Refutation of mapping mu-calculus onto second-order logic

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Abstract: When mapping mu-calculus onto second-order logic, we show use of the fixpoint operator as untenable. What follows is the effective refutation of mapping mu-calculus onto second-order logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: p, q, w, v; ~ Not; & And, ∧; \ Not And, /; > Imply, greater than, →; < Not Imply, lesser than, ∈; = Equivalent; ~(y<x) x≤y, x⊆y; % possibility, for one or some, ∃; # necessity, for every or all, ∀.

From: Carreiro, F.; Facchini, A.; Venema, Y.; Zanasi, F. (2018). The power of the weak. arxiv.org/pdf/1809.03896.pdf

Remark: We ignore phi-asterisk (φ*) as a constant/scalar in this demonstration.

[W]e would inductively translate $(\mu p.\varphi)^*$ as $\forall p\ \forall w\ (\varphi^*[w/v] \rightarrow p(w)) \rightarrow p(v))$ (29.1)

#p&((#r&((r\s)>(p&r)))>(p&s)) ; **F**N**F**N **FFFF** **F**N**F**N **F**N**F**N (29.2)

$(\mu p.\varphi)^* := \exists q\ \forall p \subseteq q.\ \underline{p \in PRE((\varphi^*_{p})_{\restriction q})} \rightarrow p(v),$ (32.1)

where $p \in PRE((\varphi^*_{p})_{\restriction q})$ expresses that $p \subseteq q$ is a prefixpoint of the map $(\varphi^*_{p})_{\restriction q}$, that is:

$\underline{p \in PRE((\varphi^*_{p})_{\restriction q})} := \forall w\ (q(w) \land \varphi^*[w/v]) \rightarrow p(w).$

%q&(#p<((q&(#r&(((q&r)&(r\s))>(p&s))))>(p&s))) ; **FFFF FFFF FFFF FFFF** (32.2)

The conjecture to be tested is if Eqs. 29.1 is equivalent to 32.1. (1.1)

(#p&((#r&((r\s)>(p&r)))>(p&s)))=
(%q&(#p<((q&(#r&(((q&r)&(r\s))>(p&s))))>(p&s)))) ; TCTT TTTC TCTT TCCT (1.2)

Eq. 1.2 is *not* tautologous meaning the fixpoint operator cannot map mu-calculus onto second-order logic. This effectively refutes the mapping of mu-calculus onto second-order logic.

Remark: We attempt to rehabilitate the conjecture of Eq. 1.1 by testing for implication (> replaces =). (2.1)

(#p&((#r&((r\s)>(p&r)))>(p&s)))>
(%q&(#p<((q&(#r&(((q&r)&(r\s))>(p&s))))>(p&s)))) ; TCTC TTTT TCTC TCTC (2.2)

which is less desirable value-wise, with six C (contingency or falsity) instead of five C.