Path Related n-Cap Cordial Graphs

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Abstract: Let $G = (V, E)$ be a graph with $p$ vertices and $q$ edges. A $n$-$cap$ $(\nabla)$ cordial labeling of a graph $G$ with vertex set $V$ is a bijection from $V$ to $\{0, 1\}$ such that if each edge $uv$ is assigned the label

$$f(uv) = \begin{cases} 0, & \text{if } f(u) = f(v) = 1 \\ 1, & \text{otherwise.} \end{cases}$$

with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a $n$-$cap$ $(\nabla)$ cordial labeling is called a $n$-$cap$ $(\nabla)$ cordial graph $(nCCG)$. In this paper, we proved that Path $P_n$, Comb $(P_n \odot K_1)$, $P_m \odot 2K_1$ and Fan $(F_n = P_n + K_1)$ are $n$-$cap$ $(\nabla)$ cordial graphs.

Key Words: $n$-$cap$ $(\nabla)$ cordial labeling, Smarandachely cordial labeling, $n$-$cap$ $(\nabla)$ cordial labeling graph.

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§1. Introduction

A graph $G$ is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of $G$ which is called edges. Each pair $e = \{uv\}$ of vertices in $E$ is called an edge or a line of $G$. In this paper, we proved that Path $P_n$, Comb $(P_n \odot K_1)$, $P_m \odot 2K_1$ and Fan $(F_n = P_n + K_1)$ are $n$-$cap$ $(\nabla)$ cordial graphs.

§2. Preliminaries

Let $G = (V, E)$ be a graph with $p$ vertices and $q$ edges. A $n$-$cap$ $(\nabla)$ cordial labeling of a graph $G$ with vertex set $V$ is a bijection from $V$ to $\{0, 1\}$ such that if each edge $uv$ is assigned the
label

\[ f(uv) = \begin{cases} 
0, & \text{if } f(u) = f(v) = 1 \\
1, & \text{otherwise.} 
\end{cases} \]

with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1, and it is said to be a Smarandachely cordial labeling if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at least 1 and the number of edges labeled with 0 or the number of edges labeled with 1 differ by at least 1.

The graph that admits a \( \bigwedge \) cordial labeling is called a \( \bigwedge \) cordial graph. We proved that Path \( P_n \), Comb \( (P_n \odot K_1) \), \( P_m \odot 2K_1 \) and Fan \( (F_n = P_n + K_1) \) are \( \bigwedge \) cordial graphs.

**Definition 2.1** A path is a graph with sequence of vertices \( u_1, u_2, \cdots, u_n \) such that successive vertices are joined with an edge, denoted by \( P_n \), which is a path of length \( n - 1 \).

A closed path of length \( n \) is cycle \( C_n \).

**Definition 2.2** A comb is a graph obtained from a path \( P_n \) by joining a pendent vertex to each vertices of \( P_n \), it is denoted by \( P_n \odot K_1 \).

**Definition 2.3** A graph obtained from a path \( P_m \) by joining two pendent vertices at each vertices of \( P_m \) is denoted by \( P_m \odot 2K_1 \).

**Definition 2.4** A fan is a graph obtained from a path \( P_n \) by joining each vertices of \( P_n \) to a pendent vertex, it is denoted by \( F_n = P_n + K_1 \).

§3. Main Results

**Theorem 3.1** A path \( P_n \) is a \( \bigwedge \) cordial graph

*Proof* Let \( V(P_n) = \{u_i : 1 \leq i \leq n\} \) and \( E(P_n) = \{u_iu_{i+1} : 1 \leq i \leq n - 1\} \) Define \( f : V(P_n) \to \{0, 1\} \) with the vertex labeling determined following.

**Case 1.** \( n \) is odd.

Define

\[ f(u_i) = \begin{cases} 
0, & 1 \leq i \leq \frac{n-1}{2}, \\
1, & \frac{n+1}{2} \leq i \leq n.
\end{cases} \]

The induced edge labeling are

\[ f^*(u_iu_{i+1}) = \begin{cases} 
1, & 1 \leq i \leq \frac{n}{2}, \\
0, & \frac{n}{2} \leq i \leq n.
\end{cases} \]

Here \( V_0(f) + 1 = V_1(f) \) and \( e_0(f) = e_1(f) \). Clearly, it satisfies the condition \( |V_0(f) - V_1(f)| \leq 1 \) and \( |e_0(f) - e_1(f)| \leq 1 \).
Case 2. \( n \) is even.

Define
\[
f(u_i) = \begin{cases} 
0, & 1 \leq i \leq \frac{n}{2}, \\
1, & \frac{n}{2} + 1 \leq i \leq n.
\end{cases}
\]

The induced edge labeling are
\[
f^*(u_iu_{i+1}) = \begin{cases} 
1, & 1 \leq i \leq \frac{n}{2}, \\
0, & \frac{n}{2} + 1 \leq i \leq n.
\end{cases}
\]

Here \( V_0(f) = V_1(f) \) and \( e_0(f) + 1 = e_1(f) \) which satisfies the condition \( |V_0(f) - V_1(f)| \leq 1 \) and \( |e_0(f) - e_1(f)| \leq 1 \). Hence, a path \( P_n \) is a cordial graph. \( \square \)

For example, \( P_5 \) and \( P_6 \) are cordial graph shown in the Figure 1.

![Figure 1](image)

\[\text{Figure 1}\]

**Theorem 3.2** A comb \( P_n \odot K_1 \) is a cordial graph

**Proof** Let \( G \) be a comb \( P_n \odot K_1 \) and let \( V(G) = \{(u_i, v_i) : 1 \leq i \leq n\} \) and \( E(G) = \{[(u_iu_{i+1}) : 1 \leq i \leq n-1]\} \cup [(u_iv_i) : 1 \leq i \leq n]\} \). Define \( f : V(G) \to \{0,1\} \) with a vertex labeling
\[
f(u_i) = 1, \ 1 \leq i \leq n,
\]
\[
f(v_i) = 0, \ 1 \leq i \leq n.
\]

The induced edge labeling are
\[
f^*(u_iu_{i+1}) = 1, \ 1 \leq i < n,
\]
\[
f^*(u_iv_i) = 0, \ 1 \leq i \leq n.
\]

Here \( V_0(f) = V_1(f) \) and \( e_0(f) = e_1(f) + 1 \) which satisfies the condition \( |V_0(f) - V_1(f)| \leq 1 \) and \( |e_0(f) - e_1(f)| \leq 1 \). Hence, a comb \( P_n \odot K_1 \) is a cordial graph. \( \square \)

For example, \( P_5 \odot K_1 \) is a cordial graph shown in Figure 2.
Theorem 3.3 A graph $P_m \odot 2K_1$ is a cordial graph.

Proof Let $G$ be a $P_m \odot 2K_1$ with $V(G) = \{u_i, v_{1i}, v_{2i}, 1 \leq i \leq n\}$ and $E(G) = \{(u_iu_{i+1}) : 1 \leq i < n\} \cup \{(u_iv_{1i}) : 1 \leq i \leq n\} \cup \{(u_iv_{2i}) : 1 \leq i \leq n\}$. Define $f : V(C_n) \to \{0, 1\}$ by a vertex labeling $f(u_i) = \{1, 1 \leq i \leq n\}$, $f(v_{1i}) = \{0, 1 \leq i \leq n\}$ and if $n$ is even,

$$f(v_{2i}) = \begin{cases} 1, & 1 \leq i \leq \frac{n}{2}, \\ 0, & \frac{n}{2} + 1 \leq i \leq n, \end{cases}$$

if $n$ is odd

$$f(v_{2i}) = \begin{cases} 1, & 1 \leq i \leq \frac{n+1}{2}, \\ 0, & \frac{n+1}{2} + 1 \leq i \leq n. \end{cases}$$

The induced edge labeling are

$$f^*(u_iu_{i+1}) = \{0, 1 \leq i \leq n\},$$
$$f^*(u_iv_{1i}) = \{1, 1 \leq i \leq n\}$$

and if $n$ is even

$$f^*(u_iv_{2i}) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{2}, \\ 1, & \frac{n}{2} + 1 \leq i \leq n. \end{cases}$$

Here $V_0(f) = V_1(f)$ and $e_0(f) + 1 = e_1(f)$ which satisfies the condition $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$, and if $n$ is odd

$$f^*(u_iv_{2i}) = \begin{cases} 0, & 1 \leq i \leq \frac{n+1}{2}, \\ 1, & \frac{n+1}{2} + 1 \leq i \leq n. \end{cases}$$

Here $V_0(f) + 1 = V_1(f)$ and $e_0(f) = e_1(f)$ which satisfies the condition $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$. Hence, $P_m \odot 2K_1$ is a cordial graph. \qed

For example, $P_5 \odot 2K_1$ is a cordial graph shown in the Figures 3.
Theorem 3.4 A fan $F_n = P_n + K_1$ is a $\bigwedge$ cordial graph if $n$ is even.

Proof Let $G$ be a fan $F_n = P_n + K_1$ and $n$ is even with $V(G) = \{u, v_i : 1 \leq i \leq n\}$ and $E(G) = \{(u, v_i) : 1 \leq i \leq n\}$. Define $f : V(G) \to \{0, 1\}$ with a vertex labeling $f(u) = \{1\}$ and

$$f(v_i) = \begin{cases} 1, & 1 \leq i \leq \frac{n}{2}, \\ 0, & \frac{n}{2} + 1 \leq i \leq n. \end{cases}$$

The induced edge labeling are

$$f^*(uv_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{2}, \\ 1, & \frac{n}{2} + 1 \leq i \leq n, \end{cases} \quad \text{and} \quad f^*(v_iv_{i+1}) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{2}, \\ 1, & \frac{n}{2} \leq i \leq n. \end{cases}$$

Here $V_0(f) + 1 = V_1(f)$ and $e_0(f) + 1 = e_1(f)$ which satisfies the conditions $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$. Hence, a fan $F_n = P_n + K_1$ is a $\bigwedge$ cordial graph if $n$ is even. \qed

For example, a fan $F_6 = P_6 + K_1$ is $\bigwedge$ cordial shown in Figure 4.
References


