On the impossibility of a Yang-Mills finite mass gap in the flat spacetime
Open letter to the Clay Mathematics Institute from V. Paromov, PhD

There is an unproven conjecture that a non-trivial quantum Yang-Mills theory exists on \( \mathbb{R}^4 \) and its vacuum state excitations in the absence of matter fields have a finite mass gap. Notably, both Abelian and non-Abelian Lie group-based gauge theories naturally produce massless quantum fields, and hence, the introduction of mass-inducing quantum fields is always \textit{ad hoc}. Moreover, taken to the account the fundamental physical quality of any massive field, the ability to induce geometrical curvature on flat spacetime, such quantum Yang-Mills theory, if exists, cannot be complete when formulated on \( \mathbb{R}^4 \). In case the mass gap exists, the quantum field corresponding to this gap induces a non-zero geometrical curvature on \( \mathbb{R}^4 \) transforming the latter into a curved hypersurface in \( \mathbb{R}^5 \). Although the effect may be infinitesimal, it cannot be neglected completely. Hence, the theory formulated on \( \mathbb{R}^4 \) will be incomplete requiring additional manipulations, e.g. renormalization. Thus, it is logical to expect the theory to be initially formulated on a manifold with a higher dimensionality (\( n > 4 \)), and then translated to \( \mathbb{R}^4 \) or, more conveniently, to \( \mathbb{R}^3 \) and absolute time.

Alternatively, the theory may be formulated assuming a higher dimensionality of the spacetime. Such an approach avoids \textit{ad hoc} introductions of mass-inducing fields. As an example, the quantum electrodynamics (QED) can be initially formulated on a simple symmetrical hypersurface in \( \mathbb{R}^5 \), \( S^4 \) (in case time curvature is disregarded) and translated to \( \mathbb{R}^3 \) (and absolute time) using the Hopf fibration: \( U(1) \rightarrow S^3 \rightarrow \mathbb{C}P^1 \). [1]. Then, the Higg’s field comes naturally as the geometrical curvature of \( S^2 \), the geometrical intersection of \( S^3 \) with \( \mathbb{R}^3 \)[1]. Furthermore, the Yang-Mills theory can be initially formulated on \( S^7 \) (in case time curvature is disregarded) and then translated to \( \mathbb{R}^3 \) and absolute time using two consecutive Hopf fibrations: \( SU(2) \rightarrow S^7 \rightarrow S^4 \) and \( U(1) \rightarrow S^3 \rightarrow \mathbb{C}P^1 \) [1]. Then, the postulated geometry of “total” spacetime with the global space topology \( S^3 \times S^1 \times S^3 \) appears as the geometry behind the Lie algebras used in the Standard Model. This geometry can explain naturally the gauge symmetry, color confinement, renormalization requirement, and weak interactions’ chirality.

Reference:

1. V. Paromov. Fractal Structure of the Spacetime, the Fundamentally Broken Symmetry. 
http://vixra.org/abs/1806.0181