The cordiality for the conjunction of two paths

S. Nada*, A. Elrokh* and R. Hamza*
*Dept. of Math., Faculty of Science, Menoufia University, Egypt.
E mail: shokrynada@yahoo.com, el-rokh@excite.com and
rehabhamza9933@yahoo.com.

Abstract
A graph is called cordial if it has a $0-1$ labeling such that the number of vertices (edges) labeled with ones and zeros differ by at most one. The conjunction of two graphs (V_1, E_1) and (V_2, E_2) is the graph $G = (V, E)$, where $V = V_1 \times V_2$ and $u = (a_1, a_2)$, $v = (b_1, b_2)$ are two vertices, then $uv \in E$ if $a_i b_i \in E_i$ for $i = \{1, 2\}$. In this paper, we present necessary and sufficient condition for cordial labeling for the conjunction of two paths, denoted by $P_n \wedge P_m$. Also, we drive an algorithm to generate cordial labeling for the conjunction $P_n \wedge P_m$.

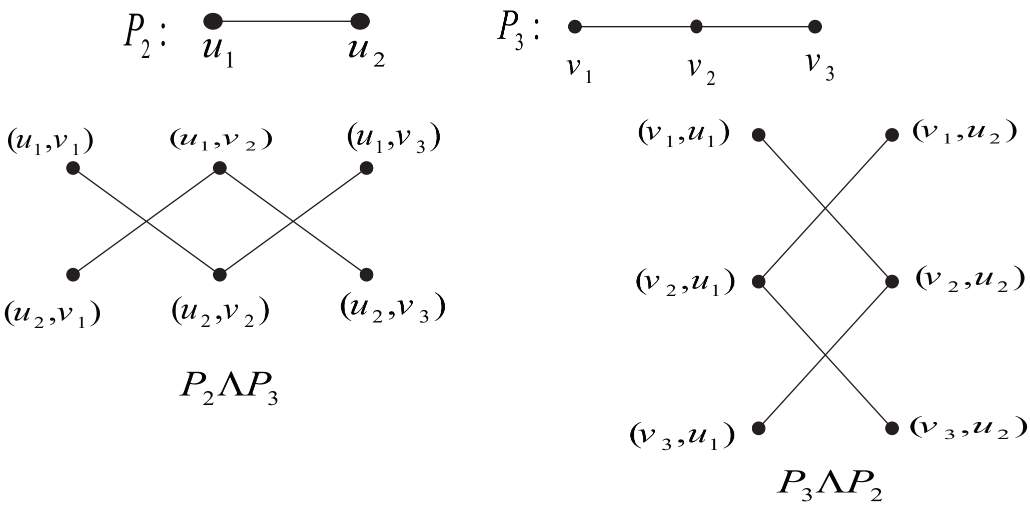
AMS Classification: 05C78.

Keywords: Graph labeling, Conjunction of two paths, Cordial labeling.

1 Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs are playing vital role especially in the field of computer science. It plays an important role in networking channels [7], data mining, cryptography, SQL query solving, etc... This vast range of applications motivated us towards labeled graphs. The concept of graph labeling was introduced by Rosa [9]. A useful survey to know about numerous graph labeling methods is found in [3]. In 1990, Cahit [1] introduced the cordial labeling of graphs. Cordiality behavior of numerous graphs was studied by several authors [5, 6, 8]. Suppose that $G = (V, E)$ is a graph, where V is the set of its vertices and E is the set of its edges. Throughout, it is assumed G is connected, finite, simple and undirected. A mapping $f : V \to \{0, 1\}$ is called binary vertex labeling of G and $f(u)$ denotes the label of vertex u of G under f. For an edge $e = uv \in E$, where $u, v \in V$, the induced edge labeling $f^* : E \to \{0, 1\}$ is defined by the formula $f^*(uv) = (f(u) + f(v))(mod 2)$, which of course equals $|f(u) - f(v)|$. Let us denote v_0 and v_1 be the numbers of vertices labeled 0 and 1 respectively, and let e_0 and e_1 be the corresponding numbers of edges. A binary vertex labeling f of G is said to be cordial if $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$ hold. A graph G is cordial if it has a cordial labeling.

Let $G_i = (V_i, E_i), i \in \{1,2\}$ be two graphs. The conjunction $G_1 \wedge G_2$ is defined to be $G = (V, E)$, where $V = V_1 \times V_2$ and if $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are two vertices in V then $uv \in E$ if and only if $u_i v_i \in E_i, \forall \{i = 1, 2\}$ [4]. The conjunction $P_n \wedge P_m$ has nm vertices and $2(n-1)(m-1)$ edges. It is easy to conclude that $P_n \wedge P_m$ is isomorphic to $P_m \wedge P_n$. As an example, $P_2 \wedge P_3$ is isomorphic to $P_3 \wedge P_2$ (see Figure(1)).



Figure(1)

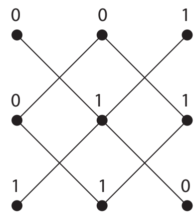
2 Terminologies and Notations

A path with n vertices and $n-1$ edges is denoted by P_n. Given a path with $4r$ vertices, we let L_{4r} denote the labeling 1100 ...1100 (repeated r-times). Sometimes, we then modify this by adding symbols at one end or the other (or both); thus $L_{4r}010$ denotes the labeling 1100...1100 010. We let O_n denote the labeling 0..0 (n-times), 1_n denotes the labeling 1...1 (n-times), M_{4r} denotes the labeling 1001...1001 (repeated r-times), A_{4r} denotes the labeling 1010...1010 (repeated r-times), $\overline{L}_{4r}$ denotes the labeling 0011...0011 (repeated r-times), $\overline{M}_{4r}$ denotes the labeling 0110...0110 (repeated r-times) and $\overline{A}_{4r}$ denotes the labeling 0101...0101 (repeated r-times).

3 Main results

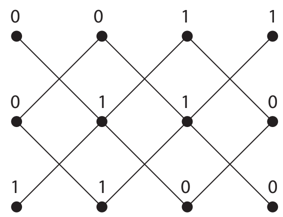
In this section, we show that the conjunction graph of two paths is cordial if and only if $(n, m) \neq (2, 2)$.

Let us first prove the easy cases by the following:
Lemma 1.3. The conjunction $P_n \wedge P_m$ where $n = 2$ is cordial except if $m = 2$ and the conjunction $P_n \wedge P_m$ where $n = 3$ is cordial for all $m \geq 2$.
Proof. We have two cases:
Case (1.1): Let $n = 2$. Since $P_2 \wedge P_m$ is equivalent to $P_m \cup P_m$ for all $m > 2$, it follows that $P_2 \wedge P_m$ is cordial except if $m = 2$ [2].
Case (2.1): Let us first discuss the easy situations which are at (i) $m = 2$. $P_3 \wedge P_2$ is cordial since it's isomorphic to $P_2 \wedge P_3$.
(ii) $m = 3$.
We choose the following labeling:
The first row is labeled by $0_2 1$, the second row is labeled by 01_2 and the third row is labeled by $1_2 0$ for $P_3 \wedge P_3$. Therefore $v_0 = 4$, $v_1 = 5$ and $e_0 = e_1 = 4$. Hence $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$. Consequently, $P_3 \wedge P_3$ is cordial. Figure(2) illustrates $P_3 \wedge P_3$.



$v_0 - v_1 = -1$ and $e_0 - e_1 = 0$
$P_3 \wedge P_3$ is cordial
Figure(2)

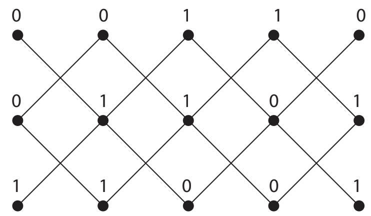
Now, we need to study the following subcases:
Subcase (1.2.1): $m \equiv 0 (mod 4)$.
Suppose that $m = 4r$, where $r \geq 1$. We choose the following labeling:
The first row is labeled by $\overline{L}_{4r}$, the second row is labeled by $\overline{M}_{4r}$ and the third row is labeled by L_{4r} for $P_3 \wedge P_{4r}$. Therefore $v_0 = v_1 = 6r$ and $e_0 = e_1 = 8r - 2$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_3 \wedge P_{4r}$ is cordial. Figure(3) illustrates $P_3 \wedge P_4$.



$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$

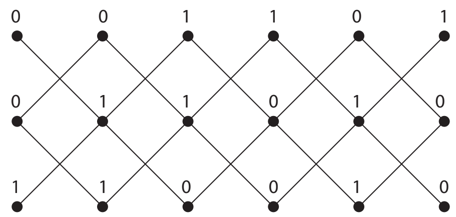
$P_3 \wedge P_4$ is cordial
Figure(3)

Subcase (2.2.1): $m \equiv 1(mod4)$.
Suppose that $m = 4r + 1$, where $r \geq 1$. We choose the following labeling: The first row is labeled by $\overline{L}_{4r}0$, the second row is labeled by $\overline{M}_{4r}1$ and the third row is labeled by $L_{4r}1$ for $P_3 \wedge P_{4r+1}$. Therefore $v_0 = 6r + 1$, $v_1 = 6r + 2$ and $e_0 = e_1 = 8r$. Hence $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$. Consequently, $P_3 \wedge P_{4r+1}$ is cordial. Figure(4) illustrates $P_3 \wedge P_5$.



$v_0 - v_1 = -1$ and $e_0 - e_1 = 0$
$P_3 \wedge P_5$ is cordial
Figure(4)

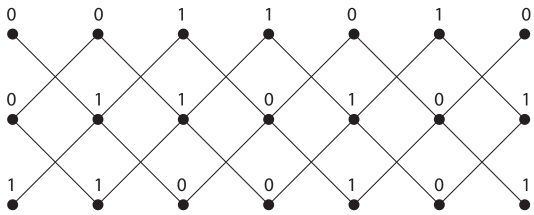
Subcase (3.2.1): $m \equiv 2(mod4)$.
Suppose that $m = 4r + 2$, where $r \geq 1$. We choose the following labeling: The first row is labeled by $\overline{L}_{4r}01$, the second row is labeled by $\overline{M}_{4r}10$ and the third row is labeled by $L_{4r}10$ for $P_3 \wedge P_{4r+2}$. Therefore $v_0 = v_1 = 6r + 3$ and $e_0 = e_1 = 8r + 2$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_3 \wedge P_{4r+2}$ is cordial. Figure(5) illustrates $P_3 \wedge P_6$.



$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$
$P_3 \wedge P_6$ is cordial
Figure(5)

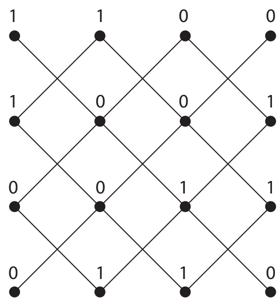
Subcase (4.2.1): $m \equiv 3(mod4)$.
Suppose that $m = 4r + 3$, where $r \geq 1$. We choose the following labeling: The first row is labeled by $\overline{L}_{4r}010$, the second row is labeled by $\overline{M}_{4r}101$ and

the third row is labeled by $L_{4r}101$ for $P_3 \wedge P_{4r+3}$. Therefore $v_0 = 6r + 4$, $v_1 = 6r + 5$ and $e_0 = e_1 = 8r + 4$. Hence $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$. Consequently, $P_3 \wedge P_{4r+3}$ is cordial. Figure(6) illustrates $P_3 \wedge P_7$.



$v_0 - v_1 = -1$ and $e_0 - e_1 = 0$
$P_3 \wedge P_7$ is cordial
Figure(6)

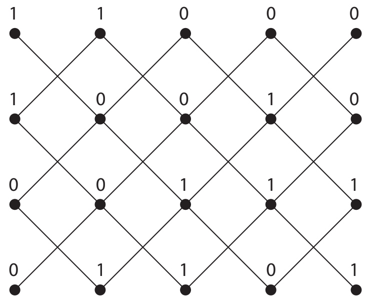
Lemma 2.3. The conjunction $P_n \wedge P_m$ is cordial where $n \equiv 0(mod4)$ for all $m \geq 2$.
Proof. Suppose that $n = 4r$, where $r \geq 1$. $P_{4r} \wedge P_2$ is cordial since it is isomorphic to $P_2 \wedge P_{4r}$ and $P_{4r} \wedge P_3$ is cordial since it is isomorphic to $P_3 \wedge P_{4r}$. Now, we need to study the following cases for $m \geq 4$.
Case (1.2): $m \equiv 0(mod4)$.
Suppose that $m = 4r$, where $r \geq 1$. We choose the following labeling:
The vertices which are in rows of order $4r - 3$ take the labeling L_{4r}, the vertices which are in rows of order $4r - 2$ are labeled by M_{4r}, the vertices that are in rows of order $4r - 1$ take the labeling $\overline{L}_{4r}$ and the vertices that are in rows of order $4r$ are labeled by $\overline{M}_{4r}$. Therefore $v_0 = v_1 = 8r^2$ and $e_0 = e_1 = 16r^2 - 8r + 1$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_{4r} \wedge P_{4r}$ is cordial. Figure(7) illustrates $P_4 \wedge P_4$.



$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$
$P_4 \wedge P_4$ is cordial
Figure(7)

Case (2.2): $m \equiv 1(mod4)$.

Suppose that $m = 4r + 1$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r - 3$ take the labeling $L_{4r}0$, the vertices which are in rows of order $4r - 2$ are labeled by $M_{4r}0$, the vertices that are in rows of order $4r - 1$ take the labeling $\overline{L}_{4r}1$ and the vertices that are in rows of order $4r$ are labeled by $\overline{M}_{4r}1$. Therefore $v_0 = v_1 = 8r^2 + 2r$ and $e_0 = e_1 = 16r^2 - 4r$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_{4r} \wedge P_{4r+1}$ is cordial. Figure(8) illustrates $P_4 \wedge P_5$.



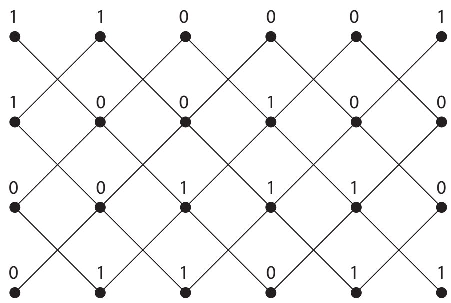
$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$

$P_4 \wedge P_5$ is cordial

Figure(8)

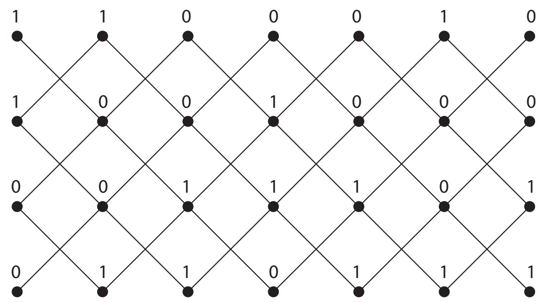
Case (3.2): $m \equiv 2(mod4)$.

Suppose that $m = 4r + 2$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r - 3$ take the labeling $L_{4r}01$, the vertices which are in rows of order $4r - 2$ are labeled by $M_{4r}0_2$, the vertices that are in rows of order $4r - 1$ take the labeling $\overline{L}_{4r}10$ and the vertices that are in rows of order $4r$ are labeled by $\overline{M}_{4r}1_2$. Therefore $v_0 = v_1 = 8r^2 + 4r$ and $e_0 = e_1 = 16r^2 - 1$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_{4r} \wedge P_{4r+2}$ is cordial. Figure(9) illustrates $P_4 \wedge P_6$.



$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$
$P_4 \wedge P_6$ is cordial
Figure(9)

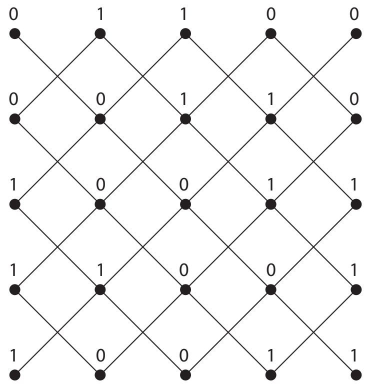
<u>**Case (4.2):**</u> $m \equiv 3(mod 4)$.
Suppose that $m = 4r + 3$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r - 3$ take the labeling $L_{4r}010$, the vertices which are in rows of order $4r - 2$ are labeled by $M_{4r}0_3$, the vertices that are in rows of order $4r - 1$ take the labeling $\overline{L}_{4r}101$ and the vertices that are in rows of order $4r$ are labeled by $\overline{M}_{4r}1_3$. Therefore $v_0 = v_1 = 8r^2 + 6r$ and $e_0 = e_1 = 16r^2 + 4r - 2$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_{4r} \wedge P_{4r+3}$ is cordial. Figure(10) illustrates $P_4 \wedge P_7$.



$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$
$P_4 \wedge P_7$ is cordial
Figure(10)

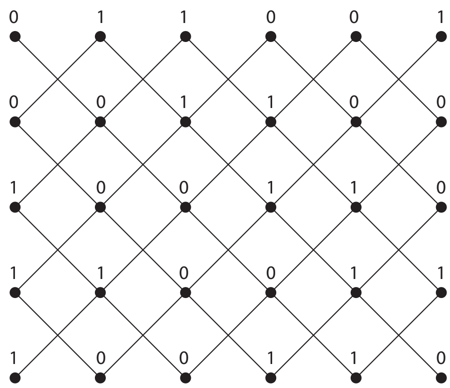
Lemma 3.3. The conjunction $P_n \wedge P_m$ is cordial where $n \equiv 1(mod 4)$ for all $m \geq 2$.
Proof. Suppose that $n = 4r+1$, where $r \geq 1$. $P_{4r+1} \wedge P_2$ is cordial since it is isomorphic to $P_2 \wedge P_{4r+1}$ and $P_{4r+1} \wedge P_3$ is cordial since it is isomorphic to $P_3 \wedge P_{4r+1}$. Now, we need to study the following cases for $m \geq 4$.
<u>**Case (1.3):**</u> $m \equiv 0(mod 4)$.
$P_{4r+1} \wedge P_{4r}$ is cordial since it's isomorphic to $P_{4r} \wedge P_{4r+1}$.
<u>**Case (2.3):**</u> $m \equiv 1(mod 4)$.
Suppose that $m = 4r + 1$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r - 3$ take the labeling $\overline{M}_{4r}0$, the vertices which are in rows of order $4r - 2$ are labeled by $\overline{L}_{4r}0$, the vertices that are in rows of order $4r - 1$ take the labeling $M_{4r}1$ and the vertices that are in rows of order $4r$ are labeled by $L_{4r}1$. Finally, the last row in $P_{4r+1} \wedge P_{4r+1}$ takes the labeling $M_{4r}1$. Therefore $v_0 = 8r^2 + 4r$, $v_1 = 8r^2 + 4r + 1$ and $e_0 = e_1 = 16r^2$. Hence $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$.

Consequently, $P_{4r+1} \wedge P_{4r+1}$ is cordial. Figure(11) illustrates $P_5 \wedge P_5$.



$v_0 - v_1 = -1$ and $e_0 - e_1 = 0$
$P_5 \wedge P_5$ is cordial
Figure(11)

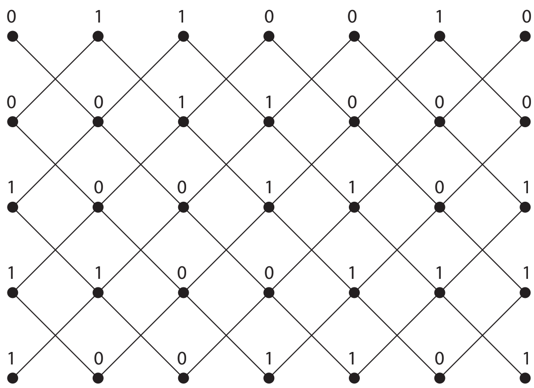
Case (3.3): $m \equiv 2(mod4)$.
Suppose that $m = 4r + 2$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r - 3$ take the labeling $\overline{M}_{4r}01$, the vertices which are in rows of order $4r - 2$ are labeled by $\overline{L}_{4r}0_2$, the vertices that are in rows of order $4r - 1$ take the labeling $M_{4r}10$ and the vertices that are in rows of order $4r$ are labeled by $L_{4r}1_2$. Finally, the last row in $P_{4r+1} \wedge P_{4r+2}$ takes the labeling $M_{4r}10$. Therefore $v_0 = v_1 = 8r^2 + 6r + 1$ and $e_0 = e_1 = 16r^2 + 4r$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_{4r+1} \wedge P_{4r+2}$ is cordial. Figure(12) illustrates $P_5 \wedge P_6$.



$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$

$P_5 \wedge P_6$ is cordial
Figure(12)

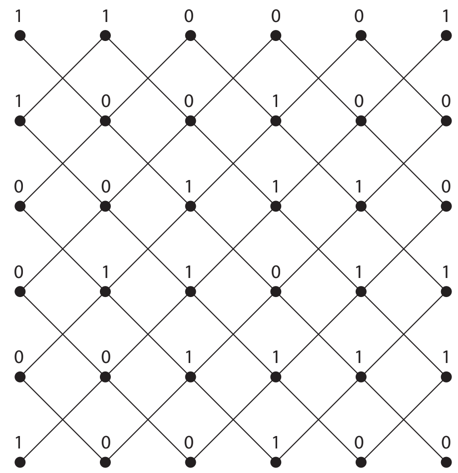
Case (4.3): $m \equiv 3(mod4)$.
Suppose that $m = 4r + 3$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r-3$ take the labeling $\overline{M}_{4r}010$, the vertices which are in rows of order $4r-2$ are labeled by $\overline{L}_{4r}0_3$, the vertices that are in rows of order $4r-1$ take the labeling $M_{4r}101$ and the vertices that are in rows of order $4r$ are labeled by $L_{4r}1_3$. Finally, the last row in $P_{4r+1} \wedge P_{4r+3}$ takes the labeling $M_{4r}101$. Therefore $v_0 = 8r^2 + 8r + 1$, $v_1 = 8r^2 + 8r + 2$ and $e_0 = e_1 = 16r^2 + 8r$. Hence $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$. Consequently, $P_{4r+1} \wedge P_{4r+3}$ is cordial. Figure(13) illustrates $P_5 \wedge P_7$.



$v_0 - v_1 = -1$ and $e_0 - e_1 = 0$
$P_5 \wedge P_7$ is cordial
Figure(13)

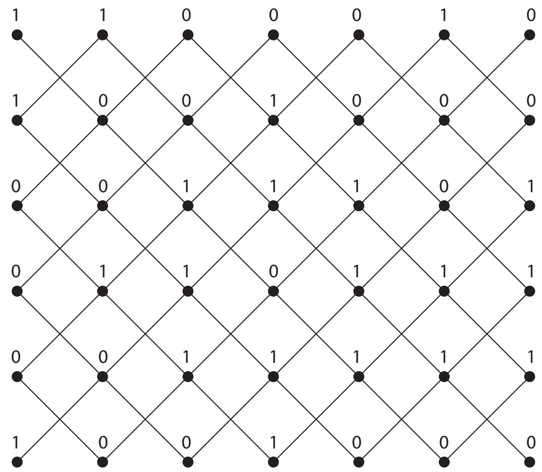
Lemma 4.3. The conjunction $P_n \wedge P_m$ is cordial where $n \equiv 2(mod4)$ for all $m \geq 2$.
Proof. Suppose that $n = 4r + 2$, where $r \geq 1$. $P_{4r+2} \wedge P_2$ is cordial since it's isomorphic to $P_2 \wedge P_{4r+2}$ and $P_{4r+2} \wedge P_3$ is cordial since it's isomorphic to $P_3 \wedge P_{4r+2}$. Now, we need to study the following cases for $m \geq 4$.
Case (1.4): $m \equiv 0(mod4)$.
$P_{4r+2} \wedge P_{4r}$ is cordial since it's isomorphic to $P_{4r} \wedge P_{4r+2}$.
Case (2.4): $m \equiv 1(mod4)$.
$P_{4r+2} \wedge P_{4r+1}$ is cordial since it's isomorphic to $P_{4r+1} \wedge P_{4r+2}$.
Case (3.4): $m \equiv 2(mod4)$.
Suppose that $m = 4r + 2$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r-3$ take the labeling $L_{4r}01$, the

vertices which are in rows of order $4r-2$ are labeled by $M_{4r}0_2$, the vertices that are in rows of order $4r-1$ take the labeling $\overline{L}_{4r}10$ and the vertices that are in rows of order $4r$ are labeled by $\overline{M}_{4r}1_2$. Finally, the last two rows in $P_{4r+2} \wedge P_{4r+2}$ take respectively the labeling $\overline{L}_{4r}1_2$ and $M_{4r}0_2$. Therefore $v_0 = v_1 = 8r^2 + 8r + 2$ and $e_0 = e_1 = 16r^2 + 8r + 1$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_{4r+2} \wedge P_{4r+2}$ is cordial. Figure(14) illustrates $P_6 \wedge P_6$.



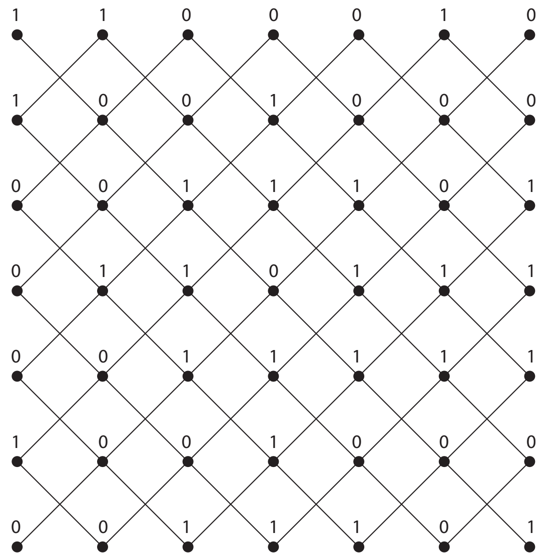
$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$
$P_6 \wedge P_6$ is cordial
Figure(14)

Case (4.4): $m \equiv 3(mod 4)$.
Suppose that $m = 4r + 3$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r - 3$ take the labeling $L_{4r}010$, the vertices which are in rows of order $4r - 2$ are labeled by $M_{4r}0_3$, the vertices that are in rows of order $4r - 1$ take the labeling $\overline{L}_{4r}101$ and the vertices that are in rows of order $4r$ are labeled by $\overline{M}_{4r}1_3$. Finally, the last two rows in $P_{4r+2} \wedge P_{4r+3}$ take respectively the labeling $\overline{L}_{4r}1_3$ and $M_{4r}0_3$. Therefore $v_0 = v_1 = 8r^2 + 10r + 3$ and $e_0 = e_1 = 16r^2 + 12r + 2$. Hence $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Consequently, $P_{4r+2} \wedge P_{4r+3}$ is cordial. Figure(15) illustrates $P_6 \wedge P_7$.



$v_0 - v_1 = 0$ and $e_0 - e_1 = 0$
$P_6 \wedge P_7$ is cordial
Figure(15)

Lemma 5.3. The conjunction $P_n \wedge P_m$ is cordial where $n \equiv 3(mod4)$ for all $m \geq 2$.
Proof. Suppose that $n = 4r + 3$, where $r \geq 1$. $P_{4r+3} \wedge P_2$ is cordial since it's isomorphic to $P_2 \wedge P_{4r+3}$ and $P_{4r+3} \wedge P_3$ is cordial since it's isomorphic to $P_3 \wedge P_{4r+3}$. Now, we need to study the following cases for $m \geq 4$.
Case (1.5): $m \equiv 0(mod4)$.
$P_{4r+3} \wedge P_{4r}$ is cordial since it's isomorphic to $P_{4r} \wedge P_{4r+3}$.
Case (2.5): $m \equiv 1(mod4)$.
$P_{4r+3} \wedge P_{4r+1}$ is cordial since it's isomorphic to $P_{4r+1} \wedge P_{4r+3}$.
Case (3.5): $m \equiv 2(mod4)$.
$P_{4r+3} \wedge P_{4r+2}$ is cordial since it's isomorphic to $P_{4r+2} \wedge P_{4r+3}$.
Case (4.5): $m \equiv 3(mod4)$.
Suppose that $m = 4r + 3$, where $r \geq 1$. We choose the following labeling: The vertices which are in rows of order $4r-3$ take the labeling $L_{4r}010$, the vertices which are in rows of order $4r-2$ are labeled by $M_{4r}0_3$, the vertices that are in rows of order $4r-1$ take the labeling $\overline{L}_{4r}101$ and the vertices that are in rows of order $4r$ are labeled by $\overline{M}_{4r}1_3$. Finally, the last three rows in $P_{4r+3} \wedge P_{4r+3}$ take respectively the labeling $L_{4r}1_3$, $M_{4r}0_3$ and $\overline{L}_{4r}101$. Therefore $v_0 = 8r^2+12r+4$, $v_1 = 8r^2+12r+5$ and $e_0 = e_1 = 16r^2+16r+4$. Hence $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$. Consequently, $P_{4r+3} \wedge P_{4r+3}$ is cordial. Figure(16) illustrates $P_7 \wedge P_7$.

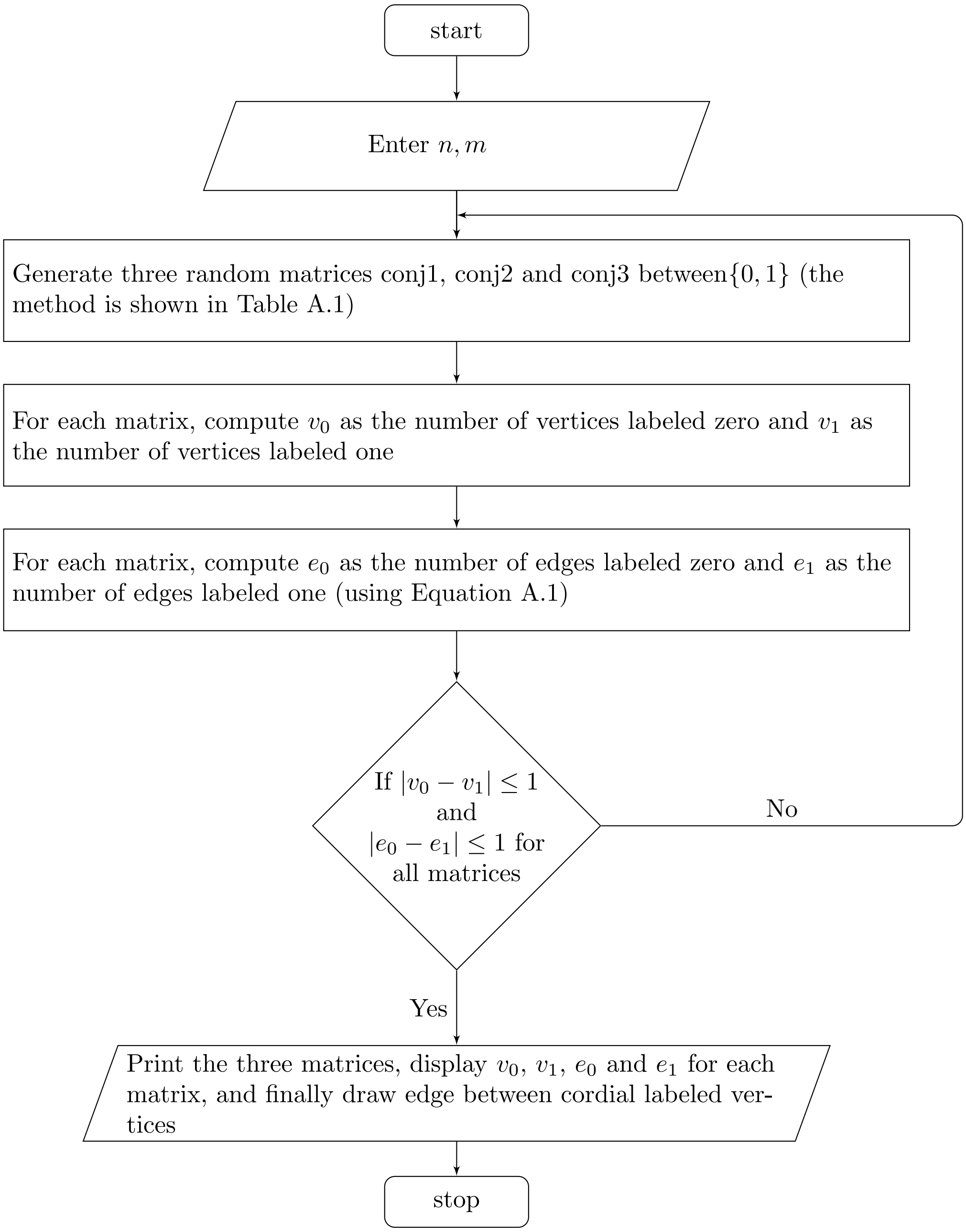


$P_7 \wedge P_7$ is cordial
$v_0 - v_1 = -1$ and $e_0 - e_1 = 0$
Figure(16)

As a consequence of all previous lemmas, one can establish the following theorem.
Theorem 1.3. The conjunction $P_n \wedge P_m$ is cordial if and only if $(n, m) \neq (2, 2)$.

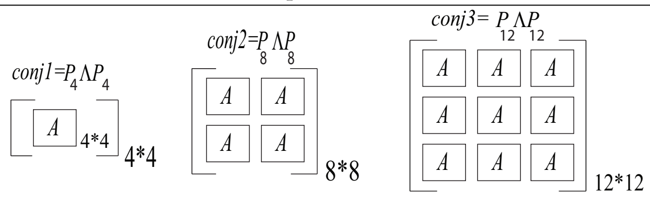
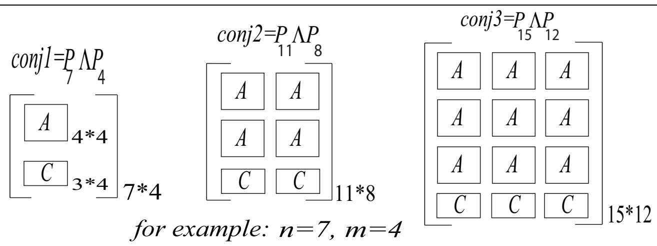
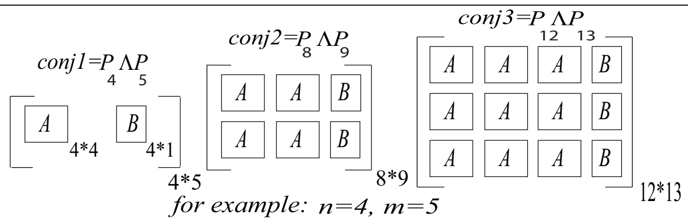
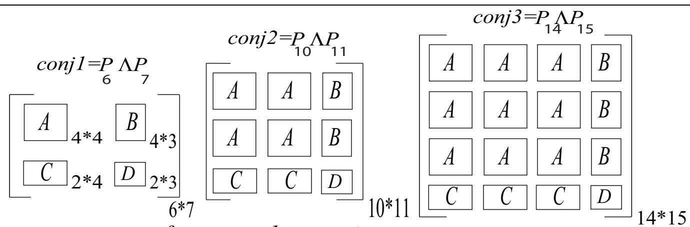
4 Algorithm

This section contains an algorithm to generate cordial labeling for the conjunction of two paths which is symbolized by $P_n \wedge P_m$.

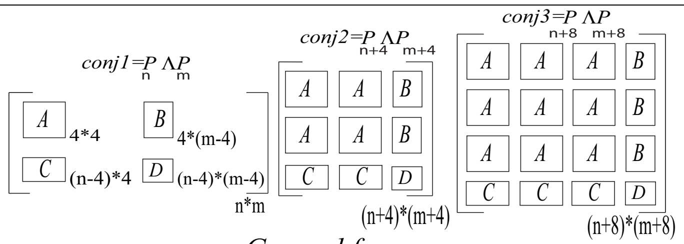

start
Enter n, m
Generate three random matrices conj1, conj2 and conj3 between$\{0,1\}$ (the method is shown in Table A.1)
For each matrix, compute v_0 as the number of vertices labeled zero and v_1 as the number of vertices labeled one
For each matrix, compute e_0 as the number of edges labeled zero and e_1 as the number of edges labeled one (using Equation A.1)
If $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$ for all matrices
No
Yes
Print the three matrices, display v_0, v_1, e_0 and e_1 for each matrix, and finally draw edge between cordial labeled vertices
stop

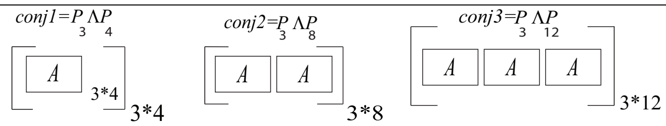
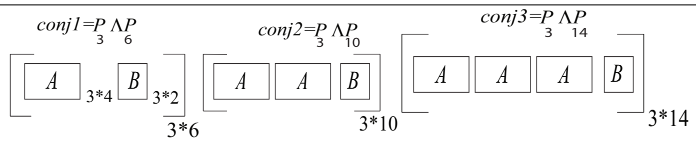
5 Appendix A

We can deal with the conjunction of two paths $P_n \wedge P_m$ as blocks of matrices whose elements represent the cordial labeled vertices. In Table A.1, we illustrate the method of generating matrices and supporting them with examples.

CASE	Description of matrices
$n = m = 4$	
$n > 4, m = 4$	 *for example: n=7, m=4*
$n = 4, m > 4$	 *for example: n=4, m=5*
$n, m > 4$	 *for example: n=6, m=7*

Continued

<table>
<tr><td>$\forall n, m > 3$</td><td>

General form</td></tr>
<tr><td>$n = 3, m = 4$</td><td>
</td></tr>
<tr><td>$n = 3, m > 4$</td><td>

for example: n=3, m=6</td></tr>
<tr><td>$n \geq 4, m = 3$</td><td>The conjunction $P_n \wedge P_3$ is isomorphic to the conjunction $P_3 \wedge P_n$</td></tr>
</table>

Table A.1
Matrices generation

<u>Notes</u>

1. The elements of the matrix represent the cordial labeled vertices of the conjunction $P_n \wedge P_m$ and are denoted by f.

If $f = 0$ then
  $v_0 = v_0 + 1$
Else
  $v_1 = v_1 + 1$
End if

2. The lines that connect each two vertex (see Figure(1)) represent edges of the conjunction $P_n \wedge P_m$ and its value denoted by f^*, define the variable i and j to cover all vertices. Using Equation A.1, we can calculate f^*.

$$f^* = \begin{cases} |v_{(i,j)} - v_{(i+1,j+1)}|; & j < m-1, i < n-1 \\ |v_{(i,j)} - v_{(i+1,j-1)}|; & i < n-1, j > 0 \end{cases}$$

$$f^* = \begin{cases} 0 \\ & or \\ 1 \end{cases}$$

- If $f^* = 0$ then
  $e_0 = e_0 + 1$
  Else
  $e_1 = e_1 + 1$
  End if

Equation A.1

References

[1] Cahit, I., On cordial and 3-equitable labelings of graphs, Utilities Math., 37 (1990).

[2] Diab, A. T. and Elsakhawi, E. A., Some Results on Cordial Graphs, Proc. Math. Phys.Soc. Egypt, No.7, pp. 67-87 (2002).

[3] Gallian, J. A., "A dynamic survey on graph labeling", The Electronic Journal of Combinatorics, Sixteenth Edition, (2013).

[4] Harary, F. and Wilcox, G. W., Boolean operations on graphs, Math.Scand.20, pp.41-51(1967).

[5] Hefnawy, A. I., Elmshtaye, Y., Cordial labeling of corona product of paths and lemniscate graphs accepted paper in Ars Combin (2017).

[6] Kaneria, V. J., Vaidya, S. K., Index of cordiality for complete graphs and cycle, IJAMC, 2(4)(2010), 38-46.

[7] Lakshmi Prasanna, N., Sravanthi, K. and NagallaSudhakar, "Applications Of Graph Labeling In Communication Networks" Oriental Journal Of Computer Science And Technology, Volume 7 No. 01 Page No. 139-145 Apr 2014.

[8] Nada, S., Diab, A. T., Elrokh A. and Sabra, D. E., The corona between paths and cycles, accepted paper in Ars Combin (2015).

[9] Rosa, A., On certain valuations of the vertices of a graph, Theory of graphs (Internat. Symposium, Rome, July 1996), Gordon and Breach, N. Y. and Dunod Paris (1967)349 – 355.