands also apply to the gravitons that are emitted from a collapsing binary due to formations of gravitational waves.

For gravitonic oscillation within a binary, each of the strands of one mass-system intertwines with each of the strands of the other mass-system. One of the Natural Hypotheses states that gravitons self gravitate, meaning, within a collapsing binary, the oscillating gravitons are attracted to each other (just as stable gravitons react to each other in unbent spacetime). But instead of forming stable bonds, the strands loop about each other and become tangled, which, as earlier stated, can be represented as quantum foam. This also applies to the scenario in which oscillating gravitons in the same single mass-system respond to each other.

For an increment of a graviton path strand $ds$, it is equal to the following metric: $dr + r \sin(\phi) d\phi$, where $dr$ is the linear tangent increment of the strand, and $\phi$ is the angle about the linear tangent $r$ - the projection angle of the strand increment. However, this equation is for 2 dimensions. To encompass 3 dimensions, the strand increment must be in spherical coordinates. By doing so, the strand increment $ds$ must be squared to form $ds^2 = dr^2 + 2r \sin(\phi) d\phi dr + r^2 \sin^2(\phi) d\phi^2$.

The strand metric expression is quite similar to the metric of a self-gravitating quantum string $ds^2 = dt^2 + dR^2 + dz^2 + e^{2\omega} d\phi^2$, where $e^{\omega} = r \sin(\phi)[\delta]$. The strand equation is not time dependent, for the motions of the oscillating graviton are instantaneous. The length of the quantum string $dz$ is identical to the linear tangent increment $dr$ (not to be confused with $dR$ in the string metric - the string’s inner radius), and $e^{\omega}$ is consistent in both string and strand equations.

It is hypothesized that gravitons are self-gravitating particles. Therefore, if the instantaneous motions of a graviton can be visualized as strands, then gravitonic strands can also be seen as self-gravitating strings. This is the closest attempt to form a connection with quantum loop theory.

IV. THE BASIS OF GRAVITONIC MECHANICS

In a system of masses, i.e. a binary or an orbital system, the largest of the masses has the strongest amount of gravitational strength, according to Newton - supported by Einstein. Therefore, the stronger the attraction of gravity, the more gravitons that are associated with the corresponding mass. Furthermore, considering proximity, gravitons that are close to any object with mass are part of the mass-system. This, classically, describes the gravitational field, that it is present for all objects with mass. Yet, for particles that reside between two masses of various quantities, the particle is drawn to the most proximate, relatively-heaviest mass past the Lagrangian Point, where the dualing gravity fields of the masses are in equilibrium (where oscillating gravitons are less energetic while in oscillation).

The graviton is the quantum counterpart of the classical field of gravitation; it must be seen that gravitons (as particles and as waves) do not disfigure through various forms of medium.

The hypotheses, theories, and models that follow are in the perspective of one particle in the most convenient dimension, on behalf of all particles in all possible dimensions.

V. GRAVITONIC FLOW AND INTENSITY

A. Gravitonic Flow

Usually, sources of high gravitational attraction are caused by objects with astronomical masses. As stated in Einstein’s Theory of General Relativity, gravity is the manifestation of mass and energy. Oscillating gravitons make up the spherical shell that is associated with gravitational attraction, meaning that per unit of distance these gravitons are from the astronomical mass, there is a varying amount of flow that brings rise to a form of flux.

According to the Gaussian Flux Theorem, the traditional equation of the flux of any field $f$ is expressed as:

$$\Phi_f = \oint f \cdot dA$$ (1)

Carl Friedrich Gauss, himself, calculated the equation of classical gravitational flux, which he derived it to be $\Phi_g = 4\pi Gm_1$.

However, the perception of gravitonic flux is different from that of gravitational flux. The one main assumption of Gauss’ derivation is that; the amount of gravitational attraction is constant from the center to the planetary surface, hence the usage of the constant gravitational acceleration in his derivation.

Gravitonic flux proposes that flux is variant; the farther away a graviton particle is from its associating mass, the less flux is present. The closer that particle is, the intense the flux. Therefore, if there is a variant flux, there is a variant level of gravitonic flow. As gravitons are the origins of the gravitational field, the varying flow of a graviton means a varying field of gravitation. The mass of the object is presumed to be constant, as well as its volume.

The Gravitonic Flux Identity is expressed as:

$$\Phi_T = \oint \frac{Gm_1}{\delta^2} \cdot dA$$ (2)
where $\delta$ is the instantaneous distance of a graviton from the object’s center. The integral is evaluated, resulting to the equation for Gravitonic Flux:

$$\Phi_T = \frac{4\pi G m_1 r^2}{\delta^2} \quad (3)$$

1. Maximal Distance of Gravitational Influence

Using the equation of the Gravitonic Flux, it is possible to decipher the maximal radial distance of gravitational influence. The one application to gravitational influence is within the heliosphere. In a great distance from a central star of a heliosphere, away from its gravitational attraction, there is still gravitational influence; objects that may be out of reach of the maximal distance of gravitational attraction are, nevertheless, under its influence. That is perhaps how and why planets many, many kilometers away from our Sun are in orbit around it.

To derive the equation of the maximal gravitational influence, the equation of gravitonic flux must be differentiated once, with respect to displacement. The equation is so:

$$I_T = \frac{8\pi G m r^2}{\delta^3} \quad (4)$$

This illustrates that the amount of gravitation has a domain of influence over a certain value of radial displacement. So that the farthest, certain value of displacement from an influencing object is where there is minimal influence, $I_T$ shall equal one. Mathematically, this does not jump to the conclusion that an infinite, undetermined distance away is the solution, by setting the equation equal to zero. Ideally, it gives a theoretical model on how far the edge of a heliosphere is from the central star.

Therefore, the equation of maximal distance of gravitational influence is:

$$\rho_I = \sqrt[3]{\frac{8\pi G m r^2}{I_1}} \quad (5)$$

Applying the Sun’s heliosphere to gravitational influence, using the Sun’s mass ($2 \times 10^{30}$ kg) and radius ($6.96 \times 10^8$ m), the distance of the Sun’s heliospherical boundary (theoretically) is $\rho_I = 1.18 \times 10^{13}$ m.

The image above is the graph of the Sun’s heliospherical boundary (bold circle). Therein, each of the planetary positions in the solar system are marked by the set of vertical lines (dotted are terrestrial, bold are gas giants - and Pluto). The hashed curve is the gravitonic flux curve.

The actual distance of the Sun’s heliosphere is measured to be 100AU ($1.5 \times 10^{13}$ m)$^6$, leaving an error percentage of 21.3 percent.

2. Maximal Distance of Gravitational Attraction

For any object with a mass and radius, there is a maximal value of displacement from deep space to that object’s center, along which a gravity-attracted secondary object travels, towards the attractive object. For this value of displacement encompasses the region of gravitational attraction, the displacement is therefore the maximum displacement for a relativistic graviton as it oscillates to and from a central mass-energy node curving spacetime.

To consider the maximal distance of gravitational attraction, the gravitonic flux equation must be differentiated twice, with respect to displacement, to have the Gravitonic Identity of Gravitational Field. It is seen as:

$$H_T = \frac{24\pi G m r^2}{\delta^4} \quad (6)$$

To decipher the farthest, certain value of displacement from an object, where there is minimal attraction, $H_T$ shall also equal one.
Therefore, the maximal distance of gravitational attraction is:

\[ \rho = \sqrt[4]{\frac{24\pi Gmr^2}{H_1}} \]  

(7)

B. Intensity Theory

The concept of gravitonic intensity is much like walking under a lamp post on a wet, reflective surface. While walking, the more the person approaches the light source, the more intense the light reflecting from the surface gets. When the walker stands directly underneath the light source, the light on the surface is at its most intense.

This concept, in terms of gravitons, is that the distance of \( \rho \) (regarding one mass) or the distance between two astronomical masses (\( r \leq \rho_1 + \rho_2 \)) can be cut into cross sections containing particles of different intensities. The farther away the cross section is from the mass-energy node, at a specific point some distance (\( \delta_1 \)) away from the heaviest of the masses, the less intense the gravitons are in that space. Whereas, the closer the cross section gets towards the mass-energy node, the more intense the gravitons become. If graphed, the intensity can be described as a bell curve, where the flat ends are where gravitons have minimal intensities, and at the peak is where gravitons have maximal intensities.

This specific point at distance \( \delta_1 \) within \( r \) is the Lagrangian Point: the point in space where the gravitational fields of each of the two masses are equal (where oscillating gravitons from each of the masses are less energetic while in oscillation). The distance of the Lagrangian Point within \( r \) depends on each of the binary masses, for \( \frac{r}{2} \leq \delta_1 \), if \( m_1 \geq m_2 \).

In a collapsing black hole/neutron star binary, the masses will collide at the center of mass, therefore proposing that the distance \( \delta_1 \) will decrease over the course of time. The following equations are derived to describe the volumetric space between two colliding masses in the simplest form. Each of the models assumes that mass and volume of each astronomical object are synchronously and directly variable, proposing a constant density.

1. The Shrinking Caterpillar Model

For two masses with the same volume creating a caterpillar-like cylindrical geometry,

\[ \frac{4\pi}{6} \left( \Gamma^3 + \lambda^3 \right) + \pi\Gamma\lambda\delta_1 \]  

(8)

In this model, the two radii \( \Gamma \) and \( \lambda \) are equal, and the diminishing distance \( \delta_1 \) is with respect to the first mass of radius \( \Gamma \).

2. The Snow Cone Model

For two masses with different volumes creating a snow cone geometry,

\[ \frac{\pi}{3} \left[ 2\Gamma^3 + 2\lambda^3 + \Gamma^2\delta_1 \right] \]  

(9)

In this model, \( \Gamma \) is the radius of the larger object (\( m_1 \)), hence having the radius \( \lambda \) being the smaller. Just like in the previous model, \( \delta_1 \) is decreasing over time.

3. The Density of the Cross Section

When dealing with a single mass in an independent system or in a binary, each cross section from the Lagrangian Point will have a varying amount of density, which is hypothesized by the Intensity Theory. In locations of low gravitonic intensity, the cross sectional density will be maximal based on the knowledge that gravitons near the Lagrangian Point have little energies while in oscillation. The equation of cross sectional density is seen as follows:

\[ P = \frac{m\tilde{g}}{c^2\alpha(\rho)} \]  

(10)

where \( m \) is the mass of the astronomical object, \( \tilde{g} \) is the classical gravitational field of the object (which the gravitons produce based on relativistic graviton-mass interactions), and \( \alpha \) is the thickness of the cross-section.

As a function of \( \alpha \), the density equation becomes a decaying curve seen below:
VI. GRAVITONIC OSCILLATION

A. The Mechanical Wave Function $\chi(\tau)$

In a single-mass system, the gravitons would oscillate. If the oscillating motion of a graviton were to be graphed based on time, it could be viewed as a sinusoidal function.

The generalized function would be viewed as:

$$\chi(t) = A_0 \sin(\omega t) + A_1 \cos(\omega t) \quad (11)$$

Recalling the Hypothesis of Gravitonic Oscillation, where gravitons begin oscillation from a node where mass curves spacetime and travels to a maximal distance $\rho$, the function of oscillation is a sine wave. Because the graviton cycles from acceleration to deceleration, the angular frequency, $\omega$, shall be viewed as the ratio between the linear velocity of the graviton particle $v$ and the wave constant $k$, where $k$ equal to $2\pi\lambda_o$, and $\lambda_o$ is the oscillator wavelength. Therefore,

$$\omega = \frac{v}{2\pi\lambda_o} \quad (12)$$

Because the time is within the perspective of a stationary observer, the time $t$ shall be rewritten as the Time Dilation Equation of Special Relativity. The time $t$ can further be simplified as:

$$t = \tau\gamma = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (13)$$

The amplitude $A$ is defined as the quotient of the linear velocity $v$ and the angular frequency $\omega$. Therefore, the amplitude $A$ is:

$$A = \frac{v}{\omega} = \frac{v}{2\pi\lambda_o} = 2\pi\lambda_o \quad (14)$$

Essentially, the amplitude $A$ is the radial wavelength, which must however equal the maximum distance of gravitational attraction.

$$\rho = 2\pi\lambda_o \quad (15)$$

Therefore, the oscillator wavelength $\lambda_o$ is derived as:

$$\lambda_o = \frac{\rho}{2\pi} \quad (16)$$

This derivation of the oscillator wavelength, although derived only to be eliminated from the sine function, suggests the dependency on the distance of maximal gravitational attraction. Astronomical masses calculate an unaltered oscillator wavelength of that belonging to a low-frequency EM wave. If gravitonic waves within oscillation were to be compared to an electromagnetic wave, that suffices the fact that we cannot visually see gravity as we see light, but we all nevertheless experience gravity as a field.

The oscillator wavelength can also make a rough, theoretical measurement of gravitational wave wavelengths. The graph of wavelength ($\lambda$) vs. strain ($h$) is given below\[7\].

For a black hole that was merged from a collapsing binary, the oscillator wavelength of the combined black hole (having a minimal mass of two stellar black holes $6M_\odot - (1 \text{ or } 2)M_\odot$, considering $E = mc^2$), the wavelength is well in the black hole binary (BHB) range, given that the maximal strain of the gravitational waves falls between $10^{-21}$ and $10^{-20}$. With the minimal mass, the oscillator wavelength can also be detected by LIGO.

Ultimately, the characteristics of the gravitonic wavelength within the oscillation function are insignificant, for $\lambda_o$ will be modified by the multiple of $\frac{1}{\gamma}$ or $\sqrt{1 - \frac{v^2}{c^2}}$. Also, the derivation of the oscillator wavelength was made only to be eliminated from the sine function, therefore focusing on the $\rho$ as the amplitude of the function.

The final equation of the sinusoidal function for an oscillating graviton is:

$$\chi(\tau) = \rho \sin\left(\frac{\tau v}{\rho \sqrt{1 - \frac{v^2}{c^2}}}\right) \quad (17)$$

where $\tau$ is the independent variable of time, and $v$ is the velocity for an oscillating graviton (which is the speed of light $c$. This, therefore, brings rise to the uncertainty principle for oscillating gravitons).

B. The Quantum Wave Function

This function is always applicable when a graviton particle is undergoing mechanical oscillation. As the particle moves in a straight line along $\rho$, it also oscillates in a position-dependent quantum wave function.
Graviton Theory, Part 1: The Classical Framework

The Time-Independent Schrödinger Equation is well needed to derive the Gravitonic Wave Function.

\[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x)\psi(x) = E\psi(x) \tag{18}\]

The starting factor of gravitonic oscillation is at the node where the astronomical mass curves spacetime. That is where gravitons become excited and build up a net energy: a potential energy. As the gravitons oscillate, their potential energy transfers into kinetic energy, indicating a potential variance. In conclusion, the potential function of the Schrödinger Equation must be some form of potential curve of the particle in oscillation.

1. **The Potential Function** \(V(x)\)

The functionality of \(V(x)\) must describe the variance in potential energy as the particle oscillates a mass system from the central node to the maximal distance \(\rho\). The Gaussian distribution curve best describes the potential variance of the oscillating particle, where energy build-up begins at the central mass-energy node, at the origin, and minimizes at the left and right measurements of \(\rho\).

The purpose of using a distribution curve to describe an energy function cannot be a mere coincidence. In reference to the Intensity Theory, when graviton particles are away from the mass of interest, there are low levels of intensity. As the particle gets closer to the mass, intensity increases. The roles of the Potential Function \(V(x)\) is not exclusive in the focus of gravitonic potential; the function plays a significant role in graphing gravitonic intensity.

The finalized form of the Gaussian Potential Function is:

\[V(x) = \left(\frac{2\pi\hbar g}{\gamma^* c}\right) e^{-\frac{\rho^2}{\sigma^2}} \tag{19}\]

where \(\gamma^*\) is the Lorentz Factor with a value of 26.889 (the speed that is used for the oscillating graviton is the exact value of the speed of light \(c^* = 299,792,458\) m/s, therefore having a Lorentz factor a defined value). The Potential Function for the graviton that oscillates in a single-mass system is seen as:

The curve given is the Gaussian Potential Function of a graviton oscillating under the influence of Earth, where the \(x\)-axis is displacement in meters, and the \(y\)-axis is the amount of potential energy in Joules \((U_{max} = 5.06 \times 10^{-33}\text{J} and \mu_{Earth} = 3.33 \times 10^7\text{m})\).

The amplitude of the curve is the classical gravitational potential \(U = mgh\), where \(h = \rho, g\) is the strength of the gravitational field that relativistic gravitons are forming, and the mass is that of the graviton. Using the energy of the photon \(E_\gamma = \frac{kE}{\gamma}\), and the oscillator wavelength \(\lambda_o\), the total energy of an oscillating graviton is \(E_\gamma = \frac{2\pi\hbar c}{\rho}\). Because the oscillating graviton is a relativistic particle, the graviton’s total energy is equal to the relativistic energy \(\gamma m_c^2\). Letting the Lorentz Factor be \(\gamma^* = 26.889\), the mass is therefore \(m_\gamma = \frac{2\pi\hbar}{\gamma^* \rho c}\).

This does imply that gravity has mass. However, the mass is significantly small to where mass is neglected. Regardless of the small mass, because oscillating gravitons are relativistic particles as they are quantum particles, their momentum is expected to increase as they travel at the speed of light \(c\), for particle-mass interactions are needed to form gravitation. Although it is understood that relativistic gravitonic motions results to gravitation, the classical strength of the gravitational field is proportional to the astronomical object’s mass: the one physical quantity that gravitons are drawn to, aside energy.

The standard deviation of the curve is \(\sigma = \frac{1}{4}\rho\). It is mentioned earlier that the Gaussian Potential has two roles: mapping out the potential energy of a graviton particle during oscillation, and graphically describing the Intensity Theory.

In reference to the graph above, in a spherical gravitational shell with diameter of \(2\rho\), it is likely to find a graviton with a high amount of intensity within the subshell of radius \(\frac{1}{4}\rho\) (diameter of \(\rho\)). Furthermore, it is highly probable to find the highest levels of gravitonic intensity in the subshell radius of \(\frac{1}{4}\rho\). The Standard Deviation of the Gaussian Potential is the region of displacement where a graviton particle with immense intensity can be found.
2. The Wave Function \( \psi_0(x) \)

Using the graviton mass in the Schrödinger Equation, and setting the wave function \( \psi(x) = e^{i\alpha x} \), the differential becomes:

\[
\frac{-\hbar c\gamma^* \rho}{4\pi^2 4\pi} \alpha^2 + V(x) = E
\]

(20)

The exponent of the \( \psi \) eigenfunction is therefore:

\[
\alpha = i 4\pi \sqrt{\frac{\epsilon\pi}{\hbar c\gamma^* \rho}}
\]

(21)

where \( \epsilon = E - V(x) \). This turns the \( \psi \) eigenfunction into a sinusoidal oscillator function, using Euler mathematics. Therefore, the generalized function of the wave function is:

\[
\psi(x) = A_0 \sin\left(4\pi x \sqrt{\frac{\epsilon\pi}{\hbar c\gamma^* \rho}}\right) + A_1 \cos\left(4\pi x \sqrt{\frac{\epsilon\pi}{\hbar c\gamma^* \rho}}\right)
\]

(22)

For normalization purposes, the constants inside the sine/cosine functions become:

\[
\frac{n\pi}{\rho} = 4\pi \sqrt{\frac{\epsilon\pi}{\hbar c\gamma^* \rho}}
\]

(23)

Through this, the matter wave of a graviton inside the gravitational shell of radius \( \rho \) is a sine wave just as the standing wave on a string is a sine wave. In the beginning of oscillation, the particle must already be in equilibrium at a node before it begins its quantum oscillation. Thus, the generalized function is only a sine function. Using the boundaries from 0 to \( \rho \), the normalization of the graviton’s wave function is:

\[
1 = \int_0^\rho A_0^2 \sin^2\left(\frac{n\pi}{\rho} x\right) dx
\]

(24)

The amplitude is derived to be \( A_0 = \sqrt{\frac{2}{\rho}} \), and the finalized quantum wave function of the graviton particle in oscillation is:

\[
\psi_0(x) = \sqrt{\frac{2}{\rho}} \sin\left(\frac{n\pi}{\rho} x\right)
\]

(25)

The function of the graviton is similar to the wave function of a particle in the square well. Imagining the radial distance from zero to \( \rho \) being the length of a box, the probability of finding an oscillating graviton within \( \rho \) is one.

3. The Energy Difference \( \epsilon \)

The difference in energy \( \epsilon = E - V(x) \) becomes:

\[
E - V(x) = \frac{n^2 \hbar c\gamma^*}{16\pi \rho}
\]

(26)

Therefore, the Hamiltonian definition of the energy difference is:

\[
E = \frac{n^2 \hbar c\gamma^*}{16\pi \rho} + \left(\frac{2\pi \hbar g}{\gamma^* c}\right) e^{\frac{-x^2}{\sigma^2}}
\]

(27)

Applying the boundaries of zero and \( \rho \) into the Potential Function, it is possible to verify if the quantum number \( n \) varies as the particle oscillates within gravitational subshells. The subshells are: \( x = \rho, \frac{1}{2} \rho, \frac{1}{4} \rho \) (or \( \sigma \), in reference to the Potential Function’s standard deviation), \( r \), and 0.

After calculating the variances in the potential function, it is found that there is no variance in quantum number, for the potential energy is already a significantly small value. Because of this, the potential variance does not affect the kinetic energies of the vibrating graviton. Therefore, the energy eigenvalue is always equal to the vibrational kinetic energy \( E_v = \frac{\hbar c\gamma^*}{16\pi \rho} \), no matter the location of the graviton while in oscillation.

However, the quantum vibrations of an oscillating graviton are less energetic than the graviton’s standard (linear) energy \( E_\ell \). Using the conservation of relativistic energy, setting the vibrational energy as the kinetic energy, and the standard energy as the rest energy, the vibrational energy has 91.5 percent less energy than the standard energy. This calculates that gravitons vibrate at the fixed velocity of \( 3.9 \times 10^7 \) m/s while in oscillation, or 0.13c.

For relativity is in effect in the vibrations, although not significantly, the overall speed of an oscillating graviton is 0.93c, taking account of the linear velocity of c and the vibrational velocity of 0.13c. The overall speed is close to the speed of light itself; the speed of light can still be acknowledged as the natural speed of the graviton, although caution should still be exercised.

For the vibrational energy never changes while in oscillation, the quantum number of the graviton is always \( n = 1 \). Therefore, the azimuthal number \( (l = n - 1) \) is always \( l = 0 \).

C. The Grand Wave Function \( \Psi(x, \tau) \)

Using superposition, the grand wave function of the graviton particle is:

\[
\Psi(x, \tau) = \chi(\tau) + \psi_0(x)
\]

(28)

VII. GRAVITONIC UNCERTAINTY

For there are two classifications of gravitons, there are two ways to carry out gravitonic uncertainty. The first shall mention the uncertainty of oscillating gravitons; the second shall mention the uncertainty of stable gravitons.
A. The Uncertainty of Oscillating Gravitons

For oscillating gravitons, two velocities will be applied to the oscillation function: the linear speed of \( c \), and the overall speed of 0.93c, even if it was addressed that the linear speed can still be viewed as the natural speed.

Inserting \( c \) in place of \( v \) into the Mechanical Oscillation Function results to:

\[
\chi(\tau) = \rho \sin \left( \frac{ct}{\rho \sqrt{1 - \frac{v^2}{c^2}}} \right) \quad (29)
\]

\[
\chi(\tau) = \rho \sin (\infty) = \rho [0 < x < 1] \quad (30)
\]

Using the overall relativistic speed of 0.93c to replace \( v \), the result is:

\[
\chi(\tau) = \rho \sin \left( \frac{0.93c\tau}{\rho \sqrt{1 - (0.93^2)}} \right) \quad (31)
\]

\[
\chi(\tau \rightarrow \infty) = \rho \sin (\infty) = \rho [0 < x < 1] \quad (32)
\]

Therefore, in both scenarios, there is an uncertainty to the particle’s true value of position as the graviton particle oscillates within a mass-energy system over time - much like the Heisenberg Uncertainty of the electron within an atom.

\[
\Delta x \Delta p \geq \frac{\hbar}{2} \quad (33)
\]

B. The Uncertainty of Stable Gravitons

For gravitons in the stable lattice, their uncertainties are not located based on position and momentum. Instead, it is based on energy and time. Although stable gravitons do not have a net energy, the quantum vibrations within their energy bonds suggest otherwise. These gravitons are not affiliated with an astronomical object with mass \( m \) and radius \( r \). Therefore, these vibrations are how the stable gravitons maintain a zero net energy; by neutralizing an uncertain amount of energy given by an interacting graviton over an increment of time.

The time-energy Heisenberg uncertainty is describing the uncertainty of stability within a two-graviton bond.

\[
\Delta E \Delta t \geq \frac{\hbar}{2} \quad (34)
\]

In a single stable bond between two gravitons, quantum fluctuations of vibrational energy are present. Representing a standing wave, these vibrations are the actions of zero net energy maintenance, each having a wavelength \( \lambda \) separating the two gravitons. In essence, the wavelength of fluctuation is the length of the stable bond between the two particles.

A two-graviton energy bond can be envisioned as a single polymer band of elastic energy. For there is no net energy in the graviton bond, the following equivalence must be true:

\[
\frac{2\hbar c}{\lambda} = \frac{1.3k_b T}{2 N\hbar^2 \lambda^2} \quad (35)
\]

where \( N = 2 \) for two gravitons, \( b = x = \lambda \) for the bond length, and \( k_b T = 2.348 \times 10^{-4}\text{eV} \), as the temperature of the system is the universal temperature 2.725K.

The length of the energy bond shall be constant, for the equation of the length is composed of constant values, given that the universe maintains a constant temperature. Therefore,

\[
\lambda = \frac{8 \hbar c}{3 k_b T} = 0.019\text{m} \quad (36)
\]

For quantum particles, this length is very large in their scale. Gravitons, as rest particles, possess large amounts of energy, to which other gravitons are attracted. In the gravitonic lattice, because gravitons respond to each other, it requires energy to stretch out any deuterium or tritium graviton cluster into two or three singular gravitons separated by an energy bond of a constant length \( \lambda \). This forms only a small fragment of the gravitonic lattice, for four-dimensional spacetime is made of infinite layers of gravitonic lattices.

For stable gravitons are rest particles, a rest mass should be calculated. Gravitons, previously thought to be massless, should have a mass whose value is significantly small to be neglected as zero, as it is for the mass of an oscillating graviton. The mass of oscillating gravitons was previously derived while solving for the Gaussian Potential, so that \( m_{\gamma} = \frac{\pi \hbar}{c \sqrt{2} \rho} \). For oscillating gravitons and stable gravitons are different in stability, the oscillating graviton mass does not apply to stable gravitons. As rest particles, \( m_{\omega}c^2 = \frac{h c}{\lambda} \), using \( \lambda \) as the bond length, so that:

\[
m_{\omega} = \frac{h c}{\lambda c} = 6.53 \times 10^{-11}\text{MeV} \quad (37)
\]

To solve the uncertainty principle for stable gravitons, the wave function of their energy fluctuations is needed to be derived. Therefore, the Time-Independent Schrödinger Equation is revisited. However, the only difference between the quantum vibrational wave function earlier derived and this wave function (aside the energy difference) is the mass. The same procedures were taken to derive the wave function for energy fluctuations, seen as:

\[
\Psi_B(x) = a \sin \left( \frac{\pi}{\lambda} x \right) \quad (38)
\]

where \( a = 10.26 \text{ m}^{-1} \).

The energy difference \( \epsilon \), which is also equal to the energy fluctuation \( \Delta E \), is calculated to be:

\[
\epsilon = \Delta E = \frac{\hbar^2}{8\lambda^2 m_{\omega}} = 8.16 \times 10^{-6}\text{eV} \quad (39)
\]
For the fluctuation range is very small, per stable bond between two gravitons, an uncertain amount of energy with neglected fluctuation is put against one particle by the other. Each of the stable gravitons has to neutralize this unknown value of energy in order to maintain its stability. Using the time-energy Heisenberg uncertainty, the amount of time it takes to neutralize the energy is:

\[
\Delta t = \frac{\hbar}{2\Delta E} = 4.033 \times 10^{-11} s \quad (40)
\]

This supports the resulting hypothesis regarding gravitonic oscillation, that (in the Planck scale) the graviton lattice is elastic, whereas (in the scale of astronomical masses) space remains inertial. The elasticity of gravitons neutralizes an uncertain amount of energy in less than one second. This time interval is beneficial for the deceleration of gravitons in oscillation moving linearly at c. In the scale of astronomical masses, the objects would not experience the resistance from the graviton’s energy bonds. Therefore, the spacial vacuum remains inertial.

VIII. GRAVITONIC SPIN

It is stated earlier that the azimuthal number of gravitons is always \( l = 0 \), in reference to the proof that \( n \) is always equal to 1. Therefore, this allows the explicit focus on only the graviton’s 2-spin when reviewing the particle’s total angular momentum \( j \).

In the case of gravitons, \( j = s \).

A. Identifying the Spin

One of the Natural Hypotheses state that, because of the interactions between gravitons and photons, gravitational and electrostatic mechanics are equal. Therefore, to describe the 2-spin mechanics of the graviton, the mechanics of gravitoelectromagnetism is adopted, and one of the four Maxwell-like equations must be applied: the curl of the gravitoelectric force field.

Like spinning electrons within a current having rotational magnetic fields, gravitons behave as such - letting the graviton move while in its oscillation and having a curved vector describing its complete intrinsic spin. When imagining a graviton as a stationary object at an arbitrary moment of time, its intrinsic motions are visualized as a planet rotating about its axis, for gravitoelectromagnetism (the proposed approach to interpret gravitonic spin) best applies to rotating astronomical masses.

To keep a consistent relation between classical and quantum gravitation, because gravity is an irrotational conservative field, the curl of the gravitational field is always equal to zero. This implies that the magnitude of the graviton’s curl is non-existent as well. The graviton’s curl is brought to rise due to its quantum spin - one of the fundamental properties of all particles. To disregard quantum spin altogether is impulsive. If spin cannot be neglected, then the classical interpretation of a zero-curl for gravity fields is moot, unless it is defined (in terms of quantum mechanics) why it is seen as zero.

It is hypothesized that, as the graviton - the quantum mediator of the gravitational force - oscillates (forming the gravitational field), it has a distinctive Poynting vector, which is influenced by the varying amount of intensity. The graviton’s Poynting vector is labeled as \( \vec{S}_h \), which shall be identical to, if not directly proportional to, the classical gravitational field \( \vec{g} \).

B. The Full-Spin Mechanics

Although spin is a quantum property of particles, the 2-spin mechanics of the graviton will be described using the framework of classical physics. A quantum mechanical interpretation of the following theorems is possible to derive.

Adopting the curl of the gravitoelectric field\[8][9], and applying the Graviton Poynting vector \( S_h \) in the stead of the gravitoelectric field, the result is the time differential of half of the Lense-Thirring precession rate:

\[
\nabla \times S_h = -\frac{\partial}{\partial t} \frac{G\vec{L}}{2c^2r^3} = \frac{-G}{2c^2r^2} \vec{F} \sin(\theta) \quad (41)
\]

where \( r \) is the radius of the particle, and \( \vec{F} \) is the centripetal force of the graviton (the presence of the sine function indicates torsion).

\( r \) can be derived by applying centripetal acceleration, the kind of acceleration the particle is undergoing as it spins. Using the gravitational acceleration \( g \) in place of \( a \), for gravitons self-gravitate (as stated in the Natural Hypotheses), and using the oscillating graviton mass, the radius of the oscillating graviton is derived to be:

\[
r = \frac{2\pi G\hbar}{\gamma \rho c^3} \quad (42)
\]

After rewriting the curl equation in terms of gravitonic mass and radius, the result is the halved-square of the gravitonic frequency multiplied by the negative sine of the angle of projection:

\[
\nabla \times S_h = -\frac{c^2}{2r^2} \sin(\theta) \quad (43)
\]

For the spin number of the graviton is 2, there are 5 projection values of \( m_s \), with respect to the axis of motion: \( \pm 2h, \pm h, 0 \). The ratio between the projection of spin and the spin magnitude \( |\vec{S}| = h\sqrt{6} \) becomes the ratio between a specific \( m_s \) value and the value of \( \sqrt{6} \). Because this ratio is the same as the negative sine of the
angle of projection, this equivalence holds true:
\[
m_s \frac{f^2}{\sqrt{6}} = -\frac{c^2}{2r^2} \sin(\theta)
\] (44)

Therefore, the revised curl equation is expressed as follows:
\[
\nabla \times \left| \mathbf{S} \right| = \frac{c^2}{2r^2} m_s \frac{\sqrt{6}}{\sqrt{}}
\] (45)

and the frequency of the spinning graviton is:
\[
f = \frac{c}{r \sqrt{2}}
\] (46)

To support the classical claim that the curl of the conservative gravitational field is zero, for a \( m_s \) projection value of zero, the entire curl is therefore equal to zero.

In the macroscopic perspective, the gravitational field is a dull field that is straight, pointing from all directions towards the center of an astronomical mass. As the perspective becomes quantized in the Planck scale, the classical gravitational field becomes the Gravitonic Poynting vector field with a defined curl.

C. The Spin Probability Function \( \Phi(r, \theta) \)

As the graviton oscillates, forming the gravitational field, at \( \chi(\tau) + \psi(x) \), it is spinning about the axis of motion at \( \nabla \times \mathbf{S}_h \). To describe the overall motion of a graviton particle over the course of displacement and time change, it undergoes intrinsic motions, as the particle moves in a straight line.

This further means that the curl is based on the perspective of the observer, since it was derived that the curl is dependent on the projection value. If the reference is that of the graviton, it cannot distinguish its own projection; it thinks there is no projection. To observers outside the graviton’s reference, they can differentiate the projection of the particle, therefore proposing that there are instances where the graviton’s full-spin mechanics is variant based on orientation.

The purpose of the Spin Probability Function is to determine the percentage of magnitude of the graviton’s full-spin mechanics in the perspective of an outside observer. To consider the graviton’s intrinsic motion, the variables are \( r \), the radius with a constant magnitude, and \( \theta \), the rotational angle. Disregarding the angle of axial projection \( \phi \) (treating it as \( 0^\circ \)), the best way to describe gravitonic spin as a function is as follows:
\[
\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} U_r + \frac{1}{r} \frac{\partial}{\partial \theta} U_\theta = \partial \Phi(r, \theta)
\] (47)

Being that the circular spin motions have no drastic changes, for the only variables present are the radial variable and the angular variable, the differential equation that describes the graviton spin is equal to zero. Thus, after using the separable method and the Fundamental Theorem of Calculus, the resulting multi-variable function is:
\[
\frac{r}{U_r} = -\frac{1}{U_\theta \sin(\theta)}
\] (48)

Hence, two functions arise from this equivalence: \( r(\theta) \), and \( \Theta(r) \). It is hypothesized that the full-spin function \( \Phi(r, \theta) \) is the product of the two functions, which are:
\[
r(\theta) = -\frac{U_r}{U_\theta} \csc(\theta)
\] (49)
\[
\Theta(r) = \sin^{-1}\left(\frac{-U_r}{U_\theta} \frac{1}{r(\theta)}\right)
\] (50)

For the function is of spin probability, the function \( \Phi(r, \theta) = r\Theta \), like all probability functions, must be normalized. The set-up of the \( \Phi \) function is as follows:
\[
\Phi(r, \theta) = r\Theta = \left(-\frac{U_r}{U_\theta} \frac{1}{r(\theta)}\right)
\] (51)

Now as a function with only one variable \( \theta \):
\[
\Phi(\theta) = -\frac{U_r}{U_\theta} \csc(\theta) \sin^{-1}(\sin(\theta))
\] (52)

Therefore, the ratio \( -\frac{U_r}{U_\theta} \) must be some normalizing constant:
\[
-\frac{U_r}{U_\theta} = \mathbb{N}
\] (53)

The spin probability function \( \Phi(r, \theta) \) is rewritten as follows:
\[
\Phi(\theta) = \mathbb{N} \csc(\theta) \sin^{-1}(\sin(\theta))
\] (54)

This normalizing constant \( \mathbb{N} \) has a numerical value of 0.6366 meters-per-radian, and the complete function is graphed as shown below:
The function $\Phi(\theta)$ verifies that the magnitude of maximum spin probability is within the angles that make $\sin(\theta) = \pm 1$, which are the projection values $m_s = \pm 2$. There are four points of interest listed on the graph, which indicates four degrees of freedom for gravitonic spin orientation. The projection value $m_s = 0$ is the restriction.

**THE MECHANICS OF GRAVITONS WITHIN A BINARY**

In either circumstance where two revolving masses cause gravitational waves, or are in orbit about each other, graviton particles are always interacting with each of these two masses. The mechanics of each individual graviton in a single-mass system are now in the perspective of two masses, therefore proposing how all interacting gravitons contribute to: 1) the angular momenta of a binary (either collapsing or orbital), and importantly, 2) gravitational wave formation.

Above is a simulated image of binary mechanics in terms of classical angular momentum and graviton interactions. The two objects of assumed-identical mass revolve about the center of mass, which in this case is half of the distance between them. They trace a circular field in space, within which are oscillating gravitons interacting with each other. This circular field shall be dubbed as either the Graviton Field or the Graviton Mesh.

It is within this field/mesh that gravitons, which are conflicted due to the rapid motions of the masses and changing domains of attraction, contribute to gravitational wave formation through their systematic energies, which are expected to vary in quantity per graviton within the field due to their intensities (which in retrospect relies on the mass and energy of an astronomical object).

**IX. THE CONSERVATION LAW OF ROTATIONAL AND QUANTIZED MOMENTA**

The total angular momentum of the binary equals the sum of the two individual momenta, which varies upon the radial distance of the center of mass and the orbital velocity. To illustrate that the classical conservation of momentum can transcend into the angular momenta of all graviton particles, this sum of astronomical momenta shall be equal to the sum of all quantized momenta of the gravitons within, seen as $\Sigma \eta \hbar$, letting $n$ equal 1, and $\hbar$ is the reduced Planck constant $1.055 \times 10^{-35}$ J · s.

The conservation of momentum equation looks as so:

$$\frac{\delta}{2} (m_1 \nu + m_2 V) = \sum_{\eta=1}^{\Upsilon} \eta \hbar$$

where $\delta$ is the instantaneous distance between the two masses at a certain moment of time (initially at measurement $r$) coming from the time equation of orbit decay of binaries, $\nu$ and $V$ are the instantaneous orbital velocities of each of their corresponding masses, and $\Upsilon$ is the maximum number of gravitons in between the masses.

Rewriting the sum as an integral from zero to $\Upsilon$, the equivalence between sum and integral looks as so:

$$\sum_{\eta=1}^{\Upsilon} \eta \hbar \equiv \int_0^{\Upsilon} \hbar k$$

where the differential notation of $dk$ is that of particle amount.

Therefore, the conservation law of momenta is also expressed as

$$\frac{\delta}{2} (m_1 \nu + m_2 V) = \int_0^{\Upsilon} \hbar k$$

**X. QUANTITATIVE LAW**

Quantitative Law is the theory behind the calculation of a precise, finite number of graviton particles in a collapsing binary or orbital system, without being restricted by three variables and the constant angular momenta of the two masses. The instantaneous number of gravitons in a binary is dubbed $\Upsilon_\eta$.

**A. The Value of $\Upsilon$ for Orbital Masses**

Applying to orbital masses with a near-steady orbital velocity and displacement of masses, to solve $\Upsilon$, the
quantum momentum integral must be evaluated, and the number of gravitons in an orbital system of two masses is equal to:

\[ \Upsilon_D = \frac{r}{2\hbar} (m_1 v_1 + m_2 v_2) \]  

(58)

### B. The Beta Factor

The Beta Factor is a very simple and elegant equation that helps in the calculation of the ever-changing maximum number of gravitons between the two masses, which are getting closer and closer together until the distance reaches zero. This can be represented by the following limit:

\[ \beta_T = \lim_{\delta \rightarrow 0} \frac{\delta}{2\hbar} \]  

(59)

where \( \delta \) is the instantaneous distance between the two objects, and \( \hbar \) is the quantized momentum with \( n = 1 \). The momentum is multiplied by two, as there are two astronomical masses the graviton particles are interacting with.

This equation shows that, as the distance approaches some finite number, the number of particles also becomes a finite number.

### C. The Quantitative Expression

The definition and derivation of the Quantitative (Q, for later use) Expression is that Quantitative Theory can also be sufficed by the masses of the two colliding objects. As the Natural Hypotheses state: the heaviest mass has the most amount of oscillating gravitons.

The derivation of the expression sets the goal of neutralizing the units of inverse-Newton-seconds. Being that inverse-Newton-seconds are the units of the Beta Factor, and Newton-seconds are the units of impulse, the expression is written as:

\[ \beta * F \Delta t = Q \]  

(60)

where \( \beta = \frac{\delta}{2\hbar} \) and \( Q \) is the unitless Quantitative Solution.

The force of the impulse, when dealing with the interactions between astronomical masses, must be that of gravitation, and the time interval of the impulse must be expressed by the form of Special Relativistic Time Dialation. The expression is rewritten as:

\[ \frac{\delta}{2\hbar} \frac{G m_1 m_2}{\delta^2} \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}} = Q \]  

(61)

The next mystery to solve is the velocity of the particle in the Lorentz Transformation Factor. It is also thought that, even though the natural speed of the graviton is the speed of light, gravitons in the mesh have conflicted velocities. Meaning that particles in certain positions within the circular field have varying velocities due to the variance in intensity.

To solve for the “Model Velocity,” a parameter that must be derived only to insert into the Lorentz Factor, the Q Expression shall be written in forms of differentials, like so:

\[ \frac{G m_1 m_2}{2 \hbar} \frac{dt}{dx} = dk \]  

(62)

Being that the Q Solution is attempting to solve the Quantitative Theory, Q is therefore equal to the instantaneous number of gravitons over the course of binary collapse. The differential of particle amount is \( dk \).

Reorganizing this expression in terms of velocity, or \( \frac{dx}{dt} \), it is resulted to:

\[ \frac{G m_1 m_2}{2 \hbar k} = \frac{dx}{dt} \]  

(63)

Therefore, the “Model Velocity” of the Lorentz Factor is:

\[ v = \frac{G m_1 m_2}{2 \hbar \Upsilon} \]  

(64)

The Q Expression shall now be rewritten as:

\[ \Upsilon \sqrt{1 - \left( \frac{G m_1 m_2}{2 \hbar} \right)^2} = \frac{G m_1 m_2 \Upsilon}{2 \hbar \delta} \]  

(65)

The expression is further derived into:

\[ \Upsilon^2 = \left( \frac{G m_1 m_2}{2 \hbar} \right)^2 \left( \frac{\tau^2}{\delta^2} + \frac{1}{c^2} \right) \]  

(66)

The Q Solution in its entirety shall be therefore:

\[ \Upsilon = \left( \frac{G m_1 m_2}{2 \hbar} \right) \sqrt{\frac{\tau^2}{\delta^2} + \frac{1}{c^2}} \]  

(67)

### D. The Value of \( \Upsilon \) of a Collapsing Black Hole Binary

Considering the Quantitative Solution, and applying it to a collapsing black hole binary, the time-distance ratio in the square root becomes the ratio between the time of merger and the distance of merger \( \tau \), which are related by the time equation of orbit decay of binaries\(^{[10]}\). The time-distance ratio results to:

\[ \frac{5}{256} \frac{c^5}{G^3} \frac{r^3}{(m_1 m_2) (m_1 + m_2)} \]  

(68)

Ignoring the inverse-square of the speed of light, for it is an infinitely small constant, the Solution becomes:

\[ \Upsilon = \left( \frac{G m_1 m_2}{2 \hbar} \right) \frac{5}{256} \frac{c^5}{G^3} \frac{r^3}{(m_1 m_2) (m_1 + m_2)} \]  

(69)
Therefore, with a varying values of displacement of merger over time, the value of gravitons in a collapsing black hole binary is equal to:

\[
\Upsilon_B = \frac{5}{2\hbar 256 G^2} \frac{c^5 \delta^3}{(m_1 + m_2)}
\]  

(70)

E. The Displacement of Binary Merger \( r \)

Setting the two Upsilon values of Orbital Masses and Collapsing Binaries (where distance is constant) equal to each other, the measurement of the displacement of binary merger \( r \) can be calculated.

\[
\frac{r}{2\hbar} c (m_1 + m_2) = \frac{5}{2\hbar 256 G^2} \frac{c^5 \delta^3}{(m_1 + m_2)}
\]  

(71)

Therefore, the initial displacement of binary merger is:

\[
r = \frac{16 G (m_1 + m_2)}{\sqrt{5} c^2}
\]  

(72)

XI. GRAVITONIC ENERGY

In the circumstance where two masses of a collapsing binary are revolving vigorously about the center of mass, gravitational waves are being formed. As stated before, the gravitons are drawn to regions of mass and energy. Therefore, as the two masses circle around each other, the gravitons undergo two forms of motion: rotational and linear motion. Gravitons are bound to a linear kinetic motion as it oscillates. The true (quantum) energy of the gravitational waves resides in the systematic energies of all interacting gravitons, which behave as a combined potential energy.

A. The Gravitonic Field Equation

As the equations of kinetic energy are set up in order to derive the equation of the Gravitonic Field, the moment of inertia of the region of all interacting gravitons must be considered to be that of a circular disk.

Yes, the rotating masses separated by some distance may appear as a dumbbell. However, the gravitons inside the two-dimensional area between the two masses represent a disk. The gravitons themselves, as non-energetic rest particles, essentially have no mass, but in classical mechanics mass is required to have energy. Therefore, it is assumed that mass from an astronomical object is distributed along a region bounded by some displacement from that object as rotation becomes more rigorous.

This said-displacement is ultimately the displacement from the center of mass[11][12].

of the rotating disk indicates that the disk’s radial distance is this distance of the center of mass, where the masses must merge. Therefore, the moment of inertia of half of the gravitonic field is that of half of a disk.

Using the initial displacement measurement \( r \), the center of mass is described as:

\[
c.m. = \frac{m_2 (r)}{m_1 + m_2}
\]  

(73)

where \( m_2 \leq m_1 \).

This forms two important values of initial displacement per mass: \( \Gamma \) for the first mass, and \( \Lambda \) for the second.

\[
\Gamma = c.m.
\]  

(74)

\[
\Lambda = r - c.m.
\]  

(75)

Lastly, the angular velocity of an individual graviton must also be considered. Although the linear velocity would be the speed of light \( c \) as it oscillates in a single-mass system (verified by deriving the equations for full-spin mechanics), within a mesh of graviton particles, each of the particles has conflicted velocities (varying velocities of dislocated particles due to their varying intensities). The net velocity of a single half of the gravitonic field would be the sum of all conflicted velocities as fractions of the speed of light \( c \), which would be the escape velocity \( v_e = \sqrt{\frac{2Gm}{R}} \). The logic behind applying the escape velocity as the sum of all fractions of \( c \) consists of the following postulate:

In a single-mass system, or in one half of the binary, for the speed of gravitation is the speed of light (due to gravitonic mechanics), and each massive object has a distinctive value of gravitational strength, the speed in which to escape gravitational strength must also be a factor of \( c \) - a factor of \( c \) whose coefficient is between 0 and 1. This speed shall be the escape velocity, i.e. the escape velocity of a black hole is the speed of light itself, and the escape velocity of Earth is \( 1.19 \times 10^4 \text{m/s} \), or 0.00004c.

This postulate implies that binary masses, whose individual values of gravitational strength is not immensely strong compared to a neutron star or a black hole, do not form energetic gravitational waves like black holes/neutron stars in a collapsing binary. It links to the idea that orbital systems do not form detectable gravitational waves.

These individual equations are:

\[
K_r = \frac{1}{2} I \omega^2
\]  

(76)

\[
\omega = \sqrt{\frac{2Gm_n}{N}}
\]  

(77)
where $N$ equals $\Gamma$ if $n = 1$, or $\Lambda$ if $n = 2$.

\[ I = \frac{1}{4}m_n\eta^2 \quad (78) \]

where $\eta$ is $\gamma$ if $n = 1$, or $\lambda$ if $n = 2$; they are instantaneous disances of their initial counterparts. If the two revolving masses are in a collapsing neutron star or black hole binary, the $\eta$ will decrease as the objects get closer due to gravitational attraction.

When dealing with orbital masses, the ratio between $\eta$ and $N$, or $O$, is the value of 1.

Note that there are two masses in a binary; the total energy of the two-mass system must be the sum of the individual energies. Therefore, the sum of the two rotational kinetic energies will decipher the energy of a field of gravitons, hence making it appear as:

\[ E_\beta = \frac{G}{4} \sum_{n=1}^{2} \frac{m_n^2 O_n^2}{R_n} \quad (79) \]

Therefore, the equation above is that of gravitonic energy, where $G$ is the gravitational constant, $m_n$ is the mass of either object, $R_n$ is the mass’ respective radius, and $O_n$ is the Displacement Ratio.

This equation indicates the decrease of gravitonic energy as the masses come closer. This is due to the decrease of interacting gravitons as the two-mass system condenses into a single-mass system upon morphing. As the linear kinetic energy of the two revolving masses increases due to gravitational attraction, the gravitonic energy decreases.

However, in a two-mass system where black holes or neutron stars are forming gravitational waves, as the gravitonic energy decreases, it is being transferred into gravitational waves. Contributing to the decrease of interacting gravitons is the forceful emission of gravitons as the particle component of the gravitational waves. In a moment of time where the Displacement Ratio $O_n$ is near zero, the gravitonic energy is also near zero, but the energy of the gravitational waves is approximately the value of initial gravitonic energy. Gravitonic energy is conserved.

B. The Summation of All “Conflicted” Quantum Energies

The purpose of this summation is to connect the mathematical concepts of classical mechanics with those of the quantum world. Derived from the total energy of an oscillating graviton $E_\gamma = \frac{2\pi h\omega}{\rho}$, the energy is rewritten as:

\[ E = 2\pi h\omega \quad (80) \]

where the $\omega$ is derived from the angular velocity of the oscillation function but in terms of each of the half lengths $\gamma$ and $\lambda$, or generally $\eta$:

\[ \omega = \frac{\nu_n}{\eta \sqrt{1 - \frac{v_n^2}{c^2}}} \quad (81) \]

Therefore, the energy equation gives the energy of a single “conflicted” graviton particle traveling along an instantaneous displacement of either $\gamma$ or $\lambda$. Taking this knowledge, and applying the mathematics of the equation for the gravitonic field energy, the “conflicted” graviton energy is put into a sum of all individual energies of each of the interacting particles.

\[ \sum_{n=1}^{\gamma} \frac{2\pi h\nu_n c}{\eta \sqrt{c^2 - \nu_n^2}} = \frac{G}{4} \sum_{n=1}^{2} \frac{m_n^2 O_n^2}{R_n} \quad (82) \]

It is thought that, within a mesh of infinitely many gravitons, the quantum-scale particle energies are summed up to equal the graviton field equation. The sum-field equation equivalence looks like so:

\[ \frac{\gamma}{\eta} \sum_{n=1}^{2} \frac{2\pi h\nu_n c}{\sqrt{c^2 - \nu_n^2}} = \frac{G}{4} \sum_{n=1}^{2} \frac{m_n^2 O_n^2}{R_n} \quad (83) \]

C. The Schwarzschild Gravitonic Field Equation

When dealing with black holes, the radii $R_1$ and $R_2$ would be replaced with the Schwarzschild Radius $R_n = \frac{2GM_n}{c^2}$. This converts the gravitonic field energy into the “Schwarzschild” gravitonic energy:

\[ E_\beta' = \frac{c^2}{8} \sum_{n=1}^{2} m_n O_n^2 \quad (84) \]

XII. THE CONSERVATION OF GRAVITONIC ENERGY

It is mentioned earlier that gravitonic energy is conserved. In relation with the two equations of generalized gravitonic and Schwarzschild gravitonic energies, the conservation law looks as so:

\[ \frac{G}{4} \sum_{n=1}^{2} \frac{m_n^2 O_n^2}{R_n} - \frac{c^2}{8} \sum_{n=1}^{2} m_n O_n^2 = 0 \quad (85) \]

Because the rotation of masses is taking place in deep space, there is no form of frictional resistance. This conservation law assures that

1) the two forms of gravitonic energy are equal,
2) the energy within a field of gravitons (the circular area encompassed by the revolving masses) is converted to the energy of gravitational waves, and that
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3) the speed of light is equal to the escape velocity of a combined black hole.

\[
\frac{G}{4} \sum_{n=1}^{2} \frac{m_n}{R_n} \sum_{n=1}^{2} m_n O_n^2 = \frac{c^2}{8} \sum_{n=1}^{2} m_n O_n^2 \tag{86}
\]

Rewritting the summation as an extended sum, \((m_1 + m_2)\) is replaced with \(M\), and \((R_1 + R_2)\) with \(R\).

\[
c^2 = \frac{2GM}{R} \tag{88}
\]

\[
c = \sqrt{\frac{2GM}{R}} \tag{89}
\]

CONCLUSION

In previous attempts to understand gravitation in the framework of quantum physics, the past theories acknowledge that the Theory of General Relativity is formulated within the framework of classical physics. As declared in the Nature of Gravitons section of the paper, gravitons shall obey the laws of General Relativity as they obey the laws of quantum mechanics. As a particle dictated by the Theories of Relativity, classical mechanics must be a contributing factor to the overall mechanics of the graviton.

The paper proposed two considerations that were addressed and identified with a mathematical theorem.

1) The roles of the graviton are vital in the overall nature of classical gravitation. It is proposed that relativistic graviton interactions with other masses are what brings rise to the classical gravitational field. Gravitons undergo oscillation, where the uncertainty principle is applied to the position of the particle.

2) The particles contribute to the formation and potential energy of gravitational waves. General Relativity depicts gravitational waves as the ripples of spacetime formed by the violent collisions of two black holes or neutron stars. In terms of gravitons, the initial potential energy that is stored within the gravitonic mesh between two binary masses is the amount of energy emitted as gravitational waves upon collision. The emission of gravitons is theorized to be the particle interpretation of the wave-particle duality of gravitational waves.

The set-backs of the paper are the following: there is still no complete quantum field theory for gravitational interactions and the graviton, most of the theories of the quantum graviton are formulated in the framework of classical physics, and General Relativity remains as not renormalized. For this paper is only one part of the Graviton Theory as a whole, future extensions to this research on gravitons consist of the quantum framework of the graviton’s nature and mechanics, and the usage of statistical mechanics and thermodynamics towards interacting gravitons in the present time and in the Big Bang itself.

If gravitons exist, how they may behave is represented by using the mathematics in this paper. The theories and laws in this paper explain everything from how gravitons may oscillate, to how gravitons may behave in a binary system, which describes many aspects of the graviton and how it interacts with other bodies.

REFERENCES

[7] International Astronomical Union, ed. (31 August 2012), "RESOLUTION B2 on the re-definition of the astronomical unit of length" (PDF), RESOLUTION B2, Beijing, Kina: International Astronomical Union, "The XXVIII General Assembly of International Astronomical Union recommends [adopted] that the astronomical unit be re-defined to be a conventional unit of length equal to exactly 149597870700 metres, in agreement with the value adopted in IAU 2009 Resolution B2"