Refutation of completeness for non-deterministic logic

Abstract: We show that the four-valued, non-deterministic semantics for modal logic are not complete. The demonstration uses contradictions based on Carnielli's paraconsistent logic.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p: p; ~ Not, ¬; & And, ∧; = Equivalent, ≡;
% possibility, ◇, for one or some, ∃; # necessity, □, for every or all, ∀.

From: Coniglio, M.; del Cerro, L.F.; Peron, N.M. (2018). Modal logic with non-deterministic semantics: Part I - Propositional case. arxiv.org/pdf/1808.10007.pdf

> **Remark:** We attribute the following equations to the paraconsistent logic system of Walter A. Carnielli since 1990, but use lower-case t, c, f, i in contrast to our bivalent `T`, `C`, `F`, `N`.

[7]These truth-values can be formalized in a modal language (assuming, as usual, the equivalences ¬□p ≡ ◇¬p and ¬◇p ≡ □¬p) as follows:

t+:	□p ∧ ◇p ∧ p ;		(7.1.1)
	(#p&%p)&p ;	`FNFN FNFN FNFN FNFN`	(7.1.2)
c+:	¬□p ∧ ◇p ∧ p ;		(7.2.1)
	(~#p&%p)&p ;	`FCFC FCFC FCFC FCFC`	(7.2.2)
f+:	□¬p ∧ ◇¬p ∧ p ;		(7.3.1)
	(#~p&%~p)&p;	`FFFF FFFF FFFF FFFF`	(7.3.2)
i+:	□p ∧ ¬◇p ∧ p ;		(7.4.1)
	(#p&~%p)&p ;	`FFFF FFFF FFFF FFFF`	(7.4.2)
t-:	□p ∧ ◇p ∧ ¬p ;		(7.5.1)
	(#p&%p)&~p ;	`FFFF FFFF FFFF FFFF`	(7.5.2)
c-:	¬□p ∧ ◇p ∧¬p ;		(7.6.1)
	(~#p&%p)&~p ;	`CFCF CFCF CFCF CFCF`	(7.6.2)
f-:	□¬p ∧ ◇¬p ∧ ¬p ;		(7.7.1)
	(#~p&%~p)&~p ;	`NFNF NFNF NFNF NFNF`	(7.7.2)
i-:	□p ∧ ¬◇p ∧ ¬p.		(7.8.1)
	(#p&~%p)&~p ;	`FFFF FFFF FFFF FFFF`	(7.8.2)

The system reduces to a three-valued logic of (f+ = i+ = t- = i-), (t+ = ~f-), and (c+ = ~c-). As such, it is not a six- or eight-valued system as claimed. We find no designated *proof* value: ~(f+ = i+ = t- = i-) is not a designated contradiction, but also is not complete as not tautologous. This is despite the alethic (T) axiom replacement by the bivalent deontic (D) axiom replacements (D1 and D2). What follows is that all infinite non-deterministic matrices are characterized by finite deterministic matrices.

The system is also *not* bivalent: i+ is not ~(i-); but rather i+ is equivalent to i-; and f+ is not equivalent to f- or to ~f-. What follows is that infinite non-deterministic matrices are by definition incomplete.