Abstract. In this paper, we state a new class of sets and called them fuzzy neutrosophic Alpha\textsuperscript{m}-closed sets, and we prove some theorem related to this definition. Then, we investigate the relation between fuzzy neutrosophic Alpha\textsuperscript{m}-closed sets, fuzzy neutrosophic \(\alpha\) closed sets, fuzzy neutrosophic closed sets, fuzzy neutrosophic semi closed sets and fuzzy neutrosophic pre closed sets. On the other hand, some properties of the fuzzy neutrosophic Alpha\textsuperscript{m}-closed set are given.

**Keywords:** fuzzy neutrosophic closed sets, fuzzy neutrosophic Alpha\textsuperscript{m}-closed sets, fuzzy neutrosophic topology.

1. Introduction:

The concept of fuzzy sets was introduced by Zadeh in 1965 [14]. Then the fuzzy set theory are extension by many researchers. The concept of intuitionistic fuzzy sets (IFS) was one of the extension sets by K. Atanassov in 1983 [2, 3, 4], when fuzzy set give the degree of membership of an element in the sets, the intuitionistic fuzzy sets give a degree of membership and a degree of non-membership. Then, several researches were conducted on the generalizations of the notion of intuitionistic fuzzy sets, one of them was Floretin Smarandache in 2010 [7] when he developed another membership in addition to the two memberships which was defined in intuitionistic fuzzy sets and called it neutrosophic set.

The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In the last year, (2017) Veereswari [13] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

Neutrosophic topological spaces and many applications have been investigated by Salama et al. in [9-12].

In this paper, the concept of Alpha\textsuperscript{m}-closed sets in double fuzzy topological spaces [6] were developed. We discussed some new class of sets and called them fuzzy neutrosophic Alpha\textsuperscript{m}-closed sets in fuzzy neutrosophic topological spaces, and we also discussed some new properties and examples based of this defined concept.

2. Basic definitions and terminologies

**Definition 2.1** [8]: A neutrosophic topology (\(NT\), for short) on a non-empty set \(X\) is a family \(\tau\) of neutrosophic subsets of \(X\) satisfying the following axioms:

i) \(\emptyset, X \in \tau\).
Definition 2.2 [1, 13]: Let $X$ be a non-empty fixed set. A fuzzy neutrosophic set (FNS, for short), $\lambda_N$ is an object having the form $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)> : x \in X\}$ where the functions $\mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N} : X \to [0, 1]$ denote the degree of membership function (namely $\mu_{\lambda N}(x)$), the degree of indeterminacy function (namely $\sigma_{\lambda N}(x)$), and the degree of non-membership function (namely $\nu_{\lambda N}(x)$) respectively, of each set $\lambda_N$. We have, $0 \leq \mu_{\lambda N}(x) + \sigma_{\lambda N}(x) + \nu_{\lambda N}(x) \leq 3$, for each $x \in X$.

Remark 2.3 [13]: FNS $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)> : x \in X\}$ can be identified to an ordered triple $<\mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N}>$ in $[0, 1]$ on $X$.

Definition 2.4 [13]: Let $X$ be a non-empty set and the FNSs $\lambda_N$ and $\beta_N$ be in the form:

$\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)> : x \in X\}$ and,

$\beta_N = \{<x, \mu_{\beta N}(x), \sigma_{\beta N}(x), \nu_{\beta N}(x)> : x \in X\}$ on $X$ then:

i. $\lambda_N \subseteq \beta_N$ if $\mu_{\lambda N}(x) \leq \mu_{\beta N}(x)$, $\sigma_{\lambda N}(x) \leq \sigma_{\beta N}(x)$ and $\nu_{\lambda N}(x) \geq \nu_{\beta N}(x)$ for all $x \in X$,

ii. $\lambda_N = \beta_N$ if $\lambda_N \subseteq \beta_N$ and $\beta_N \subseteq \lambda_N$,

iii. $\lambda_N \cup \beta_N = \{<x, \nu_{\lambda N}(x), 1 - \sigma_{\lambda N}(x), \mu_{\lambda N}(x)> : x \in X\}$,

iv. $\lambda_N \cap \beta_N = \{<x, \mu_{\lambda N}(x), \mu_{\beta N}(x)), \max(\sigma_{\lambda N}(x), \sigma_{\beta N}(x)), \min(\nu_{\lambda N}(x), \nu_{\beta N}(x)) >: x \in X\}$,

v. $\lambda_N = \{<x, 0, 0, 1>, \lambda_N = \{<x, 1, 1, 0>, \nu_{\lambda N}(x), \nu_{\beta N}(x)) >: x \in X\}$,

vi. $\beta_N = \{<x, 0, 0, 1>$ and $\lambda_N = \{<x, 1, 1, 0>$.

Definition 2.5 [13]: A fuzzy neutrosophic topology (FNT, for short) on a non-empty set $X$ is a family $\tau_N$ of fuzzy neutrosophic subsets in $X$ satisfying the following axioms.

i. $\emptyset_N, 1_N \in \tau_N$,

ii. $\lambda_{N1} \cap \lambda_{N2} \in \tau_N$ for any $\lambda_{N1}, \lambda_{N2} \in \tau_N$,

iii. $\cup \lambda_{Nj} \in \tau_N \forall \{\lambda_{Nj} : j \in J\} \subseteq \tau_N$.

In this case the pair $(X, \tau_N)$ is called fuzzy neutrosophic topological space (FNTS, for short). The elements of $\tau$ are called fuzzy neutrosophic open sets (FNS-open set, for short). The complement of FNS-open sets in the FNTS $(X, \tau_N)$ are called fuzzy neutrosophic closed sets (FNC-closed set, for short).

Definition 2.6 [13]: Let $(X, \tau_N)$ be FNTS and $\lambda_N = \{<x, \mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N}> : x \in X\}$ be FNS in $X$. Then, the fuzzy neutrosophic closure of $\lambda_N$ (FNCi, for short) and fuzzy neutrosophic interior of $\lambda_N$ (FNI, for short) are defined by:
FNCl(λ_\text{\textit{N}}) = \cap \{ \beta_\text{\textit{N}}: \beta_\text{\textit{N}} \text{ is F}_\text{\textit{N}}\text{-closed set in } X \text{ and } \lambda_\text{\textit{N}} \subseteq \beta_\text{\textit{N}} \}.

FNInt (\lambda_\text{\textit{N}}) = \cup \{ \beta_\text{\textit{N}}: \beta_\text{\textit{N}} \text{ is F}_\text{\textit{N}}\text{-open set in } X \text{ and } \beta_\text{\textit{N}} \subseteq \lambda_\text{\textit{N}} \}.

Note that FNCl(\lambda_\text{\textit{N}}) be F\text{\textit{N}}\text{-closed set and } FNInt (\lambda_\text{\textit{N}}) be F\text{\textit{N}}\text{-open set in } X.

Further,

i. \lambda_\text{\textit{N}} be F\text{\textit{N}}\text{-closed set in } X \iff FNCl (\lambda_\text{\textit{N}}) = \lambda_\text{\textit{N}},

ii. \lambda_\text{\textit{N}} be F\text{\textit{N}}\text{-open set in } X \iff FNInt (\lambda_\text{\textit{N}}) = \lambda_\text{\textit{N}}.

Proposition 2.7 [13]:

Let (X, \tau_\text{\textit{N}}) be FNTS and \lambda_\text{\textit{N}}, \beta_\text{\textit{N}} are FNSs in X. Then, the following properties hold:

i. \text{FNInt}(\lambda_\text{\textit{N}}) \subseteq \lambda_\text{\textit{N}} and \lambda_\text{\textit{N}} \subseteq \text{FNCl}(\lambda_\text{\textit{N}}),

ii. \lambda_\text{\textit{N}} \subseteq \beta_\text{\textit{N}} \Rightarrow \text{FNInt}(\lambda_\text{\textit{N}}) \subseteq \text{FNInt}(\beta_\text{\textit{N}}) and \lambda_\text{\textit{N}} \subseteq \beta_\text{\textit{N}} \Rightarrow \text{FNCl}(\lambda_\text{\textit{N}}) \subseteq \text{FNCl}(\beta_\text{\textit{N}}),

iii. \text{Int}(\text{FNInt}(\lambda_\text{\textit{N}})) = \text{FNInt}(\text{FNCl}(\lambda_\text{\textit{N}})) = \text{FNCl}(\lambda_\text{\textit{N}}),

iv. \text{FNInt}(\lambda_\text{\textit{N}} \cup \beta_\text{\textit{N}}) = \text{FNInt}(\lambda_\text{\textit{N}}) \cup \text{FNInt}(\beta_\text{\textit{N}}) and \text{FNCl}(\lambda_\text{\textit{N}} \cap \beta_\text{\textit{N}}) = \text{FNCl}(\lambda_\text{\textit{N}}) \cap \text{FNCl}(\beta_\text{\textit{N}}),

v. \text{FNInt}(1_\text{\textit{N}}) = 1_\text{\textit{N}} and \text{FNCl}(0_\text{\textit{N}}) = 0_\text{\textit{N}},

vi. \text{FNInt}(0_\text{\textit{N}}) = 0_\text{\textit{N}} and \text{FNCl}(1_\text{\textit{N}}) = 1_\text{\textit{N}}.

Definition 2.8 [2]:

FNS \lambda_\text{\textit{N}} in FNTS (X, \tau_\text{\textit{N}}) is called:

i. fuzzy neutrosophic semi-open set (FNS-open, for short) if \lambda_\text{\textit{N}} \subseteq \text{FNCl}(\text{FNInt}(\lambda_\text{\textit{N}})),

ii. fuzzy neutrosophic semi-closed set (FNS-closed, for short) if \text{FNInt}(\text{FNCl}(\lambda_\text{\textit{N}})) \subseteq \lambda_\text{\textit{N}},

iii. fuzzy neutrosophic pre-open set (FNP-open, for short) if \lambda_\text{\textit{N}} \subseteq \text{FNInt}(\text{FNCl}(\lambda_\text{\textit{N}})),

iv. fuzzy neutrosophic pre-closed set (FNP-closed, for short) if \text{FNCl}(\text{FNInt}(\lambda_\text{\textit{N}})) \subseteq \lambda_\text{\textit{N}},

v. fuzzy neutrosophic \alpha\text{-open set (FN\alpha\text{-open, for short) if } \lambda_\text{\textit{N}} \subseteq \text{FNInt}(\text{FNCl}(\lambda_\text{\textit{N}})) \subseteq \lambda_\text{\textit{N}}.

\text{FNCl}(\lambda_\text{\textit{N}}) \subseteq \lambda_\text{\textit{N}} and \lambda_\text{\textit{N}} \subseteq \text{FNCl}(\lambda_\text{\textit{N}}).

\text{FNInt}(\lambda_\text{\textit{N}}) = \lambda_\text{\textit{N}}.

3. Fuzzy Neutrosophic Alpha^m - Closed Sets in Fuzzy Neutrosophic Topological Spaces.

Now, the concept of fuzzy neutrosophic Alpha^m-closed set in fuzzy neutrosophic topological space is introduced, as follows:

Definition 3.1: Fuzzy neutrosophic subset \lambda_\text{\textit{N}} of FNTS (X, \tau_\text{\textit{N}}) is called fuzzy neutrosophic Alpha^m-closed set (FN\alpha^m\text{-closed set, for short}) if \text{FNInt}(\text{FNCl}(\lambda_\text{\textit{N}})) \subseteq \lambda_\text{\textit{N}}, wherever \lambda_\text{\textit{N}} \subseteq U_\text{\textit{N}} and U_\text{\textit{N}} be FN\alpha\text{-open set. And } \lambda_\text{\textit{N}} is said to be fuzzy neutrosophic Alpha^m\text{-open set (FN\alpha^m\text{-open, for short) if } \text{FNCl}(\text{FNInt}(\lambda_\text{\textit{N}})) \subseteq \lambda_\text{\textit{N}}.

\text{FNCl}(\lambda_\text{\textit{N}}) \subseteq \lambda_\text{\textit{N}} and \lambda_\text{\textit{N}} \subseteq \text{FNCl}(\lambda_\text{\textit{N}}).

\text{FNInt}(\lambda_\text{\textit{N}}) = \lambda_\text{\textit{N}}.

Proposition 3.2: For any FNS, the following statements satisfy:

i. Every F\text{-open set is FN\alpha\text{-open set.}

ii. Every FN\alpha\text{-closed set is FN\alpha^m\text{-closed set.}

iii. Every F\text{-closed set is FN\alpha^m\text{-closed set.}

iv. Every FNS\text{-closed set is FN\alpha^m\text{-closed set.}

\text{Proof:}

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i. Let \( \lambda_N = \{ <x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) : x \in X \} \) be FN-open set in FNTS \((X, \tau_N)\). Then, by Definition 2.6 (ii) we get,

\[ \lambda_N = \text{FNInt}(\lambda_N) \] (1)

And, by Proposition 2.7 (i) we get, \( \lambda_N \subseteq \text{FNCl}(\lambda_N) \). But, \( \lambda_N \subseteq \text{FNCl}(\text{FNInt}(\lambda_N)) \).

Then, \( \text{FNInt}(\lambda_N) \subseteq \text{FNCl}(\text{FNInt}(\lambda_N)) \).

Therefore, by (1) we get, \( \lambda_N \subseteq \text{FNInt}(\text{FNCl}(\text{FNInt}(\lambda_N))) \).

Hence, \( \lambda_N \) be FN\( \alpha \)-open set in \((X, \tau_N)\).

ii. Let \( \lambda_N = \{ <x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) : x \in X \} \) be FN\( \alpha \)-closed set in FNTS \((X, \tau_N)\).

Then, \( \text{FNInt}(\text{FNCl}(\text{FNInt}(\lambda_N))) \subseteq \lambda_N \).

Now, let \( \beta_N \) be FN\( \alpha \)-open set such that, \( \lambda_N \subseteq \beta_N \).

Since, \( \beta_N \) be FN\( \alpha \)-open set then, is FN\( \alpha \)-open set by (i). Then, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \text{FNCl}(\lambda_N) \subseteq \lambda_N \subseteq \beta_N \).

Therefore, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \beta_N \).

Hence, \( \lambda_N \) be FN\( \alpha \)-closed set in \((X, \tau_N)\).

iii. Let \( \lambda_N = \{ <x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) : x \in X \} \) be FN\( \sigma \)-closed set in FNTS \((X, \tau_N)\).

Then, by Definition 2.6 (i) we get,

\[ \lambda_N = \text{FNCl}(\lambda_N) \] (1)

And, by Proposition 2.7 (i) we get,

\[ \text{FNInt}(\lambda_N) \subseteq \lambda_N \] (2). But, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \text{FNCl}(\lambda_N) \).

Then, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \lambda_N \).

Now, let \( \beta_N \) be FN\( \beta \)-open set such that, \( \lambda_N \subseteq \beta_N \).

Since, \( \beta_N \) be FN\( \beta \)-open set then, is FN\( \beta \)-open set, by (i).

Then, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \lambda_N \subseteq \beta_N \).

Therefore, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \beta_N \).

Hence, \( \lambda_N \) be FN\( \alpha \)-closed set in \((X, \tau_N)\).

iv. Let \( \lambda_N = \{ <x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) : x \in X \} \) be FNS-closed set in FNTS \((X, \tau_N)\).

Then, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \lambda_N \).

Now, let \( \beta_N \) be FN\( \beta \)-open set such that, \( \lambda_N \subseteq \beta_N \).

Since, \( \beta_N \) be FN\( \beta \)-open set then, is FN\( \beta \)-open set, by (i).

Then, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \lambda_N \subseteq \beta_N \).

Therefore, \( \text{FNInt}(\text{FNCl}(\lambda_N)) \subseteq \beta_N \).

Hence, \( \lambda_N \) be FN\( \alpha \)-closed set in \((X, \tau_N)\).

Remark 3.3: The converse of Proposition 3.2 is not true in general and we can show it by the following examples:

Example 3.4:

i. Let \( X = \{ x \} \) define FNSSs \( \lambda_N \) and \( \beta_N \) in \( X \) as follows:

\[ \lambda_N = \{ <x, 0.7, 0.6, 0.5> : x \in X \} \]

\[ \beta_N = \{ <x, 0.8, 0.9, 0.4> : x \in X \} \]

And the family \( \tau_N = \{ \emptyset_N, 1_N, \lambda_N, \beta_N \} \) be FNT such that, \( 1_N \cap \tau_N = \{ 1_N, 0_N, <x, 0.5, 0.4, 0.7>, <x, 0.4, 0.1, 0.8> \} \).

Now if,
\[ \omega_N = \{<x, 0.8, 0.6, 0.5>: x \in X \}. \]

Then, \( \text{FNInt}(F_N) = \{<x, 0.7, 0.6, 0.5>: x \in X \} \), \( \text{FNCl}(\text{FNInt}(F_N)) = \underline{1}_N \) and \( \text{FNInt}(\text{FNCl}(\text{FNInt}(\omega_N))) = \underline{1}_N \).

Therefore, \( \omega_N \subseteq \text{FNInt}(\text{FNCl}(\text{FNInt}(\omega_N))) \).

Hence, \( \omega_N \) be FN\( \alpha \)-open set. But, not FN\( \alpha \)-open set.

\textbf{ii.} Let \( X=\{x\} \) define FNSs \( \lambda_N, \beta_N, \eta_N \) and \( \Psi_N \) in \( X \) as follows:

\( \lambda_N = \{<x, 1, 0.5, 0.7>: x \in X \} \), \( \beta_N = \{<x, 0, 0.9, 0.2>: x \in X \} \),

\( \eta_N = \{<x, 1, 0.9, 0.2>: x \in X \} \) and \( \Psi_N = \{<x, 0, 0.5, 0.7>: x \in X \} \)

And the family \( r_N = \{\Omega_N, \underline{1}_N, \lambda_N, \beta_N, \eta_N, \Psi_N \} \) be FNT such that, \( \underline{1}_N-r_N = \{\underline{1}_N, \Omega_N, <x, 0.7, 0.5, 1>, <x, 0.2, 0.1, 0>, <x, 0.2, 0.1, 1>, <x, 0.7, 0.5, 0> \}

Now if, \( \omega_N = \{<x, 0, 0.4, 0.8>: x \in X \} \) and \( U_N = \{<x, 0, 0.5, 0.7>: x \in X \} \).

Where, \( U_N \) be FN\( \alpha \)-open set such that, \( \omega_N \subseteq U_N \).

Since \( U_N \) be FN\( \alpha \)-open set then, is FN\( \alpha \)-open set by \textbf{Proposition 3.2 (i)}.

Then, \( \text{FNCl}(\omega_N) = \{<x, 0.7, 0.5, 0>: x \in X \} \) and \( \text{FNInt}(\text{FNCl}(\omega_N)) = \{<x, 0, 0.5, 0.7>: x \in X \} \).

Therefore, \( \text{FNInt}(\text{FNCl}(\omega_N)) \subseteq U_N \).

Hence, \( \omega_N \) be FN\( \alpha \)-m closed set.

But, \( \text{FNCl}(\omega_N) = \{<x, 0.7, 0.5, 0>: x \in X \} \),

\( \text{FNInt}(\text{FNCl}(\omega_N)) = \{<x, 0, 0.5, 0.7>: x \in X \} \) and

\( \text{FNCl}(\text{FNInt}(\text{FNCl}(\omega_N))) = \{<x, 0.7, 0.5, 0>: x \in X \} \)

Therefore, \( \text{FNCl}(\text{FNInt}(\text{FNCl}(\omega_N))) \subseteq \omega_N \). Hence, \( \omega_N \) be not FN\( \alpha \)-closed set.

\textbf{iii.} Take, the example which defined in \textbf{ii}. Then, we can see \( \omega_N \) be FN\( \alpha \)-m closed set. But, not FN\( \alpha \)-closed set.

Take again, the example which defined in \textbf{ii}. Then, \( \omega_N \) be FN\( \alpha \)-m closed set. But, not FN\( \alpha \)-closed set.

\textbf{Remark 3.5:} The relation between FNP-closed sets and FN\( \alpha \)-m closed sets are independent and we can show it by the following examples.

\textbf{Example 3.6: (1)} Let \( X=\{x\} \) define FNSs \( \lambda_N \) and \( \beta_N \) in \( X \) as follows:

\( \lambda_N = \{<x, 0.1, 0.2, 0.4>: x \in X \} \), \( \beta_N = \{<x, 0.7, 0.5, 0.2>: x \in X \} \),

And, the family \( r_N = \{\Omega_N, \underline{1}_N, \lambda_N, \beta_N \} \) be FNT such that, \( \underline{1}_N-r_N = \{\underline{1}_N, \Omega_N, <x, 0.4, 0.8, 0.1>, <x, 0.2, 0.5, 0.7> \} \).

Now if, \( \omega_N = \{<x, 0.1, 0.3, 0.4>: x \in X \} \) and

\( U_N = \{<x, 0.7, 0.5, 0.2>: x \in X \} \) where, \( U_N \) be FN\( \alpha \)-open set such that, \( \omega_N \subseteq U_N \).

Since \( U_N \) be FN\( \alpha \)-open set then, is FN\( \alpha \)-open set by \textbf{Proposition 3.2 (i)}

Then, \( \text{FNCl}(\omega_N) = \{<x, 0.4, 0.8, 0.1>: x \in X \} \) and

\( \text{FNInt}(\text{FNCl}(\omega_N)) = \{<x, 0.1, 0.2, 0.4>: x \in X \} \). Therefore, \( \text{FNInt}(\text{FNCl}(\omega_N)) \subseteq U_N \).

Hence, \( \omega_N \) be FNP\( \alpha \)-closed set.

But, \( \text{FNCl}(\omega_N) = \{<x, 0.1, 0.2, 0.4>: x \in X \} \),

\( \text{FNCl}(\text{FNInt}(\omega_N)) = \{<x, 0.4, 0.8, 0.1>: x \in X \} \). Therefore, \( \text{FNCl}(\text{FNInt}(\omega_N)) \subseteq \omega_N \).

Hence, \( \omega_N \) be not FNP-closed set.

\textbf{(2)} Let \( X=\{a, b\} \) define FNS \( \lambda_N \) in \( X \) as follows:

\( \lambda_N = <a, 0.5, b, 0.5), (a, 0.5, b, 0.5), (a, 0.4, b, 0.5) > \). And the family \( r_N = \{\Omega_N, \underline{1}_N, \lambda_N \} \) be FNT.

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Such that, \( \lambda_{N} \cap \tau_{N} = \{ \lambda_{N}, 0_{N}, \langle x, (a, 0.4, b, 0.5), (a, 0.5, b, 0.5), (a, 0.6, b, 0.5) \rangle \} \).

Now if, \( \omega_{N} = \langle x, (a, 0.5, b, 0.4), (a, 0.5, b, 0.5), (a, 0.6, b, 0.5) \rangle \), \( \Lambda \) \( u_{N} = \lambda_{N} \) be \( F\alpha \)-open set such that, \( \omega_{N} \subseteq U_{N} \).

Since, \( U_{N} \) be \( F\alpha \)-open set then, is \( F\alpha \) open set by **Proposition 3.2 (i)**

Then, \( F\alpha \int(F\alpha \cl(\omega_{N})) = 0_{N} \)

Therefore, \( F\alpha \cl(F\alpha \int(\omega_{N})) \subseteq \omega_{N} \). Hence, \( \omega_{N} \) be \( F\alpha \)-closed set.

But, \( F\alpha \cl(F\alpha \int) = 0_{N} \) and \( F\alpha \int(F\alpha \cl(\omega_{N})) = 1_{N} \).

Therefore, \( F\alpha \int(F\alpha \cl(\omega_{N})) \subseteq U_{N} \). Hence, \( \omega_{N} \) be not \( F\alpha \)-closed set.

**Proposition 3.7:** If \( \lambda_{N} \) be \( F\alpha \)-closed set and \( \lambda_{N} \subseteq \eta_{N} \subseteq F\alpha \int(F\alpha \cl(\lambda_{N})) \), Then, \( \eta_{N} \) be \( F\alpha \)-closed set.

**Proof:** Let \( \lambda_{N} = \langle \langle x, \mu_{N}(x), \sigma_{N}(x), v_{N}(x) \rangle : x \in X \rangle \) be \( F\alpha \)-closed set such that, \( \lambda_{N} \subseteq \eta_{N} \subseteq F\alpha \int(F\alpha \cl(\lambda_{N})) \).

Now let \( \beta_{N} \) be \( F\alpha \)-open set such that, \( \eta_{N} \subseteq \beta_{N} \).

Since, \( \lambda_{N} \) be \( F\alpha \)-closed set then, we have

\[ F\alpha \int(F\alpha \cl(\lambda_{N})) \subseteq \beta_{N}, \text{ where } \lambda_{N} \subseteq \beta_{N}. \]

Since, \( \lambda_{N} \subseteq \eta_{N} \) and \( \eta_{N} \subseteq F\alpha \int(F\alpha \cl(\lambda_{N})) \) we get,

\[ F\alpha \int(F\alpha \cl(\eta_{N})) \subseteq F\alpha \int(F\alpha \cl(F\alpha \int(F\alpha \cl(\lambda_{N})))) \subseteq F\alpha \int(F\alpha \cl(\lambda_{N})) \subseteq \beta_{N}. \]

Therefore, \( F\alpha \int(F\alpha \cl(\eta_{N})) \subseteq \beta_{N} \). Hence, \( \eta_{N} \) be \( F\alpha \)-closed set in \((X, \tau_{N})\).

**Proposition 3.8:** Let \((X, \tau_{N})\) be FNSTS. So, the intersection of two \( F\alpha \)-closed sets be \( F\alpha \)-closed set.

**Proof:** Let \( \lambda_{N} \) and \( \beta_{N} \) be \( F\alpha \)-closed sets on FNSTS \((X, \tau_{N})\)

Then, \( F\alpha \int(F\alpha \cl(\lambda_{N})) \subseteq \lambda_{N} \) \( \ldots \ldots (1) \)

And, \( F\alpha \int(F\alpha \cl(\beta_{N})) \subseteq \beta_{N} \) \( \ldots \ldots (2) \)

Consider \( \lambda_{N} \cap \beta_{N} \subseteq F\alpha \int(F\alpha \cl(\lambda_{N})) \cap F\alpha \int(F\alpha \cl(\beta_{N})) \)

\[ = F\alpha \int(F\alpha \cl(\lambda_{N}) \cap F\alpha \cl(\beta_{N})) \]

\[ \subseteq F\alpha \int(F\alpha \cl(\lambda_{N} \cap \beta_{N})). \]

Therefore, \( F\alpha \int(F\alpha \cl(\lambda_{N} \cap \beta_{N})) \subseteq \lambda_{N} \cap \beta_{N} \)

Now, let \( \eta_{N} \) be \( F\alpha \)-open set such that, \( \lambda_{N} \cap \beta_{N} \subseteq \eta_{N} \).

Since, \( \eta_{N} \) be \( F\alpha \)-open set then it is \( F\alpha \)-open set, by **Proposition 3.2 (i)**.

Then, \( F\alpha \int(F\alpha \cl(\lambda_{N} \cap \beta_{N})) \subseteq \lambda_{N} \cap \beta_{N} \subseteq \eta_{N} \).

Therefore, \( F\alpha \int(F\alpha \cl(\lambda_{N} \cap \beta_{N})) \subseteq \eta_{N} \). Hence, \( \lambda_{N} \cap \beta_{N} \) be \( F\alpha \)-closed set in \((X, \tau_{N})\).

**Remark 3.9:** The union of any \( F\alpha \)-closed sets is not necessary to be \( F\alpha \)-closed set and we can show it by the following example.

**Example 3.10:** Take, **Example 3.4 (ii)** if,

\( \omega_{N1} = \langle x, 0.4, 0.5, 1 \rangle : x \in X \rangle \) and \( \omega_{N2} = \langle x, 0.2, 0.8 \rangle : x \in X \rangle \). And \( U_{N} = \langle x, 1.0, 0.7 \rangle : x \in X \rangle \)

Then, \( F\alpha \cl(\omega_{N1}) = \langle x, 0.7, 0.5, 1 \rangle : x \in X \rangle \) and \( F\alpha \int(F\alpha \cl(\omega_{N1})) = 0_{N} \)

Therefore, \( F\alpha \int(F\alpha \cl(\omega_{N1})) \subseteq U_{N} \).

Hence, \( \omega_{N1} \) be \( F\alpha \)-closed set.
And, $\text{FNCI}(\omega_{N_2}) = \{<x, 0.2, 0.1, 0.>: x \in X\}$, $\text{FNInt}(\text{FNCI}(\omega_{N_2})) = 0_N$.

Therefore, $\text{FNInt}(\text{FNCI}(\omega_{N_2})) \subseteq U_N$. Hence, $\omega_{N_2}$ be $\text{FN}^{m}$- closed set.

Therefore, $\omega_{N_1} \cup \omega_{N_2}$ be not $\text{FN}^{m}$- closed set.

**Definition 3.11:** Let $(X, \tau_N)$ be FNTS and $\lambda_N = <x, \mu_N (x), \sigma_N (x), \nu_N (x)>$ be FNS in $X$. Then, the fuzzy neutrosophic Alpha $^{m}$ closure of $\lambda_N$ ($\text{FN}^{m}\text{Cl}$, for short) and fuzzy neutrosophic Alpha $^{m}$ interior of $\lambda_N$ ($\text{FN}^{m}\text{Int}$, for short) are defined by:

i. $\text{FN}^{m}\text{Cl}(\lambda_N) = \cap \{\beta_N: \beta_N \text{ is } \text{FN}^{m}\text{-closed set in } X \text{ and } \lambda_N \subseteq \beta_N\}$,

ii. $\text{FN}^{m}\text{Int}(\lambda_N) = \cup \{\beta_N: \beta_N \text{ is } \text{FN}^{m}\text{-open set in } X \text{ and } \beta_N \subseteq \lambda_N\}$.

**Proposition 3.12:** Let $(X, \tau_N)$ be FNTS and $\lambda_N$, $\beta_N$ are FNSs in $X$. Then the following properties hold:

i. $\text{FN}^{m}\text{Cl}(0_N) = 0_N$ and $\text{FN}^{m}\text{Cl}(1_N) = 1_N$,

ii. $\lambda_N \subseteq \text{FN}^{m}\text{Cl}(\lambda_N)$, 

iii. If $\lambda_N \subseteq \beta_N$, then $\text{FN}^{m}\text{Cl}(\lambda_N) \subseteq \text{FN}^{m}\text{Cl}(\beta_N)$, 

iv. $\lambda_N$ be $\text{FN}^{m}$-closed set if $\lambda_N = \text{FN}^{m}\text{Cl}(\lambda_N)$,

v. $\text{FN}^{m}\text{Cl}(\lambda_N) = \text{FN}^{m}\text{Cl}(\text{FNTS}(\lambda_N))$.

**Proof:**

i. by **Definition 3.11** (i) we get, 

$\text{FN}^{m}\text{Cl}(0_N) = \cap \{\beta_N: \beta_N \text{ is } \text{FN}^{m}\text{-closed set in } X \text{ and } 0_N \subseteq \beta_N \} = 0_N$

And, 

$\text{FN}^{m}\text{Cl}(1_N) = \cap \{\beta_N: \beta_N \text{ is } \text{FN}^{m}\text{-closed set in } X \text{ and } 1_N \subseteq \beta_N \} = 1_N$.

ii. $\lambda_N \subseteq \cap \{\beta_N: \beta_N \text{ is } \text{FN}^{m}\text{-closed set in } X \text{ and } \lambda_N \subseteq \beta_N \} = \text{FN}^{m}\text{Cl}(\lambda_N)$.

iii. Suppose that $\lambda_N \subseteq \beta_N$ then,

$\cap \{\beta_N: \beta_N \text{ is } \text{FN}^{m}\text{-closed set in } X \text{ and } \lambda_N \subseteq \beta_N \} \subseteq \cap \{\eta_N: \eta_N \text{ is } \text{FN}^{m}\text{-closed set in } X \text{ and } \beta_N \subseteq \eta_N \}$. Therefore, $\text{FN}^{m}\text{Cl}(\lambda_N) \subseteq \text{FN}^{m}\text{Cl}(\beta_N)$.

iv. If, $\lambda_N$ be $\text{FN}^{m}$-closed set, then

$\text{FN}^{m}\text{Cl}(\lambda_N) = \cap \{\beta_N: \beta_N \text{ is } \text{FN}^{m}\text{-closed set in } X \text{ and } \lambda_N \subseteq \beta_N \}$

And, by (ii) we get, $\lambda_N \subseteq \text{FN}^{m}\text{Cl}(\lambda_N)$........(1)

Thus, $\lambda_N = \cap \{\beta_N: \beta_N \text{ is } \text{FN}^{m}\text{-closed set in } X \text{ and } \lambda_N \subseteq \beta_N \}$,

Therefore, $\lambda_N = \text{FN}^{m}\text{Cl}(\lambda_N)$.

Conversely; Let $\lambda_N = \text{FN}^{m}\text{Cl}(\lambda_N)$ by using **Definition 3.11** (i), we get, $\lambda_N$ be $\text{FN}^{m}$-closed set.

v. Since, by (iv) we get, $\lambda_N = \text{FN}^{m}\text{Cl}(\lambda_N)$

Then, $\text{FN}^{m}\text{Cl}(\lambda_N) = \text{FN}^{m}\text{Cl}(\text{FNTS}(\lambda_N))$.

**Proposition 3.13:** Let $(X, \tau_N)$ be FNTS and $\lambda_N$, $\beta_N$ are FNSs in $X$. Then the following properties hold:

i. $\text{FN}^{m}\text{Int}(0_N) = 0_N$ and $\text{FN}^{m}\text{Int}(1_N) = 1_N$,

ii. $\text{FN}^{m}\text{Int}(\lambda_N) \subseteq \lambda_N$,

iii. If $\lambda_N \subseteq \beta_N$ then $\text{FN}^{m}\text{Int}(\lambda_N) \subseteq \text{FN}^{m}\text{Int}(\beta_N)$,

iv. $\lambda_N$ be $\text{FN}^{m}$-open set if $\lambda_N = \text{FN}^{m}\text{Int}(\lambda_N)$,
v. \( \text{FN}^\alpha \text{Int} (\lambda_N) = \text{FN}^\alpha \text{Int} (\text{FN}^\alpha \text{Int} (\lambda_N)). \)

**Proof:**

i. by **Definition 3.11 (ii)** we get, 
\[
\text{FN}^\alpha \text{Int}(\emptyset_N) = \cup \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \beta_N \subseteq 0_N \} = 0_N.
\]
And, 
\[
\text{FN}^\alpha \text{Int}(1_N) = \cup \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \beta_N \subseteq 1_N \} = 1_N.
\]

ii. Follows from **Definition 3.11 (ii).**

iii. \( \text{FN}^\alpha \text{Int}(\lambda_N) = \cup \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \beta_N \subseteq \lambda_N \}. \)
Since, \( \lambda_N \subseteq \beta_N \) then, 
\[
\cup \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \beta_N \subseteq \lambda_N \} \subseteq \{ \eta_N : \eta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \eta_N \subseteq \beta_N \}
\]
Therefore, \( \text{FN}^\alpha \text{Int}(\lambda_N) \subseteq \text{FN}^\alpha \text{Int}(\beta_N). \)

iv. We must proof that, \( \text{FN}^\alpha \text{Int}(\lambda_N) \subseteq \lambda_N \) and \( \lambda_N \subseteq \text{FN}^\alpha \text{Int}(\lambda_N). \)

Suppose that \( \lambda_N \) be \( \text{FN}^\alpha \text{-open set in X.} \)

Then, \( \text{FN}^\alpha \text{Int}(\lambda_N) = \cup \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \beta_N \subseteq \lambda_N \}. \)

by using (ii) we get, \( \text{FN}^\alpha \text{Int}(\lambda_N) \subseteq \lambda_N \) ……(1)

Now to proof, \( \lambda_N \subseteq \text{FN}^\alpha \text{Int}(\lambda_N) \), we have, For all \( \lambda_N \subseteq \lambda_N \), the \( \text{FN}^\alpha \text{Int}(\lambda_N) \subseteq \lambda_N \)
So, we get \( \lambda_N \subseteq \cup \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \beta_N \subseteq \lambda_N \} = \text{FN}^\alpha \text{Int}(\lambda_N) \) ……(2)

From (1) and (2) we have, \( \lambda_N = \text{FN}^\alpha \text{Int}(\lambda_N) \).

Conversely; assume that \( \lambda_N = \text{FN}^\alpha \text{Int}(\lambda_N) \) and by using **Definition 3.11 (ii)** we get, \( \lambda_N \) be \( \text{FN}^\alpha \text{-open set in X.} \)

v. By (iv) we get, \( \lambda_N = \text{FN}^\alpha \text{Int}(\lambda_N) \)

Then, \( \text{FN}^\alpha \text{Int}(\lambda_N) = \text{FN}^\alpha \text{Int}(\text{FN}^\alpha \text{Int}(\lambda_N)). \)

**Proposition 3.14:** Let \( (X, \tau) \) be FNTS. Then, for any fuzzy neutrosophic subsets \( \lambda_N \) of \( X. \)

i. \( \text{FN}_N^\alpha (\text{FN}^\alpha \text{Int}(\lambda_N)) = \text{FN}^\alpha \text{Cl}(\text{FN}_N^\alpha \lambda_N). \)

ii. \( \text{FN}_N^\alpha (\text{FN}^\alpha \text{Cl}(\lambda_N)) = \text{FN}^\alpha \text{Int}(\text{FN}_N^\alpha \lambda_N). \)

**Proof:**

i. \( \text{FN}^\alpha \text{Int}(\lambda_N) = \cup \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \beta_N \subseteq \lambda_N \} \), by the complement we get, 
\[
\text{FN}_N^\alpha \text{Int}(\lambda_N) = \cup \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \beta_N \subseteq \lambda_N \}. \)
So, 
\[
\text{FN}_N^\alpha (\text{FN}^\alpha \text{Int}(\lambda_N)) = \cap \{ \text{FN}_N^\alpha \beta_N \}:
\]
\[
(\text{FN}_N^\alpha \beta_N) \text{ is } \text{FN}^\alpha \text{-closed set in X and } (\text{FN}_N^\alpha \beta_N) \subseteq (\text{FN}_N^\alpha \beta_N). \}
\]

Now, replacing \( \text{FN}_N^\alpha \beta_N \) by \( \eta_N \) we get, 
\[
\text{FN}_N^\alpha (\text{FN}^\alpha \text{Int}(\lambda_N)) = \cap \{ \eta_N : \eta_N \text{ is } \text{FN}^\alpha \text{-closed set in X and } (\text{FN}_N^\alpha \lambda_N) \subseteq \eta_N \} = \text{FN}^\alpha \text{Cl}(\text{FN}_N^\alpha \lambda_N). \)

ii. \( \text{FN}^\alpha \text{Cl}(\lambda_N) = \cap \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-closed set in X and } \lambda_N \subseteq \beta_N \}, \) by the complement we get, 
\[
\text{FN}_N^\alpha (\text{FN}^\alpha \text{Cl}(\lambda_N)) = \cap \{ \beta_N : \beta_N \text{ is } \text{FN}^\alpha \text{-closed set in X and } \lambda_N \subseteq \beta_N \}. \]
So, 
\[
\text{FN}_N^\alpha (\text{FN}^\alpha \text{Cl}(\lambda_N)) = \cup \{ \text{FN}_N^\alpha \beta_N : (\text{FN}_N^\alpha \beta_N) \text{ is } \text{FN}^\alpha \text{-open set in X and } (\text{FN}_N^\alpha \lambda_N) \subseteq (\text{FN}_N^\alpha \lambda_N). \}
\]
Again replacing \( \text{FN}_N^\alpha \beta_N \) by \( \eta_N \) we get, 
\[
\text{FN}_N^\alpha (\text{FN}^\alpha \text{Cl}(\lambda_N)) = \cup \{ \eta_N : \eta_N \text{ is } \text{FN}^\alpha \text{-open set in X and } \eta_N \subseteq (\text{FN}_N^\alpha \lambda_N) \} = \text{FN}^\alpha \text{Int}(\text{FN}_N^\alpha \lambda_N). \)

**Proposition 3.15:** Fuzzy neutrosophic interior of \( \text{F}_N \)-closed set be \( \text{FN}^\alpha \text{-closed set.} \)
proof: Let $\lambda_N = \{<x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x)> : x \in X\}$ be $F_N$-closed set in $FNTS (X, \tau_N)$. Then, by **Definition 2.6** (i) we get, $\lambda_N = FNCl(\lambda_N)$. So, $FNInt(\lambda_N) = FNInt(FNCl(\lambda_N))$. …..(1)

And, by **Proposition 2.7** (i) we get,

\[ FNInt(\lambda_N) \subseteq \lambda_N \] ……..(2)

From (1) and (2) we get, $FNInt(FNCl(\lambda_N)) \subseteq \lambda_N$

Now, let $\beta_N$ be $F_N$-open set such that, $\lambda_N \subseteq \beta_N$.

Since, $\beta_N$ be $F_N$-open set, then $\beta_N$ is $FNa$-open set by **Proposition 3.2** (i)

Therefore, $FNInt(FNCl(\lambda_N)) \subseteq \beta_N$. Hence, $\lambda_N$ be $FNa^m$-closed set in $(X, \tau_N)$.

**Remark 3.16:** The relationship between different sets in FNTS can be showing in the next diagram and the converse is not true in general.

![Diagram 1](image)

**Conclusion**

In this paper, the new concept of a new class of sets and called them fuzzy neutrosophic $\alpha^m$-closed sets. we investigated the relation between fuzzy neutrosophic $\alpha^m$-closed sets, fuzzy neutrosophic $\alpha$ closed sets, fuzzy neutrosophic closed sets, fuzzy neutrosophic semi closed sets and fuzzy neutrosophic pre closed sets with some properties.

**References**


Fatimah M. Mohammed and Shaymaa F. Matar, Fuzzy Neutrosophic Alpha-open Closed Sets in Fuzzy Neutrosophic Topological Spaces

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