SunQM-3s6: Predict mass density r-distribution for Earth and other rocky planets based on \(\{N,n\}\) QM probability distribution

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Abstract

My previous papers (SunQM-3, SunQM-3s1, SunQM-3s2) have shown that the structure of our solar system can be described by Solar \(\{N,n\}\) QM model. Therefore, everything inside solar system (including all planets) should be able to be described by the QM probability density distribution. In this paper, from studying Earth’s known internal structure and mass density distribution, I have developed a method which can be used to estimate any planet’s internal structure and mass density. This method can be expressed as: Planet mass = \(4\pi \int \text{(planet’s QM probability density r-distribution)} * W * D * r^2\ dr\), where mass density \(D = a*r + b\), and \(W\) is a scaling factor. I applied this method to all four rocky planets, and predicted the internal structure and the (close to the true) mass density distribution for these planets. Besides the \(p\{-1.4/4\}\) surface and \(p\{-1.2/4\}\) core, a \(p\{-1.3/4\}\) QM structure is found to be the foundation both for Earth’s liquid iron core, and for Mars current size. Furthermore, Earth’s current mantle structure is found to be the \(p\{-1.15/16\}\) QM structure of Earth. Dynamics of this QM structure evolution has been discussed. The results from this paper (as well as from papers of SunQM-3s7 and SunQM-3s8) convinced me that all planets and stars were formed and evolved under planet’s (or star’s) QM.

Introduction

In a series papers, the quantum mechanics of Solar system has been established \([1]~[9]\). In paper SunQM-3 \([6]\), I established a non-spinning Solar system’s (or pre-Sun’s) QM model based on \(\{N,n\}\) QM structure and Schrodinger equation/solution. From it, I was able to use Schrodinger equation/solution to construct the radial probability density of mass distribution for Sun ball from \([-2,n=1...5]\) super-shell to \([0,2]\) shell (see Figure 3 of SunQM-3). In a point G-force produced spherical QM structure, the probability density function can be separated into two parts, one in r-dimension, described by \(r^2 * |R(nl)|^2\), and one in \(\theta\)-2D-dimension, described by \(\sin(\theta) * |Y(lm)|^2\). In paper SunQM-3s3 \([9]\), we see that the probability peaks of \(\sin(\theta) * |Y(lm)|^2\) in \(\theta\)-dimension generate the atmospheric band pattern on Jupiter (and Saturn, Earth, etc.)’s surface. In current paper, I will present that the probability peaks of \(r^2 * |R(nl)|^2\) in r-dimension generate the internal (core) structure for all planets and Sun. Furthermore, by combining the probability density and known total mass, I try to predict the planets’ (Earth, Jupiter, etc.) internal structure with (close to the true) mass density. Note: the content of this paper was spun-off from paper SunQM-3s3 (because it is too long), so it should have been in serial number SunQM-3s4. However, SunQM-3s4 has been assigned to QM calculation for Saturn’s ring, and SunQM-3s5 has been assigned to nL0 effect and bipolar overflow. So the current paper is assigned as SunQM-3s6. Note: due the size limitation, the same analysis for the four ice/gas planets has been spun-off from this paper, and moved to a new paper SunQM-3s7. Also, the same analysis for Sun has been spun-off from this paper, and moved to a new paper SunQM-3s8. Note: for \(\{N,n\}\) QM nomenclature as well as the general notes for \(\{N,n\}\) QM model, please see my paper SunQM-1 section VII. Note: Microsoft Excel’s number format is often used in this paper, for example: \(x^2 = x^2, 3.4E+12 = 3.4\times10^{12}, 5.6E-9 = 5.6\times10^{-9}\).

I. Earth’s mass radial distribution can be described by a simple mass integration equation of QM probability
From paper SunQM-1s3, we know that Earth can be described by a p{N,n/2} QM structure, which means its Moon’s orbit can be described by the exterior p{N,n/2} QM, and its internal structure (mainly the inner core) can be described by the interior p{N,n/2} QM. From paper SunQM-3s3, we know that if we set Earth surface as p{0,1}, then Earth main body (including mantle and outer core) mass is in p{-1,1/2}o orbit space, and this orbit covers shell space from p{-1,1/2} to p{-1,2/2} = p{0,1}. Earth inner core mass is within size of p{-1,1/2}. Now let us set Earth’s inner core size p{-1,1/2} as r₁, so r₁ ≈ 6.38E+6 / 4 = 1.6E+6 meters. Accordingly, Earth main body’s QM can be described by wave function |nlm> with n = 1 and n = 2. Therefore, Earth main body’s radial mass density distribution should follow the radial probability density of r² * |R(1,0)|² plus r² * |R(2,0)|² up to r/r₁ = 4. Figure 1a shows the plot of the original radial wave function from Schrodinger equation’s solution of r² * |R(n,l)|² vs. r/r₁, for Earth’s p{-1,1/2}o orbit shell in r-dimension. The Earth main body’s radial mass density distribution should correlate to the thick grey curve up to r/r₁ = 4.

In wiki “Structure of the Earth”, it shows a plot of “Earth’s radial density distribution”, which is the true (or close to true) mass density kg/m³. I re-plotted it in Figure 1b, and named it as “Earth true mass D”.[11] So the purpose of section I is to use Earth’s known radial density distribution as the template, 1) to find out that how to correlate QM’s radial probability density distribution to Earth’s true mass D, and then 2) to develop a practical useful method that can be used to predict all planets and even Sun’s internal structure and radial mass density distribution.

**Figure 1a (left). The original r² * |R(n,l)|² vs. r/r₁ plot for Earth’s p{-1,1/2}o and p{-1,2/2}o orbit shells in r-dimension.**

**Figure 1b (right). Comparing the scaled-up radial probability distribution to the known Earth’s radial mass density distribution. Earth’s true mass D curve obtained from wiki “Structure of the Earth”, originally created by A. M. Dziewonski & D. L. Anderson (1981) [11].**

**I-a. Directly using QM probability curve as the mass density radial distribution for the mass integration**

The simplest method is to directly scale-up the QM probability of the Earth ball and use it as the Earth’s mass density radial distribution for the mass integration (as shown below):

\[
\text{Mass (r, } \theta, \varphi) = \iiint r^2 * (|R(1,0)|^2 + |R(2,0)|^2 + |R(2,1)|^2) * W * \sin(\theta) * r^2 \, dr \, d\theta \, d\varphi, \quad [r = 0, 6.38E+6 \text{ m}; \theta = 0, \pi; \varphi = 0, 2\pi]
\]

or

\[
5.97E+24 \text{ kg} = 4\pi \int r^2 * (4/(1.6E+6)^3) * \exp(-r/1.6E+6) + 1/2/(1.6E+6)^3 * (1 - r/2/1.6E+6)^2 * \exp(-r/1.6E+6) + 1/24/(1.6E+6)^3 * (r/1.6E+6)^2 * \exp(-r/1.6E+6)) * W * r^2 \, dr, \quad [r = 0, 6.38E+6 \text{ m}]
\]
where $W$ is a scaling factor. Manually adjusting $W$ value for the integration gives $W = 2480$ (see the integration result above, using WolframAlpha online calculator). Note: the very low probability density near the sphere center (within size of $p\{-2,1\}$) is ignored due to that it has very low volume. If someone really concerning about it, you can add another $r^2|\mathcal{R}(n\ell)|^2$ with $r^2 = 6.38E+6/4^2$ m to fill this region (see paper SunQM-3s7 section V for example). The result will not make any significant difference. Note: the probability $r^2*(|\mathcal{R}(1,0)|^2 + |\mathcal{R}(2,0)|^2 + |\mathcal{R}(2,1)|^2)$ has no physics unit. It is only an $r$-dependent scaling factor.

In Figure 1b, the scaled-up radial probability density distribution is plotted against the known Earth’s radial mass density distribution. Obviously they do not match each other well. So this method (by directly using probability density radial distribution) is not acceptable.

**I-b. Using linear function as the mass density radial distribution for the mass integration**

1) If using two linear functions to fit to Earth’s true mass density curve in Figure 1b, then I obtain two mass density linear curves:

$$D = -0.0008x + 13000 \text{ for } 0 \leq r \leq 3.49E+6 \text{ m}$$

and

$$D = -0.0008x + 8600 \text{ for } 3.49E+6 \text{ m} \leq r \leq 6.38E+6 \text{ m}$$

$$\int_{0}^{3.49 \times 10^6} 4 \pi (-0.0008x + 13000) x^2 \, dx = 1.9419157473746545 \times 10^4$$

$$\int_{3.49 \times 10^6}^{6.38 \times 10^6} 4 \pi (-0.0008x + 8600) x^2 \, dx = 4.032546201362792 \times 10^4$$

The sum of these two integrated mass = 1.94E+24 kg + 4.03E+24 kg = 5.97E+24 kg, which closely equals to Earth’s total mass. Figure 2a shows that the true Earth’s mass density radial distribution can accurately fit by these two linear functions. Although this method gives a accurate fitting, it is not useful for predicting other planets (or Sun)’s internal structure, because for other planets (or Sun), the only information we have is the size, the total mass, and the $p\{N,n\}$ QM structure. So we have to use planet’s QM probability density curve because I believe it is correlates to planet’s internal structure.

2) If using a single linear function to fit to Earth’s true mass density curve in Figure 2a, then I obtain the mass density linear curve:

$$D = -0.0008x + 9300, \text{ for } 0 \leq r \leq 6.38E+6 \text{ m}.$$
I-c. Using $D = a*r + b$ multiply $r^2 * |R(nl)|^2$ as the mass density radial distribution for the mass integration

From the test in both sections I-a and I-b, we can see that if we use the single linear function (e.g., $D = -0.0008*x + 9300$) as the base line, and scaling-up at the $r < 3.49E+6$ m region (e.g., by multiplying the probability density distribution), then we can have the fitted mass density curve more closer to the true Earth’s mass density curve. Now let us try it.

First, to obtain the single linear function $D = a*r + b$, besides the mass integration $= 5.97E+24$ kg, we need to add one border condition: at $r = 6.38E+6$ m, $D \approx 3400$ kg/m$^3$. So by manually fitting the integration

\[
\text{Mass} \; (r, \theta, \phi) = \int \int \int (a*r + b) * \sin(\theta) * r^2 \; dr \; d\theta \; d\phi, \; [r = 0, 6.38E+6 \; \text{m}; \; \theta = 0, \pi; \; \phi = 0, 2\pi]
\]

or

\[
5.97E+24 \; \text{kg} = 4\pi \int (a*r + b) * r^2 \; dr, \; [r = 0, 6.38E+6 \; \text{m}]
\]

we obtain $D = -0.0013 * r + 11720$ (kg/m$^3$), and it satisfy the border condition. Second, we use QM probability to weight the linear $D$ equation, and then do the mass integration:

\[
\text{Mass} \; (r, \theta, \phi) = \int \int \int r^2 * (|R(1,0)|^2 + |R(2,l)|^2) * W * D * \sin(\theta) * r^2 \; dr \; d\theta \; d\phi, \; [r = 0, 6.38E+6 \; \text{m}; \; \theta = 0, \pi; \; \phi = 0, 2\pi]
\]

or

\[
5.97E+24 \; \text{kg} = 4\pi \int r^2 * (4/(1.6E+6)^3 \exp(-2*r/(1.6E+6)) + 1/2/(1.6E+6)^3 *(1 - r/(2/1.6E+6))^2 * \exp(-r/1.6E+6) + 1/24/(1.6E+6)^3 *(r/1.6E+6)^2 * \exp(-r/(1.6E+6))) \; W * (-0.0013 * x + 11720) * r^2 \; dr, \; [r = 0, 6.38E+6 \; \text{m}]
\]

By manually adjusting the value of $W$ to fit the integration equation, we obtain $W \approx 4.4E+6$.

Table 1. Correlate Earth’s mass density radial distribution to Earth’s $\{N,n\}$ QM probability function.
### Predicting Earth’s mass density radial distribution

**Note:** if use \( r_1 = 0.16 \text{ E}+7 \) m, then the maximum Probability value = 3.77. If use \( r_1 = 1.6\text{E}+6 \) m, then the maximum Probability value = 3.77E-7. The E-7 probability must be due to that the radial wave function \( R(n,l) \) is normalized for \( a_0 = 5.299 \text{E}-11 \) m. So when using this probability, I need to scale it up to \( ~1\text{E}+7 \) times to make it around one. We can avoid this trouble by deducing out the radial wave function \( R(n,l) \) that specifically normalized to Earth’s \( r_1 = 1.6\text{E}+6 \) m. But I am only a citizen scientist of QM, it is too much work for me to do it.

In Figure 2b, the linear “\( \text{D} = -0.0013 \cdot r + 11720 \)” curve, and the “\( \text{D*Prob} \cdot 4.4\text{E}+6 \)” curve, are compared to the Earth’s true mass density curve. It gives the best fitting result when \( W \) is adjusted to be 4.4E+6 (see in Table 1, column 11 “\( \text{mass} = (\text{D*Prob} \cdot 1\text{E}+7 \cdot 0.44) \cdot \text{ΔV} \)” top line’s 5.44E+24 kg which almost equals to column 9 “\( \text{mass} = \text{D*ΔV} \)” top line’s 5.49E+24 kg). We can see that the D*Prob*4.4E+6 curve matches to the Earth’s true mass density curve reasonably well at \( r > 3.49\text{E}+6 \) m. The difference at \( r \leq 3.49\text{E}+6 \) m will be explained in section I-e.

**Figure 2b.** Predicting Earth’s mass density radial distribution using \([N,n]\) QM probability function. Earth’s true mass D curve obtained from wiki “Structure of the Earth”, originally created by A. M. Dziewonski & D. L. Anderson (1981) [111].

### Inspired by the stepped curve of Earth’s true mass density distribution, I manually draw a stepped line based on the D*Prob*4.4E+6 curve (see the thick grey line). According to this line, I can predict that there are two (major) layers with one interfaces for Earth’s internal structure: The inner core (\( 0 < r < 2.2\text{E}+6 \) m) with \( D \approx 14500 \text{ kg/m}^3 \), and the main body (\( -3\text{E}+6 \text{ m} < r < -6.4\text{E}+6 \) m) with \( D \approx 7000 \rightarrow 3400 \text{ kg/m}^3 \) (see columns 12 to 13 in Table 1). By using this newly predicted

### Predict Earth’s mass density vs. r using \([N,n]\) QM probability function

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<table>
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<th>r (meter)</th>
<th>Mass Density, kg/m^3</th>
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<td>1.00+07</td>
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</table>

predicted D = Earth surface

\( \text{Prob}(n=1..2) \cdot 4.4\text{E}+6 \)
\( \text{D} = -0.0013 \cdot r + 11720 \)
\( \text{D*Prob*4.4E+6} \)
\( \text{Earth’s true mass D} \)
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mass density to re-calculate Earth’s mass, it has been proved that this prediction is accurate enough (the calculation is not shown here).

I-d. Reconstitute the original planets’ atmosphere \( p\{1,1/2\} \), main body \( p\{0,1/2\} \), and core \( p\{-1,1/2\} \)

Many calculations and discussions below will have to use the original Earth (or original planets)’s data. In my paper SunQM-1s3 Table 9, I reconstituted the original planets’ atmosphere \( p\{1,1/2\} \), main body \( p\{0,1/2\} \), and core \( p\{-1,1/2\} \) for all eight planets. There, with the known (modeled) total mass for each planet, I used the guessed mass density \((3340 \text{ kg/m}^3) \) for all four rocky planets, and \(1326 \text{ kg/m}^3 \) for all four ice-gas planets) to calculate the size of each original planet. Now I realize that all eight original planets were ice-gas planet, so they should have ice-gas planet’s mass density. In the new calculation (see Table 2 below), I pick Neptune’s mass density \((1638 \text{ kg/m}^3) \) and applied to all eight original planets. This is because I believe that Neptune is the only one (among eight planets) that still keeps the original state after its formation. So from now on, Table 2 of SunQM-3s6 should replace Table 9 of SunQM-1s3 for calculations in all my SunQM series papers.

It is unlikely that all mass in each n shell had been accreted into a planet, especially in the \( \{1,n=2..6\} \) super-shell. So in Table 2 I assume that for \( \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{1,6\} \) shells, each accreted 50\%, 60\%, 70\%, 80\%, 90\% of total mass in its shell space, while for \( \{2,n=2..5\} \) orbits and beyond, they accreted 100\% of total mass in its shell. The grey out bottom lines in Table 2 are supposed that the mass (in Kuiper belt) in each of \( \{4,n=1..5\} \) orbit shells had been accreted to be planets.

Table 2. Reconstitute the original planets’ atmosphere \( p\{1,1/2\} \), main body \( p\{0,1/2\} \), and core \( p\{-1,1/2\} \) (A correction for Table 9 in paper SunQM-1s3).

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I-e. Why the biggest drop of mass density of Earth is not at 1/4 of \(6.38E+6 \text{ m} = 1.6E+6 \text{ m} \), but at \( r \approx 3.49E+6 \text{ m} \)?

One possible reason is: the original Earth had \(\sim 20\times\) of current Earth mass (see Table 2), so it had much higher gravity pressure at center, and formed much higher mass density (> \(16000 \text{ kg/m}^3\)) in the inner core \(\{-1,1/2\}\). After Earth’s outer atmosphere was stripped off, the \(20\times\) Earth mass decreased to today’s 1x Earth mass, the gravity pressure at...
center greatly decreased, and the original core expanded its r from 1.62E+6 m to today’s 3.49E+6 m, with its D decreased to 13000 ~10000 kg/m^3. If this hypothesis is correct, then we can calculate out the original inner core mass density as D = 1.94E+24 / (4 / 3 * π * 1.62E+6^3) = 9.1E+4 kg/m^3, where 1.94E+24 kg is the mass of current Earth outer core with r = 3.49E+6 m (see section I-b). Comparing to the current Neptune, which has 17× of Earth mass and the predicted inner core mass density = 2.15E+4 kg/m^3 (see paper SunQM-3s7, Figure 2b, column 15), the value of 9.1E+4 kg/m^3 seems too high for the inner core of the original Earth (20× Earth mass). So the expansion of 1.62E+6 m original core to today’s 3.49E+6 m must have included and mixed with some mass that was originally inside r = 3.49E+6 m sphere.

Interestingly, this 3.49E+6 m is not a random value, it is one of the current Earth’s weak QM structure at p{-1,1.5//2} = p{-1.3//4}, calculated as r = 6.38E+6 *(3/4)^2 = 3.59E+6 m. (see paper SunQM1s3, Table 2b, columns 9 ~ 10, Earth has a strong QM structure of p{N,n/2}, and a weak QM structure p{N,n/4}).

A p{N,n/2} QM can always be naturally described in a p{N,n/4} QM. So let us use p{N,n/4} QM to explain the Earth inner core’s evolution: When Earth was originally formed (with ~20× Earth mass), it was under the original Earth’s p{N,n/4} QM, with the original atmosphere surface p{0,1//4} = p{-1.4//4} at r = 2.58E+7 m. The original Earth inner core was at p{-1,1//4} QM structure, with r = 1.62E+6 m. After lost over 19/20 = 95% of mass, the original p{N,n} QM of Earth was gone. The new (current) Earth established a new p{N,n/4} QM with the new p{0,1//4} at the new surface r = 6.38E+6 m. The 95% mass decreasing removed most of the gravity pressure at Earth’s center, so the original inner core expanded to a p{-1,3//4} QM structure of current Earth. Meanwhile, to adapt the current Earth’s (strong) QM of p{N,n/2}, a new core was formed at r = 1.62E+6 m as solid iron core p{-1,2/4}, with mass density only slightly higher than the old expanded (liquid iron) core. So it is p{N,n} QM that determines the dynamics of Earth inner core’s evolution. If this is correct, then all four rocky planets will have the expanded inner core and the lower D, due to all of their original p{1,1//2} outer atmospheres (from ~20× to ~ 50× of Earth mass, see Table 2) were stripped off.

Furthermore, because of the existence of a major cliff-fall of mass density at r = 3.49E+6 m which is at p{-1,3//4} QM structure, the p{N,n/4} QM seems to be more accurate for current Earth’s internal structure (than that of p{N,n/2}). Under a p{N,n/4} QM, current Earth’s surface is at p{0,1//4} = p{-1,4//4}, the liquid iron core is at p{-1,3//4}, the inner (solid iron) core is at p{-1,2/4}. Then it has to be a p{-1,1//4} QM structure with r = 6.38E+6 /16 = 4.0E+5 m (let’s name it as inner-inner core), or 1/4 of Earth’s inner core in radius. Since there is no inner-inner core of Earth has been reported so far, the mass density difference between inner-inner core and inner core must be very small (e.g., 13000 kg/m^3 vs. 12500 kg/m^3). Wiki “Earth” mentioned that “The inner core was ... composed primarily of iron and some nickel”. From this I predict that Earth’s inner-inner core p{-1,1//4} is primarily made of (compressed, solid) Nickel, Au, Pb, etc., and Earth’s inner core p{-1,2/4} is primarily made of (compressed, solid) iron. I hope that with improved technology, scientists will find the inner-inner core of Earth in the future. I also believe that this inner-inner core also exists in all four rocky planets.

I-f. The p{N,n/4} QM’s probability density (stepped) curve forces elements to reposition themselves according to their weight through convection

After all leftover mass in {1,5} orbit shell accreted to be planet Earth, all mass in the original Earth was completely melted. The mass density in r-distribution was mainly caused by the gravity attraction, so it had a D = A / r^B - C like (smooth, non-stepped) curve. However, through previous analysis, we know that Earth formation was strictly follow the p{N,n/2} QM (or p{N,n/4} QM). The probability density of p{N,n/4} QM produced peaks at n = 1, 2, 3, 4, or r/r_1 = 1, 4, 9, 16, which in turn caused the mass density r-distribution to become a stepped curve. This QM property not only compressed the mass at the Earth center, but also differentiated the elements’ distribution by their weight, and drove the heavy elements to the center of Earth. I just learned that this elements weight differentiation process is called “Planetary differentiation” (see wiki “Planetary differentiation”). So the planetary differentiation process is driven by planet’s QM, and it is through the heat induced mass convection process. After fully differentiated, the original Earth then had a iron inner core p{-1,1//4} with r ~ 1.62E+6 m, a p{-1,1//4} orbit shell up to r = 6.46E+6 m mainly with C, O, Mg, Si, etc., a p{-1,2/4} orbit shell up to r = 1.62E+6^3 *3^2 = 1.46E+7 m mainly with H_2O and CH_4, and a p{-1,3//4} orbit shell up to r = 2.58E+7 m with mainly H atoms. After the ice-evap-line expanded to {1,8}, all mass in Earth’s p{-1,3//4}o and p{-1,2//4}o orbit shells was stripped.
we can use \( (\frac{363}{364})^2 \) = 35000 m. So Earth's averaged crust thickness is 35000 m, and it matches 6.38E+6 * [(n - 1) / n] = 35000. Solving this equation, we obtain n’ = 1 / [1 - sqrt(1 - 35000 / 6.38E+6)] = 364.07. Let n’ ≥ 364. Let us choose n’ = 364. Then 6.38E+6 * [1 - (363/364)] = 35000 m. So Earth’s crust starts at p{-1.363/364} QM size, and ends at p{-1.364/364} = p{0.1} QM size. Or, we can use some other n’ values. For example, let us use n’ = 2^16 = 65536. Then after solving the equation, we can off, and was excited to \{2,2\} and \{2,3\} orbits. Meanwhile, the release of gravity pressure (from 20\times to 1\times Earth mass) caused the center iron shell expanded. Then the Earth updated its p{N,n/4} QM according to its current mass and size, with \( r \) of p{-1,4/4} shrank to \( r = 6.38E+6 m \). So a new p{-1.1/4} inner core was formed mainly with heavy elements like Ni, Au, Pb, etc., a new (solid) iron core was formed at p{-1,2/4}, part of the old iron core was expanded up to p{-1.3/4} as the liquid iron core, and the rest O, Mg, Si, etc. filled in the p{-1.3/4}o orbit shell and forming today’s mantle shell with \( r \) up to 6.38E+6 m.

The classical physics tells us that heat transfer in three ways: conduction, radiation, and convection. I believe that the conduction (through molecular collision) should belong to radiation because it can be explained as the infrared photon emitted from one molecule and absorbed by another one molecule. So there are only two major ways to transfer heat: radiation and convection. According to the “photon thermos core effect”, or PTC effect (see paper SunQM-3s8, section II), each orbit shell is heated or cooled as a whole unit, so that it has the same temperature everywhere inside the shell. According to the “lower mass density causes lower point of convection” effect (see paper SunQM-3s8, section II), the p{-1,2/4}o orbit shell, due to it has lower mass density (according to QM probability \( r \) distribution) than that of p{-1,1/4}o orbit shell, should have lower melt/convection point than that of p{-1,1/4}o orbit shell. So iron in p{-1,1/4}o orbit shell is in the pre-melt (or solid) form, while iron in p{-1,2/4}o orbit shell is in the melted/convective (or liquid) form. Using Solar structure’s terminology, current Earth’s p{-1,1/4} solid iron core is equivalent to a radiative zone, while current Earth’s p{-1,2/4}o orbit shell (liquid iron) is equivalent to a convective zone (see paper SunQM-3s8 section II for details).

I-g Earth’s tectonic movement can be explained by using the Earth mantle p{-1,3/4}o orbit shell’s QM dynamics

Wiki “Mantle (geology)” mentioned that Earth’s mantle “is predominantly solid but in geological time it behaves as a viscous fluid”. This is largely because the p{-1,3/4}o orbit shell (the mantle) has the lower mass density than that of p{-1,2/4}o orbit shell. Therefore, Earth’s tectonic movement can be explained by using the Earth mantle p{-1,3/4}o orbit shell’s \{N,n\} QM dynamics (See paper SunQM-3s9 for details).

I-h Earth’s magnetic field can be explained by using the Earth liquid iron p{-1,2/4}o orbit shell’s QM dynamics

See paper SunQM-3s9 for details.

I-i Using multiplier n’ and \{N,n\} QM to analyze Earth’s internal structure of mantle and crust

We always can use multiplier n’ for \{N,n\} QM structure analysis to get better accuracy. In paper SunQM-2 Table 4b, I explained how to use the multiplier n’ to explain the Earth orbit \{1,5\} in a more accurate way. Here I am going to give one more example: how to use multiplier n’ in \{N,n\} QM to analyze Earth’s internal structure of mantle and crust. First, let us try to use n’ = 2\^16 = 16, or p{N,n/16} QM structure to analyze Earth’s internal structure. From wiki “Earth”, Earth inner core’s \( r = 1.28E+6 m \), and it matches 6.38E+6 * (7/16) \^2 = 1.22E+6 m, so the inner core is a p{-1,7/16} QM structure of Earth (notice that before we always approximate Earth’s inner core as p{-1,2/4} = p{-1,18/16}, its accurate size is actually a p{-1,7/16} QM structure). Earth outer core’s \( r = 3.49E+6 m \), and it matches 6.38E+6 * (12/16) \^2 = 3.59E+6 m, so the outer core is a p{-1,12/16} = p{-1,3/4} QM structure of Earth. Earth mantle’s \( r = 5.68E+6 m \), and it matches 6.38E+6 * (15/16) \^2 = 5.61E+6 m, so Earth’s mantle structure is a p{-1,15/16} QM structure of Earth.

According to wiki “Earth”, the averaged crust thickness is 35000 m, then what n’ should we use for \{N,n\} QM structural calculation? We can build an equation: 6.38E+6 *[1 - ((n’-1)/n’)] \^2 = 35000. Solving this equation, we obtain n’ = 1 / [1 - sqrt(1 - 35000 / 6.38E+6)] = 364.07. Let n’ ≥ 364. Let us choose n’ = 364. Then 6.38E+6 *[1 - (363/364)] \^2 = 35000 m. So Earth’s crust starts at p{-1.363/364} QM size, and ends at p{-1.364/364} = p{0.1} QM size. Or, we can use some other n’ values. For example, let us use n’ = 2\^16 = 65536. Then after solving the equation, we can
calculate out $6.38 \times 10^6 \times [1 - (65356/65536)^2] = 34998$ m $\approx 35000$ m. So with multiplier $n' = 2^{16}$, Earth’s crust starts at $p[-1,65356/65536] = p[-1, (2^{16} - 180)/2^{16}]$ QM size, and ends at $p[-1,65536/65536] = p[0,1]$ QM size.

I-j. A practical useful method that can be directly used to predict all planets (and even Sun’)s internal structure and radial mass density distribution

Now, from studying Earth’s internal structure and mass density distribution, I have developed a method which can be directly used to estimate any planet’s internal structure and mass density.

1) Simplify a planet’s mass density $r$-distribution as a linear equation $D = a \cdot r + b$, manually fit the integration equation $\int D \ dV = \text{Mass}$ for a planet (with border condition at surface), to obtain the values of $a$ and $b$. For example, for Earth, manually adjust $a$ and $b$ to fit

$$\int (a \cdot r + b) \ 4\pi r^2 \ dr = 5.97 \times 10^{24} \text{kg},$$

with condition that at Earth surface, $D = 3400 \text{kg/m}^3$, to obtain $D = -0.0013 \times r + 11720$.

2) Obtain the $p\{N,n\}$ QM structure of this planet (from paper SunQM-1s3), and plot its radial probability density $r^2 \times |R(n_l)|^2$ against $r$ in a figure. For example, Earth’s $p\{N,n/2\}$ has the radial probability density distribution of $r^2 \times (|R(1,0)|^2 + |R(2,l)|^2)$ if using Earth’s inner core as $r_1$. So plot $r^2 \times (|R(1,0)|^2 + |R(2,l)|^2)$ vs. $r_n$.

3) Scale up $r^2 \times |R(n_l)|^2$ by manually fit the integration equation

$$\int r^2 \times |R(n_l)|^2 \times W \times (a \cdot r + b) \ dV = \text{Mass of planet},$$

where $W$ is the scaling factor. For example,

$$4\pi \int r^2 \times (|R(1,0)|^2 + |R(2,l)|^2) \times W \times (-0.0013 \times r + 11720) \times r^2 \ dr, \ [r = 0, 6.38 \times 10^6 \text{m}] = 5.97 \times 10^{24} \text{kg}.$$

Note: as a citizen scientist, I don’t have the time to figure out the complicated integration calculation, so I use an alternative method: simply plot both $D = a \cdot r + b$ and $r^2 \times |R(n_l)|^2$ in the same plot, and then scale up $r^2 \times |R(n_l)|^2$ curve manually according to $D = a \cdot r + b$ curve. Then use the Excel spreadsheet to calculate the “integrated” mass of the scaled-up $r^2 \times |R(n_l)|^2$ (see column 11 top line in Table 1) to match the Earth mass (see column 9 top line in Table 1). In both calculations, the mass of Earth is decreased by ~10% due to that in the mass “integration”, the $\Delta r$ is not infinity.

4) Then manually draw a line with multiple steps to match the (scaled up) probability curve. This stepped line gives the information (not only the size, but also the mass density) of the internal QM structure of this planet with reasonable accuracy.

In the rest part of this paper (and papers SunQ-3s7, and SunQm-3s8), I will use this method to analyze all other planets (including the four undiscovered planets), and plus Sun.

II. Predict Mars’ internal structure and the mass density radial distribution using $\{N,n\}$ QM probability function

So far no experimental determined mass density radial distribution (like Earth’s) has been found for Mars. According to section I-j, let us first constitute the mass density linear equation $D = a \cdot r + b$ for Mars. After manual fitting, it is $D = -0.00059 \times x + 5400$. It satisfy both conditions 1) $\int D \ dV = \text{mass of Mars}$; 2) at surface $r = 3.40 \times 10^6 \text{m}, D = 3400 \text{kg/m}^3$.

$$\int_{0}^{3.4 \times 10^6} 4\pi(-0.00059 \times x + 5400) \times x^2 \ d\ x = 641 \ 340 \ 356 \ 886 \ 865 \ 383 \ 202 \ 816$$
According to paper SunQM-1s3, Mars has a \{N,n/2\} QM structure similar as that of Earth, so I follow the same calculation in Section I for Earth. Therefore I only need to analyze \(p(-1,1/2)\) inner core and \(p(-1,1/2)\) orbit shell, using radial probability function \(R(1,0)r^2 + |R(2,0)r^2|\).

\[
Mass (r, \theta, \varphi) = \frac{4}{\pi} \int r^2 \left(|R(1,0)|^2 + |R(2,0)|^2\right) * W * D * \sin(\theta) * r^2 \, dr \, d\theta \, d\varphi, \quad [r = 0, 3.40E+6 \text{ m}; \theta = 0; \varphi = 0, 2\pi]
\]
or

\[
6.42E+23 \text{ kg} = \frac{4}{\pi} \int r^2 \left(|R(1,0)|^2 + |R(2,0)|^2\right) * W * D * r^2 \, dr, \quad [r = 0, 3.4E+6 \text{ m}]
\]

Now the problem is where we should choose as \(r_1\) for Mars’ \(p(N,n)\) QM structure? From wiki “Mars”, “Current models of its interior imply a core region about 1,794 km in radius”\(^{12}\). So I can’t choose 1/4 of Mars’ surface \(r = (3.4E+6 / 4 = 8.5E+5 \text{ m})\) as \(r_1\). According to Table 2, the original formed Mars core \(p(-1,1)\) ’s r was at 1.55E+6 m. So I use \(r_1 = 1.55E+6 \text{ m here.}\)

Table 3. Predict Mars’ internal structure and mass density \(r\)-distribution using \(p\{N,n\}\) QM probability function.

![Figure 3. Predict Mars’ internal structure and the mass density radial distribution by using \(p\{N,n\}\) QM probability function and a linear \((D = a*r + b)\) scaling up.](image)
From the calculation in Table 3, I predict that there are two layers with one interface for Mars’ internal structure (see the thick grey line in Figure 3): The inner core (0 < r < 1.86E+6 m) with D ≈ 6000 kg/m³, the main body (1.86E+6 m < r < 3.4E+6 m) with D = 3750 → 3300 kg/m³. So {N,n} QM predicts that Mars has an r = 1.86E+6 m inner core. It matches to the wiki mentioned result reasonably well.

Now according to the p{N,n} QM dynamics, I am able to guess out the evolution history of Mars’ p{N,n} QM structure for the past ~4 billion years: (Note: byr = billion years, bya = billion years ago, myr = million years, mya = million years ago). At ~4.6 bya, the Sun formed. At ~4.3 bya, ~90% of mass in {3,6}o orbit shell (0.9* 1.16E+26 kg, see Table 2) accreted to form the original Mars, with the original atmosphere up to p{0,1/4} at r = 2.48E+7 m, the original (Earth-sized) core p{-1.2//4} at r = 6.19E+6 m, and the original inner core p{-1,1//4} at r = 1.55E+6 m. This is a good p{N,n//4} QM structure, so it was stable for ~1 byr (from 4.3 bya to 3.3 bya, purely from assumption).

During this period, the Sun’s ice-evap-line expanded to {1,6} shell, so it was keeping to warm up the Mars frozen atmosphere shell. At ~3.3 bya, the atmosphere was heated and passed the critical point, so the whole atmosphere evaporated quickly within 0.1 byr (3.3 ~ 3.2 bya). The leftover Mars (Earth-sized core) had a p{-1.2//4} = p{-1,1//2} QM structure with r ≈ 6.19E+6 m. It is a good p{N,n} QM structure so that it was stable for another ~1 byr (3.2 ~ 2.2 bya). During this period, the Sun’s ice-evap-line expanded to ~ {1,8} shell, and it continues to warm up the Mars frozen surface. Then the (ice) melted Mars surface start to re-distribute its mass density in r-dimension, and transformed its p{N,n/2} QM into a p{N,n/4} QM with r = 6.19E+6 m unchanged! The original p{-1,1//2} o orbit shell from r = 1.62E+6 m to r = 6.19E+6 m which was filled with the mixture of frozen ice and rock, now is differentiated into two sub-shells: a new p{-1,2//4}o orbit shell from r = 6.19E+6 * (2/4)^2 = 1.55E+6 m to r = 6.19E+6 * (3/4)^2 = 3.48E+6 m, which now is filled with (the heavier) rocks (named as rock shell). Another new (sub-) p{-1,3//4}o orbit shell from r = 3.48E+6 m to r = 6.19E+6 m, which now is filled with (the lighter) water (named as water shell). The evaporation of the original atmosphere (> 15x of Earth’s mass) removed most of the gravity pressure at Mars center, so all r(s) should expand correspondingly. At ~2.2 bya, the water shell was so warmed and passed a critical point, so the whole water shell evaporated quickly within 0.1 byr (2.2 ~ 2.1 bya). After the whole p{-1,3//4}o orbit shell (water shell) stripped off, the leftover [-1,2//4]o orbit shell (or the rock shell) formed today’s Mars r = 3.44E+6 m. Therefore today’s Mars (r = 3.44E+6 m) is equivalent to today’s Earth outer liquid iron core (r = 3.49E+6 m) in turn of its QM origin.

Then today’s Mars again forms a new p{N,n/4} QM with p{0,1/4} at its current surface r = 3.44E+6 m. It also formed a core with r = 1.86E+6 m. Interestingly, this 1.86E+6 m is not a random value, it is one of the current Mars’ weak QM structure at p{-1,3//4}, calculated as r = 3.44E+6 * (3/4)^2 = 1.94E+6 m. Therefore the Mars evolution story continues as following: to accommodate today’s Mars QM, the original p{-1,1/2} core has further restructured to be the current p{-1,3//4} QM structure with r = 3.44E+6* (3/4)^2 = 1.94E+6 m. So today’s Mars (r = 3.44E+6 m) is a good p{N,n} QM structure, and it will last for many byr. Furthermore, if we apply p{N,n/4} QM to the current Mars, inside a p{-1,3//4} core with r = 1.94E+6 m, there should have a p{-1,2//4} core with r = 3.4E+6 * (2/4)^2 = 8.4E+5 m, and a p{-1,1//4} core with r = 3.4E+6 * (1/4)^2 =2.13E+5 m. Like that in Earth inner cores, the mass density difference between these inner cores may be very small.

The timeline of this evolution process is not important (due to it is purely from my guess). The most important is that the whole evolution process of Mars follows p{N,n} QM dynamics. Or, the whole strip-off process of Mars’ mass is actually a quantum dynamic process. It has several QM states which are relative stable and last for long time (≥ 1 bys). Between two stable QM states there is a (relatively) short transitional phase (≤ 0.1 bys). This is the similar QM dynamics that we have seen in the classical QM theory for atom. It is also the same QM dynamics that we have seen for the Solar QM [N,n] structure evolution (see paper SunQM-1s1).

It is interesting to see that p{N,n} QM structure of Mars changes as the total mass of Mars decrease: from the original p{0,1/4} at r1 = 2.48E+7 m, to a new p{0,1/4} at r1 = 6.19E+6 m, then to today’s p{0,1/4} at r1 = 3.44E+6 m. In comparison, Solar system’s total mass did not change during the quantum collapse from {6,1} to {0,1}, so its {N,n} QM structure (the big frame) does not change (except the H-fusion increased the radius of N = 0, 1 super-shells by 26%).

What is the physics reason that Mars had two long period of stable p{N,n} QM states (the first one was before the original atmosphere evaporation, and second one was before the water shell evaporation) during its evolution? Both relative
stable states were due to the large heat capacity, one came from the original atmosphere, and another one came from the water shell. It took ~1 byr to warm the whole original Mars’ atmosphere shell (or the whole water shell) to the critical point before the large scale evaporation start. This gives us a chance to explain why some extra-planetary systems have Jupiter-sized planet much closer to the Sun (see wiki “hot Jupiter”): it is because these extra-planetary systems are still young so they have the young Mars (or young Earth, or young Venus, etc.) with the original atmosphere shell.

If strictly follow the Solar \([N,n]\) QM structure and the simplified rule “all mass between \(r_n\) and \(r_{n+1}\) belongs to \(n\) shell”, due to the “ball-torus-7-11-gap effect” (see paper SunQM-1s1 section V), \([2,1]\) pre-Sun ball would have zero mass density in the \([2,1]_0\) orbit space, then there would be no Mars because there was no mass in \([1,6]_0\) orbit shell. Therefore the rule “all mass between \(r_n\) and \(r_{n+1}\) belong to \(n\) shell” is too rough for \([N,6]\) \(0\) orbit shell. The \([2,1]\) pre-Sun ball seems did tail the mass distribution (in \(r\)-dimension) into \([1,6]_0\) orbit shell. Therefore, \([1,6]_0\) orbit shell contains only the tailed-in mass from \([2,1]\) pre-Sun ball, so Mars should be expected to have much less original mass than what was estimated in table 2. How to accurately estimate the original mass in \([1,6]_0\) orbit shell is beyond the scope of this paper. The same thing is valid also for the \([2,6]\) \(0\) Kuiper belt.

### III. Predict Mercury’s internal structure and the mass density radial distribution using \([N,n]\) QM probability function

So far no experimental determined mass density radial distribution (like Earth’s) has been found for Mercury. Mercury has a similar \([N,n]\) QM structure as that of Mars, so I follow the same method. First, constitute the mass density linear equation \(D = a \cdot r + b\) for Mercury. After manual fitting, it is \(D = -0.0034x + 11700\). It satisfies both conditions 1) \(\int D dV = \text{mass of Mercury};\) 2) at surface \(r = 2.44E+6 \text{ m}, D \approx 3400 \text{ kg/m}^3\).

\[
\int_0^{2.44E+6} 4\pi (-0.0034x + 11700) x^2 \, dx = 333334.385582785100251136
\]

So I only need to analyze \([1,1//2]_0\) inner core and \([1,1//2]_0\) \(0\) orbit shell, using radial probability function \(|R(1,0)|^2 + |R(2,0)|^2\).

\[
\text{Mass (r, } \theta, \varphi) = \int\int\int r^2 *([|R(1,0)|^2 + |R(2,0)|^2] * D *\sin(\theta) * r^2 \, dr \, d\theta \, d\varphi, [r = 0, 2.44E+6 \text{ m}; \theta = 0, \pi; \varphi = 0, 2\pi]
\]

or

\[
3.3E+23 \text{ kg} = 4\pi \int r^2 *([|R(1,0)|^2 + |R(2,0)|^2] * (-0.0034x + 11700) *\sin(\theta) * r^2 \, dr, [r = 0, 2.44E+6 \text{ m}]
\]

Again, where we should choose as \(r_1\) for Mercury’s \([N,n]\) QM structure? Wiki “Mercury (planet)” mentioned “Mercury’s core occupies about 55% of its volume ... Surrounding the core is a 500–700 km mantle consisting of silicates”. So Mercury’s core has \(r \approx 2.44E+6 \cdot 6E+5 = 1.84E+6 \text{ m}\). Hence I can’t choose 1/4 of Mercury’s surface \(r (= 2.44E+6 /4 = 6.1E+5 \text{ m})\) as \(r_1\).

According to Table 2, the original formed Mercury core \([1,1]_0\) ‘s \(r_1\) was at 1.87E+6 m. So I use \(r_1 = 1.87E+6 \text{ m}\) here.

Table 4. Predict Mercury’s internal structure and mass density distribution using \([N,n]\) QM probability function.
Figure 4. Predict Mercury’s internal structure and the mass density radial distribution using p{N,n} QM probability function and a linear (D = a*r + b) scaling up.

From the calculation in Table 4, I predict that there are two layers with one interface for Mercury’s internal structure (see the thick grey line in Figure 4): The inner core (0 m < r < 1.7E+6 m) with D = 7200 kg/m^3, the main body (1.7E+6 m < r < 2.44E+6 m) with D = 4000 → 3400 kg/m^3. So [N,n] QM predicts that Mars has an r = 1.7E+6 m inner core. It matches to the wiki mentioned result (r = 1.84E+6 m) reasonably well.

A p{-1.3/4} QM structure of the current Mercury should have r = 2.44E+6 *(3/4)2 = 1.37E+6 m. So the current core with r = 1.84E+6 m is not at the p{-1.3/4} of current Mercury.

As I mentioned in paper SunQM-1x3, Mercury orbit’s high inclination and high eccentricity implies that Mercury must have accreted only partial mass (~60% in Table 2, or even less) in orbit space of [1,3]o. Large amount (~40% in Table 2, or even more) of the pre-planets in the shell of [1,3]o must have lost (into the Sun) during accretion (due to they were too close to the Sun). After Mercury was formed, its outer atmosphere (from p{-1.4/4} r = 2.98E+7 m to p{-1.2/4} r = 7.46E+6 m) was then stripped away when the Sun’s ice-evap line passed [1,3]. Removal of the original atmosphere (~30X of Earth’s mass) released gravity pressure at Mercury’s center, and caused Mercury’s original p{-1.2/4} r = 7.46E+6 m core expanded to r > 7.46E+6 m (unfortunately I don’t know the final value). According to the results of Earth and Mars, then Mercury changed its p[N,n] to a new p[N,n/4] structure with the new surface r > 7.46E+6 m as p{-1.4/4}. Under this new [N,n] QM, the old Mercury’s original p{-1.1/4} core r = 1.87E+6 m now becomes p{-1.2/4} core r = 1.87E+6 m, and it expanded its r from 1.87E+6 m to new p{-1.3/4} QM structure with r = 1.87E+6 *4 *(3/4)2 ≈ 4.21E+6 m. When Sun’s rock-evap line
moved close to \( \{1,3\}_o \) orbit, all mass (they were the light elements) in Mercury’s shell space between \( p\{-1,4//4\} \) \( r > 7.46 \times 10^6 \) m and \( p\{-1,3//4\} \) \( r = 4.21 \times 10^6 \) m was evaporated. So at one time Mercury had \( p\{-1,2//4\} \) core with \( r = 1.87 \times 10^6 \) m, and a \( p\{-1,3//4\} \) surface with \( r = 4.21 \times 10^6 \) m, similar as today’s Mars’ QM structure. Then with Sun’s rock-evap line further close to \( \{1,3\}_o \), most part of \( p\{-1,3//4\} \) mass has been stripped off, so today’s Mercury has only small part of \( p\{-1,3//4\} \) shell mass left, with \( r = 2.44 \times 10^6 \) m. So currently Mercury is not in a stable QM state of either \( p\{-1,3//4\} \) or \( p\{-1,2//4\} \) (either one should stable for ~1 byr). It is in the (relative) short transitional phase (\( \leq 0.1 \) byr) from \( p\{-1,3//4\} \) QM state to \( p\{-1,2//4\} \) QM state. In other words, Mercury is keep losing mass every day now. This process will end after all mass in the \( p\{-1,2//4\}_o \) orbit shell (which is made of median-light elements) have been evaporated. So within 0.1 byr, Mercury will reach the next QM stable state \( p\{-1,2//4\} \) with \( r = 1.87 \times 10^6 \) m, and it will be made of almost pure heavy elements. If the rock-evap-line does not further expand, then this \( p\{-1,2//4\} \) sized Mercury will be stable for the next ~1 byr. More discussion will be given on this topic in section V.

## IV. Predict Venus’s internal structure and the mass density radial distribution using \{N,n\} QM probability function

So far no experimental determined mass density radial distribution (like Earth’s) has been found for Venus. Venus has the similar \{N,n\} QM structure as that of Earth, so I follow the same method. First, constitute the mass density linear equation \( D = a \cdot r + b \) for Venus. After manual fitting, it is \( D = -0.0012 \times x + 10700 \). It satisfies both conditions, 1) \( \int D \, dV = \) mass of Venus; 2) at surface \( r = 6.05 \times 10^6 \) m, \( D \approx 3400 \text{ kg/m}^3 \).

\[
\int_0^{6.05 \times 10^6} 4\pi (-0.0012 \times + 10700) x^2 \, dx = 4874470.580965408965.132288
\]

For Venus I only need to analyze \( p\{-1,1//2\}_o \) inner core and \( p\{-1,1//2\}_o \) orbit shell, using radial probability function \( |R(1,0)|^2 + |R(2,l)|^2 \).

Mass \( (r, \theta, \phi) = \iiint r^2 \times (|R(1,0)|^2 + |R(2,l)|^2) \times D \times W \times \sin(\theta) \times r^2 \, dr \, d\theta \, d\phi, \ [r = 0, 6.05 \times 10^6 \text{ m}; \ \theta = 0, \pi; \ \phi = 0, 2\pi] \) or

\[
4.87 \times 10^24 \text{ kg} = 4\pi \int r^2 \times (|R(1,0)|^2 + |R(2,l)|^2) \times (-0.0012 \times + 10700) \times W \times r^2 \, dr, \ [r = 0, 6.05 \times 10^6 \text{ m}]
\]

According to Table 2, the original formed Mercury inner core \( p\{-1,1//2\}_o \)’s \( r \) is at \( 1.72 \times 10^6 \) m (close to \( 6.05 \times 10^6 / 4 = 1.51 \times 10^6 \) m). So I use \( r_1 = 1.72 \times 10^6 \) m here.

Table 5. Predict Venus’ internal structure and mass density distribution using \{N,n\} QM probability function.
Figure 5. Predict Venus’ internal structure and the mass density radial distribution using \( p{N,n} \) QM probability function and a linear (\( D = a \times r + b \)) scaling up.

From the calculation in Table 5, I predict that there are two layers with one interface for Venus’ internal structure (see the thick grey line in Figure 5): The inner core (0 m < \( r < 2.0 \times 10^6 \) m) with \( D \approx 7.76 \times 10^6 \text{ kg/m}^3 \), the main body (2.0 \( \times 10^6 \) m < \( r < 6.05 \times 10^6 \) m) with \( D \approx 7000 \rightarrow 3400 \text{ kg/m}^3 \). So \( \{N,n\} \) QM predicts that Mars has a \( r \approx 2.0 \times 10^6 \) m inner core.

According to Earth’ \( \{N,n/4\} \) QM analysis result, besides a \( p{-1.1/2} = p{-1.2/4} \) core with \( r \approx 2.0 \times 10^6 \) m, Venus should also have a \( p{-1.3/4} \) core (equivalent to Earth’s liquid iron core) with \( r = 7.76 \times 10^6 \times (3/4)^2 = 4.37 \times 10^6 \) m, and a \( p{-1.1/4} \) core (equivalent to Earth’s inner Nickel core) with \( r = 7.76 \times 10^6 \times (1/4)^2 = 4.85 \times 10^5 \) m.

V. Global analysis of rocky-planets’ \( p{N,n} \) QM structural change and the mass flow in \( N=1 \) super shell

Table 6. Global analysis of rocky-planets’ \( p{N,n} \) QM structural change. The dark grey cell means the mass in this QM structure has completely evaporated. The light grey cell means the mass in this QM structure has partially evaporated.
In Table 6, I demonstrate the global analysis for all rocky planets (including the already burned out {1,2} planet). You can see that all {1,n=2..6} planets’ \(p[N,n]\) QM changes actually follow the exactly the same dynamic pattern. So I believe this \(p[N,n]\) dynamics model does describes the true history of our four rocky planets.

The rocky-planet’s \(p[N,n]\) QM structural change in Table 6 is caused by the mass reduction for each rocky planet, which is further caused by the ice-evap (or rock-evap) line passing (or closing to) this planet’s orbit. Therefore the out-moving ice-evap line and rock-evap line from the center of Solar system causes two waves of mass out-flow in the \(\{1,n=2..6\}\) super shell. After the first wave (which happened \(-4\) billion years ago), the original atmospheres (mostly \(H_2, CH_4, NH_3, H_2O\) molecules) of all rocky planets were stripped-off and out-flew to \([2,2]\) and \([2,3]\) orbits (see paper SunQM-1s1). During the second wave (it is happening right now), large amount of light weight elements mass (maybe mostly in forms of \(CO_2\)) has been stripping off from \([1,n=2..4]\) planets. Similar as that the stripped-off water from \([1,n]\) shells was mainly captured by \([2,2]\) shell, I believe that most part of the stripped off \(CO_2\) molecules is being captured by \([1,5]\) Earth, and the small amount is being captured by \([1,6]\) Mars.

One possible evidence that an extremely large amount of \(H_2O\) & \(H_2\) must have evaporated from Mars may come from that why Mars surface is covered by iron oxide. According to Table 6, at one time Mars had a Earth-sized body with \(r \geq 6.19E+6\) m, but now has a solid surface with \(r\) only \(3.44E+6\) m. So Mars at that time could have had an ocean as deep as over \((6.19E+6 - 3.48E+6) = 2710\) km on its surface. During evaporation, beside most \(H_2O\) molecules were directly evaporated, there were significant amount of \(H_2O\) molecules decomposed into \(O_2\) and \(H_2\). 

\(H_2\) was immediately run away (due to its too light), and \(O_2\) was trapped in the Mars atmosphere (due to it is too heavy and more difficult to run away). So at the end of Mars ocean evaporation, Mars atmosphere (above surface \(r = 3.48E+6\) m) might have a high \(O_2\) concentration (could be \(-20%)\). This \(O_2\) atmosphere oxidized the iron on Mars surface. Then, after billion more years, the whole atmosphere (including \(O_2\)) on Mars has completely run away. Current Mars’ thin atmosphere may come from the captured \(CO_2\) which had run-away from those inner planets (Mercury, Venus, or even a \([1,2]\) planet which has evaporated completely)

Just like ice-evap line causes \(H_2\) gas & \(H_2O\) in \([1,n=2..6]\) evaporated to \([2,2]\) or and increased Jupiter’s (ice) mass, rock-evap line between \([1,2]\) and \([1,3]\) not only causes Mercury evaporated all crust and most of mantle’s light elements (e.g., \(C\) & \(O\) in form of \(CO_2\)), it also causes Venus start to losing its crust & mantle’s light elements (may also in form of \(CO_2\)). This explains why currently Mercury's mass < Venus' mass < Earth’s mass, which is opposite of the original mass sequence where \([1,3]\)’s mass > \([1,4]\)’s mass > \([1,5]\)’s mass. And much of the run-away \(CO_2\) has been captured by Earth, so that Earth may have more light elements (\(C\) & \(O\)) mass than it was initially formed. So across the ice-evap line there is a \(H_2\) & \(H_2O\) out-flowing, and across the rock-evap line there should have a \(CO_2\) out-flowing.

Conclusions

1) From studying Earth’s known internal structure and mass density distribution, I have developed a method which can be used to estimate any planet’s internal structure and mass density. This method can be expressed as Planet mass = \(4\pi \int (\text{planet’s QM probability density } r\text{-distribution}) \times W \times D \times r^2 dr\), where mass density \(D = a \times r^2 + b\), and \(W\) is a scaling factor.

2) I applied this method to all four rocky planets, and predicted the internal structure and the (close to the true) mass density \(r\)-distribution for these planets.
3) Besides the p{-1,4/4} surface and p{-1,2/4} core, a p{-1,3/4} QM structure is found to be the foundation for Earth’s liquid iron core, and for Mars current size. Furthermore, Earth’s current mantle structure is found to be the p{-1,15/16} QM structure of Earth, and current Earth’s solid iron inner core is found to be the p{-1,7/16} QM structure of Earth.

4) Dynamics of this QM structure evolution has been discussed.

References


[10] A series of my papers that to be published (together with current paper):
SunQM-3s6: Predict mass density r-distribution for Earth and other rocky planets based on {N,n} QM probability distribution.
SunQM-3s7: Predict mass density r-distribution for gas/ice planets based on {N,n} QM probability distribution.
SunQM-3s8: Using {N,n} QM to study Sun’s internal structure, convective zone formation, planetary differentiation and temperature r-distribution
SunQM-3s9: Using {N,n} QM to explain Sun’s and Earth’s dynamo, the sunspot drift, and the continental drift.
SunQM-5: A new version of QM based on interior {N,n}, multiplier n’, |R(n,l)|^2 |Y(l,m)|^2 guided mass occupancy, and RF, and its application from string to universe.
SunQM-5s1: White dwarf, neutron star, and black hole re-analyzed by using the internal {N,n} QM.


Major QM books, data sources, software I used for this study are:
Douglas C. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. 2009.
Wikipedia at: https://en.wikipedia.org/wiki/
Online free software: WolframAlpha (https://www.wolframalpha.com/)
Online free software: MathStudio (http://mathstud.io/)
Free software: R
Microsoft Excel.
Public TV’s space science related programs: PBS-NOVA, BBC-documentary, National Geographic-documentary, etc.
Journal: Scientific American.