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Original Article

Neutrosophic vague soft multiset for decision under uncertainty

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Abstract

The notion of classical soft multisets is extended to neutrosophic vague soft multisets by applying the theory of soft multisets to neutrosophic vague sets to be more effective and useful. We also define its basic operations, namely complement, subset, union, intersection along with illustrative examples, and study some related properties with supporting proofs. Lastly, this notion is applied to a decision making problem and its effectiveness is demonstrated using an illustrative example.

Keywords: decision making, neutrosophic soft multiset, neutrosophic vague set, soft multiset, vague soft set

1. Introduction

Fuzzy set was introduced by Zadeh (1965) as a mathematical tool to solve problems and vagueness in everyday life. Since then the fuzzy sets and fuzzy logic have been applied in many real life problems in uncertain and ambiguous environments (Mohammed & Ghareeb, 2016; Siripitukdet & Suebsan, 2015; Tripathy & Debnath, 2015; Yaqoob *et al.,* 2016). A great deal of research and applications in the literature were undertaken to deal with uncertainty like probability theory, rough set theory (Pawlak, 1982) and intuitionistic fuzzy set theory (Atanassov, 1986). However, all of these theories have their inherent difficulties and weaknesses. Molodtsov (1999) initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties. It was further extended to soft multiset (Alkhazaleh *et al.,* 2011), multiparameterized soft set (Salleh *et al.*, 2012), and multi Q-fuzzy parameterized soft set (Adam & Hassan, 2014).

Later on, Alkazaleh and Salleh (2012) introduced fuzzy soft multiset, a more general concept, which is a combination of fuzzy set and soft multisets and studied its properties and gave an application of this concept in decision making problems. Vague set theory was proposed by Gau and Buehrer (1993), followed by vague soft set (Xu *et al.,* 2010). Alhazaymeh and Hassan extended these to generalized vague soft set (2012), possibility interval-valued vague soft set (2013), vague soft multisets (2014a), and vague soft set relations and functions (2015). Vague soft expert set (Hassan & Alhazaymeh, 2013) and mapping on generalized vague soft expert set (Alhazaymeh & Hassan, 2014b) were also proposed.

The words "neutrosophy" and "neutrosophic" were introduced by Smarandache (1998). Smarandache (2005) further proposed the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise data, involving a truth membership function T, an indeterminacy membership function I and a falsehood membership function F. Recently, the works on neutrosophic set and their hybrid structure in theories and applications have been progressing rapidly (Alkhazaleh, 2016; Broumi & Deli, 2016; Broumi *et al.*, 2015, 2016a, 2016b; Sahin & Liu, 2016; Ye, 2016a, 2016b;). Maji (2013) introduced neutrosophic soft set which has been developed rapidly to neutrosophic soft multiset theory (Deli *et al.,* 2014) and interval-valued neutrosophic soft sets (Deli, 2015).

As a combination of neutrosophic set and vague set, Alkhazaleh (2015a) introduced the concept of neutrosophic vague set, followed by Al-Quran and Hassan on fuzzy parameterized single valued neutrosophic soft expert set (2016a) and neutrosophic vague soft expert set (2016b).

We will further extend the studies on soft multiset (Alkhazaleh *et al.*, 2011) and neutrosophic vague set (Alkha

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zaleh, 2015a) through the establishment of the notion of neutrosophic vague soft multiset theory which can better handle the elements of imprecision and uncertainty compared to the other generalizations of soft multiset such as fuzzy soft multisets, vague soft multisets, and neutrosophic soft multisets. In line with this, the purpose of this paper is to extend the classical soft multiset model to the neutrosophic vague soft multiset model and thereby establish a new generalization of the soft multiset model called the neutronsophic vague soft multiset.

We first present the basic definitions of neutrosophic vague set and fuzzy soft multisets that are useful for subsequent discussions. We then propose a novel concept of neutrosophic vague soft multiset theory and define some operations along with illustrative examples. We also give some related properties with supporting proofs. Finally we give a decision making method for neutrosophic vague soft multiset theory and present an application of this concept in solving a decision making problem.

2. Preliminaries

In this section, we recall some basic notions of neutrosophic set, vague set, neutrosophic vague set, and fuzzy soft multiset theory.

Definition 2.1 (Smarandache, 2005) A neutrosophic set A on the universe of discourse X is defined as

$A = \{< x; T_A(x); I_A(x); F_A(x) >; x \in X\}$, where $T; I; F : X \to]^-0; 1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.2 (Gau & Buehrer, 1993) Let X be a space of points (objects), with a generic element of X denoted by x. A vague set V in X is characterized by a truth-membership function t_v and a false-membership function f_v. $t_v(x)$ is a lower bound on the grade of membership of x derived from the evidence for x, and $f_v(x)$ is a lower bound on the negation of x derived from the evidence against x. $t_v(x)$ and $f_v(x)$ both associate a real number in the interval [0,1] with each point in x, where $t_v(x) + f_v(x) \le 1$.

Definition 2.3 (Alkhazaleh, 2015a) A neutrosophic vague set A_{nv} (NVS in short) on the universe of discourse X written as

$A_{NV} = \{< x; T_{A_{NV}}(x); I_{A_{NV}}(x); F_{A_{NV}}(x) >; x \in X\}$

Whose truth - membership, indeterminacy - membership, and falsity - membership functions are defined as

$T_{A_{NV}}(x) = [T^-, T^+]$, $I_{A_{NV}}(x) = [I^-, I^+]$ and $F_{A_{NV}}(x) = [F^-, F^+]$ where (1) $T^+ = 1 - F^-$, (2) $F^+ = 1 - T^-$ and (3) $^-0 \le T^- + I^- + F^- \le 2^+$.

Definition 2.4 (Alkhazaleh, 2015a) Let Ψ_{NV} be a NVS of the universe U where $\forall u_i \in U$, $T_{\Psi_{NV}}(x) = [1,1]$, $I_{\Psi_{NV}}(x) = [0,0]$, $F_{\Psi_{NV}}(x) = [0,0]$, then Ψ_{NV} is called a unit NVS, where $1 \le i \le n$.

Definition 2.5 (Alkhazaleh, 2015a) Let Φ_{NV} be a NVS of the universe U where $\forall u_i \in U$, $T_{\Phi_{NV}}(x) = [0,0]$, $I_{\Phi_{NV}}(x) = [1,1]$, $F_{\Phi_{NV}}(x) = [1,1]$, then Φ_{NV} is called a zero NVS, where $1 \le i \le n$.

Definition 2.6 (Alkhazaleh, 2015a) Let A_{nv} and B_{nv} be two NVSs of the universe U. If $\forall u_i \in U$,

(1) $T_{A_{NV}}(u_i) \le T_{B_{NV}}(u_i)$, (2) $I_{A_{NV}}(u_i) \ge I_{B_{NV}}(u_i)$ and (3) $F_{A_{NV}}(u_i) \ge F_{B_{NV}}(u_i)$, then the NVS is A_{nv} included by, B_{nv} denote $A_{NV} \subseteq B_{NV}$, where $1 \le i \le n$.

Definition 2.7 (Alkhazaleh, 2015a) The complement of a NVS A_{nv} is denoted by A^c andis defined by

$T^c_{A_{NV}}(x) = [1 - T^+, 1 - T^-]$, $I^c_{A_{NV}}(x) = [1 - I^+, 1 - I^-]$ and $F^c_{A_{NV}}(x) = [1 - F^+, 1 - F^-]$.

Definition 2.8 (Alkhazaleh, 2015a) The union of two NVSs A_{nv} and B_{nv} is a NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{nv} and B_{nv} given by

$$T_{C_{NV}}(x) = \left[\max\left(T^-_{A_{NV}x}, T^-_{B_{NV}x}\right), \max\left(T^+_{A_{NV}x}, T^+_{B_{NV}x}\right)\right],$$

$$I_{C_{NV}}(x) = \left[\min\left(I^-_{A_{NV}x}, I^-_{B_{NV}x}\right), \min\left(I^+_{A_{NV}x}, I^+_{B_{NV}x}\right)\right] \text{ and}$$

$$F_{C_{NV}}(x) = \left[\min\left(F^-_{A_{NV}x}, F^-_{B_{NV}x}\right), \min\left(F^+_{A_{NV}x}, F^+_{B_{NV}x}\right)\right]$$

Definition 2.9 (Alkhazaleh, 2015a) The intersection of two NVSs A_{nv} and B_{nv} is a NVS C_{NV}, written as $H_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{nv} and B_{nv} given by

$$T_{H_{NV}}(x) = \left[\min\left(T^-_{A_{NV}x}, T^-_{B_{NV}x}\right), \min\left(T^+_{A_{NV}x}, T^+_{B_{NV}x}\right)\right],$$

$$I_{H_{NV}}(x) = \left[\max\left(I^-_{A_{NV}x}, I^-_{B_{NV}x}\right), \max\left(I^+_{A_{NV}x}, I^+_{B_{NV}x}\right)\right] \text{ and}$$

$$F_{H_{NV}}(x) = \left[\max\left(F^-_{A_{NV}x}, F^-_{B_{NV}x}\right), \max\left(F^+_{A_{NV}x}, F^+_{B_{NV}x}\right)\right]$$

Definition 2.10 (Alkhazaleh & Salleh, 2012) Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \varnothing$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i, $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a *fuzzy soft multiset* over U, where F is a mapping given by $F : A \to U$.

Definition 2.11 (Alkhazaleh & Salleh, 2012) For two fuzzy soft multisets *(F, A)* and *(G, B)* over *U*, *(F, A)* is called a *fuzzy soft multisubset* of *(G, B)* if :

1. $A \subseteq B$ and
2. $\forall e_{U_{i,j}} \in a_k, \left(e_{U_{i,j}}, F_{e_{U_{i,j}}}\right)$ is a fuzzy subset of $\left(e_{U_{i,j}}, G_{e_{U_{i,j}}}\right)$

where $a_k \in A,\ k = \{1,2,...,n\},\ i \in \{1,2,...,m\}$ and $j \in \{1,2,...,r\}$.

Definition 2.12 (Alkhazaleh & Salleh, 2012) The complement of a fuzzy soft multiset *(F, A)* is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to U$ is a mapping given by $F^c(\alpha) = c(F(\alpha)), \forall \alpha \in A$ where c is any fuzzy complement.

Definition 2.13 (Alkhazaleh & Salleh, 2012) The *union* of two fuzzy soft multisets (F, A) and (G, B) over *U*, denoted by $(F, A) \cup (G, B)$, is the fuzzy soft multiset (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & ,if\ \varepsilon \in A - B \\ G(\varepsilon) & ,if\ \varepsilon \in B - A \\ \cup\ (F(\varepsilon), G(\varepsilon)) & ,if\ \varepsilon \in A \cap B, \end{cases}$$

where $\cup(F(\varepsilon), G(\varepsilon)) = s(F_{\varepsilon_{U_{i,j}}}, G_{\varepsilon_{U_{i,j}}}), \forall i \in \{1,2,...,m\}$

with *s* as an *s*-norm.

Definition 2.14 (Alkhazaleh & Salleh, 2012) The *intersection* of two fuzzy soft multisets (F, A) and (G, B) over *U*, denoted by $(F, A) \cap (G, B)$, is the fuzzy soft multiset (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & ,if\ \varepsilon \in A - B \\ G(\varepsilon) & ,if\ \varepsilon \in B - A \\ \cap(F(\varepsilon), G(\varepsilon)) & ,if\ \varepsilon \in A \cap B, \end{cases}$$

where $\cap(F(\varepsilon), G(\varepsilon)) = t(F_{\varepsilon_{U_{i,j}}}, G_{\varepsilon_{U_{i,j}}}), \forall i \in \{1,2,3,...,m\}$

with *t* as a *t*-norm.

3. Neutrosophic Vague Soft Multiset

In this section we introduce the concept of neutrosophic vague soft multiset and define some operations on a neutrosophic vague soft multiset, namely subset, equality, null, absolute, complement, union, and intersection. We also give some properties of this concept.

Now we propose the definition of a neutrosophic vague soft multiset and we give an illustrative example of it.

Definition 3.1 Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} NV(U_i)$ where $NV(U_i)$ denotes the set of all neutrosophic vague subsets of U_i, $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a *neutrosophic vague soft multiset* over *U*, where *F* is a mapping given by $F : A \to U$.

In other words, a neutrosophic vague soft multiset over *U* is a parameterized family of neutrosophic vague subsets of *U*. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε -approximate elements of the neutrosophic vague soft multiset (F, A).

Based on the above definition, any change in the order of universes will produce a different neutrosophic vague soft multiset.

Example 3.2 Suppose there are three universes U_1, U_2 and U_3. Suppose that a person has a budget to buy a house, a car, and rent a venue to hold a wedding celebration. Let us consider a neutrosophic vague soft multiset (F, A) which describes "houses", "cars" and "hotels" that are being considered for accommodation purchase, transportation purchase, and a location venue to hold a wedding celebration respectively. Let $U_1 = \{h_1, h_2, h_3\}, U_2 = \{c_1, c_2, c_3\}$, and $U_3 = \{v_1, v_2\}$. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = condominium, e_{U_1,2} = apartment, e_{U_1,3} = duplex\},$$

$$E_{U_2} = \{e_{U_2,1} = compact, e_{U_2,2} = sedan, e_{U_2,3} = sporty\} \text{ and}$$

$$E_{U_3} = \{e_{U_3,1} = uptown, e_{U_3,2} = downtown, e_{U_3,3} = suburb\}.$$

Let $U = \prod_{i=1}^{3} NV(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and $A \subseteq E$, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), a_3 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,1})\}.$$

Suppose that

$$F(a_1) = \left(\left\{\frac{h_1}{[0.2,0.5],[0.1,0.4],[0.5,0.8]},\frac{h_2}{[0.4,0.6],[0.5,0.6],[0.4,0.6]},\frac{h_3}{[0.8,0.9],[0.1,0.3],[0.1,0.2]}\right\},\left\{\frac{c_1}{[0.8,0.9],[0.5,0.6],[0.1,0.2]},\frac{c_2}{[0.5,0.8],[0.6,0.7],[0.2,0.5]},\frac{c_3}{[0.4,0.7],[0.6,0.8],[0.3,0.6]}\right\}\left\{\frac{v_1}{[0.8,0.9],[0.1,0.2],[0.1,0.2]},\frac{v_2}{[0.7,0.9],[0.8,0.9],[0.1,0.3]}\right\}\right).$$

$$F(a_2) = \left(\left\{\frac{h_1}{[0.5,0.6],[0.3,0.4],[0.4,0.5]},\frac{h_2}{[0.8,0.9],[0.2,0.5],[0.1,0.2]},\frac{h_3}{[0.2,0.4],[0.3,0.3],[0.6,0.8]}\right\},\left\{\frac{c_1}{[0.7,0.9],[0.5,0.9],[0.1,0.3]},\frac{c_2}{[0.3,0.8],[0.2,0.4],[0.2,0.7]},\frac{c_3}{[0.7,0.9],[0.3,0.5],[0.1,0.3]}\right\}\left\{\frac{v_1}{[0.1,0.5],[0.3,0.6],[0.5,0.9]},\frac{v_2}{[0.4,0.6],[0.5,0.9],[0.4,0.6]}\right\}\right).$$

$$F(a_3) = \left(\left\{\frac{h_1}{[0.4,0.7],[0.1,0.2],[0.3,0.6]},\frac{h_2}{[0.6,0.8],[0.6,0.6],[0.2,0.4]},\frac{h_3}{[0.1,0.5],[0.2,0.5],[0.5,0.9]}\right\},\left\{\frac{c_1}{[0.5,0.6],[0.1,0.5],[0.4,0.5]},\frac{c_2}{[0.2,0.8],[0.1,0.7],[0.2,0.8]},\frac{c_3}{[0.6,0.7],[0.2,0.8],[0.3,0.4]}\right\}\left\{\frac{v_1}{[0.6,0.9],[0.2,0.2],[0.1,0.4]},\frac{v_2}{[0.5,0.8],[0.5,0.9],[0.2,0.5]}\right\}\right).$$

Then we can view the neutrosophic vague soft multiset (F,A) as consisting of the following collection of approximations:

$$(F,A) = \left\{\left(a_1,\left(\left\{\frac{h_1}{[0.2,0.5],[0.1,0.4],[0.5,0.8]},\frac{h_2}{[0.4,0.6],[0.5,0.6],[0.4,0.6]},\frac{h_3}{[0.8,0.9],[0.1,0.3],[0.1,0.2]}\right\},\left\{\frac{c_1}{[0.8,0.9],[0.5,0.6],[0.1,0.2]},\frac{c_2}{[0.5,0.8],[0.6,0.7],[0.2,0.5]},\frac{c_3}{[0.4,0.7],[0.6,0.8],[0.3,0.6]}\right\}\left\{\frac{v_1}{[0.8,0.9],[0.1,0.2],[0.1,0.2]},\frac{v_2}{[0.7,0.9],[0.8,0.9],[0.1,0.3]}\right\}\right)\right),\right.$$
$$\left(a_2,\left(\left\{\frac{h_1}{[0.5,0.6],[0.3,0.4],[0.4,0.5]},\frac{h_2}{[0.8,0.9],[0.2,0.5],[0.1,0.2]},\frac{h_3}{[0.2,0.4],[0.3,0.3],[0.6,0.8]}\right\},\left\{\frac{c_1}{[0.7,0.9],[0.5,0.9],[0.1,0.3]},\frac{c_2}{[0.3,0.8],[0.2,0.4],[0.2,0.7]},\frac{c_3}{[0.7,0.9],[0.3,0.5],[0.1,0.3]}\right\}\left\{\frac{v_1}{[0.1,0.5],[0.3,0.6],[0.5,0.9]},\frac{v_2}{[0.4,0.6],[0.5,0.9],[0.4,0.6]}\right\}\right)\right),$$
$$\left.\left(a_3,\left(\left\{\frac{h_1}{[0.4,0.7],[0.1,0.2],[0.3,0.6]},\frac{h_2}{[0.6,0.8],[0.6,0.6],[0.2,0.4]},\frac{h_3}{[0.1,0.5],[0.2,0.5],[0.5,0.9]}\right\},\left\{\frac{c_1}{[0.5,0.6],[0.1,0.5],[0.4,0.5]},\frac{c_2}{[0.2,0.8],[0.1,0.7],[0.2,0.8]},\frac{c_3}{[0.6,0.7],[0.2,0.8],[0.3,0.4]}\right\}\left\{\frac{v_1}{[0.6,0.9],[0.2,0.2],[0.1,0.4]},\frac{v_2}{[0.5,0.8],[0.5,0.9],[0.2,0.5]}\right\}\right)\right)\right\}.$$

Each approximation has two parts: A predicate and an approximate value set.

Definition 3.3 For any neutrosophic vague soft multiset (F,A), a pair $\left(e_{U_{i,j}}, F_{e_{U_{i,j}}}\right)$ is called a U_i *– neutrosophic vague soft multiset part* $\forall\, e_{U_{i,j}} \in a_k$ and $F_{e_{U_{i,j}}} \subseteq F(A)$ is a *neutrosophic vague approximate value set*, where $a_k \in A$, $k \in \{1,2,...,n\}$, $i \in \{1,2,...,m\}$ and $j \in \{1,2,...,r\}$.

Example 3.4 Consider Example 3.2. Then

$(e_{U_1,j}, F_{e_{U_1,j}}) =$

$$\left\{\left(e_{U_1,1}, \left\{\frac{h_1}{[0.2,0.5],[0.1,0.4],[0.5,0.8]}, \frac{h_2}{[0.4,0.6],[0.5,0.6],[0.4,0.6]}, \frac{h_3}{[0.8,0.9],[0.1,0.3],[0.1,0.2]}\right\}\right),\right.$$

$$\left(e_{U_1,1}, \left\{\frac{h_1}{[0.5,0.6],[0.3,0.4],[0.4,0.5]}, \frac{h_2}{[0.8,0.9],[0.2,0.5],[0.1,0.2]}, \frac{h_3}{[0.2,0.4],[0.3,0.3],[0.6,0.8]}\right\}\right),$$

$$\left.\left(e_{U_1,2}, \left\{\frac{h_1}{[0.4,0.7],[0.1,0.2],[0.3,0.6]}, \frac{h_2}{[0.6,0.8],[0.6,0.6],[0.2,0.4]}, \frac{h_3}{[0.1,0.5],[0.2,0.5],[0.5,0.9]}\right\}\right)\right\}.$$

is a U_1-neutrosophic vague soft multiset part of (F, A).

In the following, we introduce the concept of the subset of two neutrosophic vague soft multisets .

Definition 3.5 For two neutrosophic vague soft multisets (F, A) and (G, B) over U, (F, A) is called a *neutrosophic vague soft multisubset* of (G, B) if

1. $A \subseteq B$, and
2. $\forall e_{U_{i,j}} \in a_k, \left(e_{U_{i,j}}, F_{e_{U_{i,j}}}\right)$ is a neutrosophic vague subset of $\left(e_{U_i,j}, G_{e_{U_i,j}}\right)$

where $a_k \in A$, $k = \{1,2,...,n\}$, $i \in \{1,2,...,m\}$ and $j \in \{1,2,...,r\}$.

This relationship is denoted by $(F, A) \subseteq (G, B)$. In this case (G, B) is called a *neutrosophic vague soft multisuperset* of (F, A).

Definition 3.6 Two neutrosophic vague soft multisets (F, A) and (G, B) over U are said to be *equal* if (F, A) is a neutrosophic vague soft multisubset of (G, B) and (G, B) is a neutrosophic vague soft multisubset of (F, A).

Example 3.7 Consider Example 3.2. Let

$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\}$, and

$B = \{b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\}$.

Clearly $A \subseteq B$. Let (F, A) and (G, B) be two neutrosophic vague soft multisets over the same universe U such that:

$(F, A) =$

$$\left\{\left(a_1, \left(\left\{\frac{h_1}{[0.2,0.5],[0.1,0.4],[0.5,0.8]}, \frac{h_2}{[0.4,0.6],[0.5,0.6],[0.4,0.6]}, \frac{h_3}{[0.8,0.9],[0.1,0.3],[0.1,0.2]}\right\}, \left\{\frac{c_1}{[0.8,0.9],[0.5,0.6],[0.1,0.2]},\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.5,0.8],[0.6,0.7],[0.2,0.5]}, \frac{c_3}{[0.4,0.7],[0.6,0.8],[0.3,0.6]}\right\}\left\{\frac{v_1}{[0.8,0.9],[0.1,0.2],[0.1,0.2]}, \frac{v_2}{[0.7,0.9],[0.8,0.9],[0.1,0.3]}\right\}\right)\right),$$

$$\left(a_2, \left(\left\{\frac{h_1}{[0.5,0.6],[0.3,0.4],[0.4,0.5]}, \frac{h_2}{[0.8,0.9],[0.2,0.5],[0.1,0.2]}, \frac{h_3}{[0.2,0.4],[0.3,0.3],[0.6,0.8]}\right\}, \left\{\frac{c_1}{[0.7,0.9],[0.5,0.9],[0.1,0.3]},\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{c_2}{[0.3,0.8],[0.2,0.4],[0.2,0.7]}, \frac{c_3}{[0.7,0.9],[0.3,0.5],[0.1,0.3]}\right\}\left\{\frac{v_1}{[0.1,0.5],[0.3,0.6],[0.5,0.9]}, \frac{v_2}{[0.4,0.6],[0.5,0.9],[0.4,0.6]}\right\}\right)\right)\right\}.$$

$(G,B)=$

$$\left\{\left(b_1,\left(\left\{\frac{h_1}{[0.3,0.5],[0.1,0.3],[0.5,0.7]},\frac{h_2}{[0.5,0.9],[0.4,0.5],[0.1,0.5]},\frac{h_3}{[0.9,0.9],[0.1,0.2],[0.1,0.1]}\right\},\left\{\frac{c_1}{[0.9,0.9],[0.4,0.5],[0.1,0.1]},\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.6,0.9],[0.5,0.6],[0.1,0.4]},\frac{c_3}{[0.5,0.7],[0.5,0.7],[0.3,0.5]}\right\}\left\{\frac{v_1}{[0.9,0.9],[0.1,0.1],[0.1,0.1]},\frac{v_2}{[0.8,0.9],[0.5,0.6],[0.1,0.2]}\right\}\right)\right),$$

$$\left(b_2,\left(\left\{\frac{h_1}{[0.6,0.6],[0.3,0.4],[0.4,0.4]},\frac{h_2}{[0.7,0.8],[0.2,0.5],[0.2,0.3]},\frac{h_3}{[0.1,0.5],[0.3,0.6],[0.5,0.9]}\right\},\left\{\frac{c_1}{[0.6,0.9],[0.8,0.9],[0.1,0.4]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.3,0.5],[0.2,0.4],[0.5,0.7]},\frac{c_3}{[0.2,0.9],[0.1,0.5],[0.1,0.8]}\right\}\left\{\frac{v_1}{[0.3,0.5],[0.5,0.6],[0.5,0.7]},\frac{v_2}{[0.3,0.6],[0.7,0.9],[0.4,0.7]}\right\}\right)\right),$$

$$\left(b_3,\left(\left\{\frac{h_1}{[0.5,0.7],[0.1,0.2],[0.3,0.5]},\frac{h_2}{[0.8,0.9],[0.1,0.4],[0.1,0.2]},\frac{h_3}{[0.3,0.5],[0.2,0.3],[0.5,0.7]}\right\},\left\{\frac{c_1}{[0.8,0.9],[0.1,0.5],[0.1,0.2]},\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{c_2}{[0.4,0.9],[0.1,0.3],[0.1,0.6]},\frac{c_3}{[0.8,0.9],[0.2,0.4],[0.1,0.2]}\right\}\left\{\frac{v_1}{[0.1,0.9],[0.2,0.2],[0.1,0.9]},\frac{v_2}{[0.5,0.8],[0.5,0.9],[0.2,0.5]}\right\}\right)\right)\right\}.$$

Therefore $(F,A)\subseteq(G,B)$.

Now, we put forward the definition of a null neutrosophic vague soft multiset and the definition of the absolute neutrosophic vague soft multiset.

Definition 3.8 A neutrosophic vague soft multiset (F,A) over U is called a *null neutrosophic vague soft multiset*, denoted by $(F,A)_{\Phi}$, if all of the neutrosophic vague soft multiset parts of (F,A) equal $\varnothing$.

Definition 3.9 A neutrosophic vague soft multiset (F,A) over U is called *absolute neutrosophic vague soft multiset*, denoted by $(F,A)_U$, if $\left(e_{U_{i,j}},F_{e_{U_{i,j}}}\right)=U_i$, $\forall i.$

In the following, we propose the definition of the complement of a neutrosophic vague soft multiset and give an example on the complement of a neutrosophic vague soft multiset.

Definition 3.10 The complement of a neutrosophic vague soft multiset (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c$ = (F^c,A) where $F^c:A\to U$ is a mapping given by $F^c(\alpha)=c(F(\alpha)),\forall\alpha\in A$ where c is a neutrosophic vague complement.

Example 3.11 Consider Example 3.2. By using the basic neutrosophic vague complement, we have

$(F,A)^c$ =

$$\left\{\left(a_1,\left(\left\{\frac{h_1}{[0.5,0.8],[0.6,0.9],[0.2,0.5]},\frac{h_2}{[0.4,0.6],[0.4,0.5],[0.4,0.6]},\frac{h_3}{[0.1,0.2],[0.7,0.9],[0.8,0.9]}\right\},\left\{\frac{c_1}{[0.1,0.2],[0.4,0.5],[0.8,0.9]},\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.2,0.5],[0.3,0.4],[0.5,0.8]},\frac{c_3}{[0.3,0.6],[0.2,0.4],[0.4,0.7]}\right\}\left\{\frac{v_1}{[0.1,0.2],[0.8,0.9],[0.8,0.9]},\frac{v_2}{[0.1,0.3],[0.1,0.2],[0.7,0.9]}\right\}\right)\right),$$

$$\left(a_2,\left(\left\{\frac{h_1}{[0.4,0.5],[0.6,0.7],[0.5,0.6]},\frac{h_2}{[0.1,0.2],[0.5,0.8],[0.8,0.9]},\frac{h_3}{[0.6,0.8],[0.7,0.7],[0.2,0.4]}\right\},\left\{\frac{c_1}{[0.1,0.3],[0.1,0.5],[0.7,0.9]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.2,0.7],[0.6,0.8],[0.3,0.8]},\frac{c_3}{[0.1,0.3],[0.5,0.7],[0.7,0.9]}\right\}\left\{\frac{v_1}{[0.5,0.9],[0.4,0.7],[0.1,0.5]},\frac{v_2}{[0.4,0.6],[0.1,0.5],[0.4,0.6]}\right\}\right)\right),$$

$$\left(a_3,\left(\left\{\frac{h_1}{[0.3,0.6],[0.8,0.9],[0.4,0.7]},\frac{h_2}{[0.2,0.4],[0.4,0.4],[0.6,0.8]},\frac{h_3}{[0.5,0.9],[0.5,0.8],[0.1,0.5]}\right\},\left\{\frac{c_1}{[0.4,0.5],[0.5,0.9],[0.5,0.6]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.2,0.8],[0.3,0.9],[0.2,0.8]},\frac{c_3}{[0.3,0.4],[0.2,0.8],[0.6,0.7]}\right\}\left\{\frac{v_1}{[0.1,0.4],[0.8,0.8],[0.6,0.9]},\frac{v_2}{[0.2,0.5],[0.1,0.5],[0.5,0.8]}\right\}\right)\right)\right\}.$$

Proposition 3.12 If (F,A) is a neutrosophic vague soft multiset over U, then

1. $((F,A)^c)^c = (F,A)$,
2. $(F,A)^c{}_{\Phi} = (F,A)_U$,
3. $(F,A)^c{}_U = (F,A)_{\Phi}$.

Proof The proof is straightforward.

In the following, we introduce the definition of the union of two neutrosophic vague soft multisets and give an example on the union of two neutrosophic vague soft multisets.

Definition 3.13 The union of two neutrosophic vague soft multisets (F,A) and (G,B) over U, denoted by $(F,A)\cup(G,B)$, is a neutrosophic vague soft multiset (H,D) where $D = A\cup B,$ and $\forall\ \alpha \in D,$

$$H(\alpha)=\begin{cases} F(\alpha) & ,\text{if } \alpha\in A-B \\ G(\alpha) & ,\text{if } \alpha\in B-A \\ F(\alpha)\cup G(\alpha) & ,\text{if } \alpha\in A\cap B, \end{cases}$$

and $\cup$ denotes the neutrosophic vague set union.

Example 3.14 Consider Example 3.2. Let

$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), a_3 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,1})\},$

$B = \{b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\}.$

Suppose (F,A) and (G,B) are two neutrosophic vague soft multisets over the same U such that

$(F,A)=$

$$\left\{\left(a_1,\left(\left\{\frac{h_1}{[0.1,0.4],[0.2,0.6],[0.6,0.9]},\frac{h_2}{[0.3,0.7],[0.8,0.8],[0.3,0.7]},\frac{h_3}{[0.1,0.2],[0.1,0.3],[0.8,0.9]}\right\},\left\{\frac{c_1}{[0.6,0.7],[0.4,0.5],[0.3,0.4]},\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.1,0.8],[0.6,0.7],[0.2,0.9]},\frac{c_3}{[0.2,0.5],[0.7,0.8],[0.5,0.8]}\right\}\left\{\frac{v_1}{[0.7,0.9],[0.1,0.2],[0.1,0.3]},\frac{v_2}{[0.8,0.9],[0.9,0.9],[0.1,0.2]}\right\}\right)\right),$$

$$\left(a_2,\left(\left\{\frac{h_1}{[0.2,0.6],[0.4,0.4],[0.4,0.8]},\frac{h_2}{[0.2,0.8],[0.2,0.5],[0.2,0.8]},\frac{h_3}{[0.3,0.5],[0.1,0.3],[0.5,0.7]}\right\},\left\{\frac{c_1}{[0.4,0.6],[0.4,0.9],[0.4,0.6]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.7,0.8],[0.1,0.4],[0.2,0.3]},\frac{c_3}{[0.6,0.9],[0.4,0.5],[0.1,0.4]}\right\}\left\{\frac{v_1}{[0.2,0.6],[0.4,0.6],[0.4,0.8]},\frac{v_2}{[0.5,0.6],[0.6,0.9],[0.4,0.5]}\right\}\right)\right),$$

$$\left(a_3,\left(\left\{\frac{h_1}{[0.3,0.5],[0.2,0.2],[0.5,0.7]},\frac{h_2}{[0.4,0.6],[0.2,0.6],[0.4,0.6]},\frac{h_3}{[0.1,0.3],[0.4,0.5],[0.7,0.9]}\right\},\left\{\frac{c_1}{[0.5,0.8],[0.3,0.5],[0.2,0.5]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.6,0.8],[0.5,0.7],[0.2,0.4]},\frac{c_3}{[0.6,0.9],[0.5,0.8],[0.1,0.4]}\right\}\left\{\frac{v_1}{[0.4,0.7],[0.2,0.2],[0.3,0.6]},\frac{v_2}{[0.4,0.7],[0.6,0.9],[0.3,0.6]}\right\}\right)\right)\right\}.$$

$(G,B)=$

$$\left\{\left(b_1,\left(\left\{\frac{h_1}{[0.1,0.4],[0.3,0.3],[0.6,0.9]},\frac{h_2}{[0.3,0.5],[0.4,0.7],[0.5,0.7]},\frac{h_3}{[0.4,0.9],[0.1,0.4],[0.1,0.6]}\right\},\left\{\frac{c_1}{[0.5,0.8],[0.4,0.6],[0.2,0.5]},\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.7,0.9],[0.5,0.6],[0.1,0.3]},\frac{c_3}{[0.2,0.7],[0.5,0.7],[0.3,0.8]}\right\}\left\{\frac{v_1}{[0.1,0.9],[0.1,0.4],[0.1,0.9]},\frac{v_2}{[0.5,0.8],[0.4,0.6],[0.2,0.5]}\right\}\right)\right),$$

$$\left(b_2,\left(\left\{\frac{h_1}{[0.3,0.5],[0.2,0.4],[0.5,0.7]},\frac{h_2}{[0.1,0.3],[0.2,0.5],[0.7,0.9]},\frac{h_3}{[0.2,0.5],[0.5,0.6],[0.5,0.8]}\right\},\left\{\frac{c_1}{[0.7,0.9],[0.8,0.9],[0.1,0.3]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.4,0.5],[0.2,0.4],[0.5,0.6]},\frac{c_3}{[0.2,0.7],[0.5,0.5],[0.3,0.8]}\right\}\left\{\frac{v_1}{[0.1,0.5],[0.5,0.6],[0.5,0.9]},\frac{v_2}{[0.6,0.6],[0.7,0.9],[0.4,0.4]}\right\}\right)\right),$$

$$\left(b_3,\left(\left\{\frac{h_1}{[0.4,0.6],[0.2,0.2],[0.4,0.6]},\frac{h_2}{[0.7,0.9],[0.3,0.4],[0.1,0.3]},\frac{h_3}{[0.2,0.4],[0.3,0.3],[0.6,0.8]}\right\},\left\{\frac{c_1}{[0.5,0.7],[0.3,0.4],[0.3,0.5]},\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{c_2}{[0.4,0.5],[0.1,0.3],[0.5,0.6]},\frac{c_3}{[0.5,0.7],[0.3,0.4],[0.3,0.5]}\right\}\left\{\frac{v_1}{[0.1,0.3],[0.2,0.3],[0.7,0.9]},\frac{v_2}{[0.4,0.5],[0.6,0.9],[0.5,0.6]}\right\}\right)\right)\right\}.$$

By using the basic neutrosophic vague union, we have

$(H,D)=$

$$\left\{\left(d_1,\left(\left\{\frac{h_1}{[0.1,0.4],[0.2,0.3],[0.6,0.9]},\frac{h_2}{[0.3,0.7],[0.4,0.7],[0.3,0.7]},\frac{h_3}{[0.4,0.9],[0.1,0.3],[0.1,0.6]}\right\},\left\{\frac{c_1}{[0.6,0.8],[0.4,0.5],[0.2,0.4]},\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.7,0.9],[0.5,0.6],[0.1,0.3]},\frac{c_3}{[0.2,0.7],[0.5,0.7],[0.3,0.8]}\right\}\left\{\frac{v_1}{[0.7,0.9],[0.1,0.2],[0.1,0.9]},\frac{v_2}{[0.8,0.9],[0.4,0.6],[0.1,0.2]}\right\}\right)\right),$$

$$\left(d_2,\left(\left\{\frac{h_1}{[0.3,0.6],[0.2,0.4],[0.4,0.7]},\frac{h_2}{[0.2,0.8],[0.2,0.5],[0.2,0.8]},\frac{h_3}{[0.3,0.5],[0.1,0.3],[0.5,0.7]}\right\},\left\{\frac{c_1}{[0.7,0.9],[0.4,0.9],[0.1,0.3]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.7,0.8],[0.1,0.4],[0.2,0.3]},\frac{c_3}{[0.6,0.9],[0.4,0.5],[0.1,0.4]}\right\}\left\{\frac{v_1}{[0.2,0.6],[0.4,0.6],[0.4,0.8]},\frac{v_2}{[0.6,0.6],[0.6,0.9],[0.4,0.4]}\right\}\right)\right),$$

$$\left(d_3,\left(\left\{\frac{h_1}{[0.3,0.5],[0.2,0.2],[0.5,0.7]},\frac{h_2}{[0.4,0.6],[0.2,0.6],[0.4,0.6]},\frac{h_3}{[0.1,0.3],[0.4,0.5],[0.7,0.9]}\right\},\left\{\frac{c_1}{[0.5,0.8],[0.3,0.5],[0.2,0.5]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.6,0.8],[0.5,0.7],[0.2,0.4]},\frac{c_3}{[0.6,0.9],[0.5,0.8],[0.1,0.4]}\right\}\left\{\frac{v_1}{[0.4,0.7],[0.2,0.2],[0.3,0.6]},\frac{v_2}{[0.4,0.7],[0.6,0.9],[0.3,0.6]}\right\}\right)\right),$$

$$\left(d_4,\left(\left\{\frac{h_1}{[0.4,0.6],[0.2,0.2],[0.4,0.6]},\frac{h_2}{[0.7,0.9],[0.3,0.4],[0.1,0.3]},\frac{h_3}{[0.2,0.4],[0.3,0.3],[0.6,0.8]}\right\},\left\{\frac{c_1}{[0.5,0.7],[0.3,0.4],[0.3,0.5]},\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{c_2}{[0.4,0.5],[0.1,0.3],[0.5,0.6]},\frac{c_3}{[0.5,0.7],[0.3,0.4],[0.3,0.5]}\right\}\left\{\frac{v_1}{[0.1,0.3],[0.2,0.3],[0.7,0.9]},\frac{v_2}{[0.4,0.5],[0.6,0.9],[0.5,0.6]}\right\}\right)\right)\right\},$$

where $D=\{d_1=a_1=b_1,\ d_2=a_2=b_2,\ d_3=a_3,\ d_4=b_3\}$.

In the following, we introduce the definition of the intersection of two neutrosophic vague soft multisets and give an example on the intersection of two neutrosophic vague soft multisets.

Definition 3.15 The intersection of two neutrosophic vague soft multisets (F,A) and (G,B) over U, denoted by $(F,A)\cap(G,B)$, is a neutrosophic vague soft multiset (H,D) where $D = A\cup B$, and $\forall\ \alpha \in D$,

$$H(\alpha) = \begin{cases} F(\alpha) & ,if\ \alpha \in A-B \\ G(\alpha) & ,if\ \alpha \in B-A \\ F(\alpha)\cap G(\alpha) & ,if\ \alpha \in A\cap B, \end{cases}$$

and $\cap$ denotes the neutrosophic vague set intersection.

Example 3.16 Consider Example 3.14. By using the basic neutrosophic vague intersection, we have

$(H,D)=$

$$\left\{\left(d_1,\left(\left\{\frac{h_1}{[0.1,0.4],[0.3,0.6],[0.6,0.9]},\frac{h_2}{[0.3,0.5],[0.8,0.8],[0.5,0.7]},\frac{h_3}{[0.1,0.2],[0.1,0.4],[0.8,0.9]}\right\},\left\{\frac{c_1}{[0.5,0.7],[0.4,0.6],[0.3,0.5]},\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.1,0.8],[0.6,0.7],[0.2,0.9]},\frac{c_3}{[0.2,0.5],[0.7,0.8],[0.5,0.8]}\right\}\left\{\frac{v_1}{[0.1,0.9],[0.1,0.4],[0.1,0.9]},\frac{v_2}{[0.5,0.8],[0.9,0.9],[0.2,0.5]}\right\}\right)\right),$$

$$\left(d_2,\left(\left\{\frac{h_1}{[0.2,0.5],[0.4,0.4],[0.5,0.8]},\frac{h_2}{[0.1,0.3],[0.2,0.5],[0.7,0.9]},\frac{h_3}{[0.2,0.5],[0.5,0.6],[0.5,0.8]}\right\},\left\{\frac{c_1}{[0.4,0.6],[0.8,0.9],[0.4,0.6]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.4,0.5],[0.2,0.4],[0.5,0.6]},\frac{c_3}{[0.2,0.7],[0.5,0.5],[0.3,0.8]}\right\}\left\{\frac{v_1}{[0.1,0.5],[0.5,0.6],[0.5,0.9]},\frac{v_2}{[0.5,0.6],[0.7,0.9],[0.4,0.5]}\right\}\right)\right),$$

$$\left(d_3,\left(\left\{\frac{h_1}{[0.3,0.5],[0.2,0.2],[0.5,0.7]},\frac{h_2}{[0.4,0.6],[0.2,0.6],[0.4,0.6]},\frac{h_3}{[0.1,0.3],[0.4,0.5],[0.7,0.9]}\right\},\left\{\frac{c_1}{[0.5,0.8],[0.3,0.5],[0.2,0.5]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.6,0.8],[0.5,0.7],[0.2,0.4]},\frac{c_3}{[0.6,0.9],[0.5,0.8],[0.1,0.4]}\right\}\left\{\frac{v_1}{[0.4,0.7],[0.2,0.2],[0.3,0.6]},\frac{v_2}{[0.4,0.7],[0.6,0.9],[0.3,0.6]}\right\}\right)\right),$$

$$\left(d_4,\left(\left\{\frac{h_1}{[0.4,0.6],[0.2,0.2],[0.4,0.6]},\frac{h_2}{[0.7,0.9],[0.3,0.4],[0.1,0.3]},\frac{h_3}{[0.2,0.4],[0.3,0.3],[0.6,0.8]}\right\},\left\{\frac{c_1}{[0.5,0.7],[0.3,0.4],[0.3,0.5]},\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{c_2}{[0.4,0.5],[0.1,0.3],[0.5,0.6]},\frac{c_3}{[0.5,0.7],[0.3,0.4],[0.3,0.5]}\right\}\left\{\frac{v_1}{[0.1,0.3],[0.2,0.3],[0.7,0.9]},\frac{v_2}{[0.4,0.5],[0.6,0.9],[0.5,0.6]}\right\}\right)\right)\right\},$$

where $D=\{d_1=a_1=b_1, d_2=a_2=b_2, d_3=a_3, d_4=b_3\}$.

Now we give some propositions on the union and intersection of two neutrosophic vague soft multisets.

Proposition 3.17 If (F,A), (G,B), and (H,C) are three neutrosophic vague soft multisets over U, then

1. $(F,A)\cup((G,B)\cup(H,C)) = ((F,A)\cup(G,B))\cup(H,C)$
2. $(F,A)\cup(F,A)=(F,A)$
3. $(F,A)\cup(G,A)_{\Phi}=(F,A)$
4. $(F,A)\cup(G,A)_{U}=(G,A)_{U}$

Proof The proof is straightforward.

Proposition 3.18 If (F,A), (G,B), and (H,C) are three neutrosophic vague soft multisetsover U, then

1. $(F,A)\cap((G,B)\cap(H,C)) = ((F,A)\cap(G,B))\cap(H,C)$
2. $(F,A)\cap(F,A)=(F,A)$
3. $(F,A)\cap(G,A)_{\Phi}=(G,A)_{\Phi}$
4. $(F,A)\cap(G,A)_{U}=(F,A)$

Proof The proof is straightforward.

4. An Application in Decision Making

In this section we recall the algorithm designed for solving a neutrosophic soft set. We also propose a new algorithm to solve neutrosophic vague soft multiset decision-making problems based on the algorithm for fuzzy soft multiset proposed by Alkhazaleh and Salleh (2012). We then apply this new algorithm to the neutrosophic vague soft multiset model to solve a decision making problem. We will use the abbreviation MA for Maji's Algorithm (Maji, 2013) and abbreviation RMA for Roy and Maji's Algorithm (Roy & Maji, 2007).Maji (2013) used the following algorithm to solve a decision-making problem.

1. input the Neutrosophic Soft Set (F, A).
2. input P, the choice parameters which is a subset of A.
3. consider the NSS (F, P) and write it in tabular form.
4. compute the comparison matrix of the NSS (F, P).
5. compute the score S_i of h_i, $\forall i$.
6. find $S_k = max_i\ S_i$.
7. if k has more than one value then any one of h_i could be the preferable choice.

Alkhazaleh and Salleh (2012) proposed the following algorithm for fuzzy soft multiset.

1. Input the fuzzy soft multiset (H,C) which is introduced by making any operations between (F, A) and (G, B).
2. Apply RMA to the first fuzzy soft multiset part in (H,C) to get the decision S_{k_1}.
3. Redefine the fuzzy soft multiset (H,C) by keeping all values in each row where S_{k_1} is maximum and replacing the values in the other rows by zero to get $(H,C)_1$.
4. Apply RMA to the second fuzzy soft multiset part in $(H,C)_1$ to get the decision S_{k_2}.
5. Redefine the fuzzy soft multiset $(F, A)_1$ by keeping the first and second parts and apply the method in step 3 to the third part to get $(H,C)_2$.
6. Apply RMA to the third fuzzy soft multiset part in $(H,C)_2$ to get the decision S_{k_3}.
7. The decision is $(S_{k_1}, S_{k_2}, S_{k_3})$.

Now we construct a neutrosophic vague soft multiset decision making method by the following algorithm.

Input the neutrosophic vague soft multiset (H,C) which is introduced by making any operation between (F, A) and (G, B).

1. Apply MA to the first neutrosophic vague soft multiset part in (H,C) to get the decision S_{k_1}.
2. Redefine the neutrosophic vague soft multiset (H,C) by keeping all values in each row where S_{k_1} is maximum and replacing the values in the other rows by zero to get $(H,C)_1$.
3. Apply MA to the second neutrosophic vague soft multiset part in $(H,C)_1$ to get the decision S_{k_2}.
4. Redefine the neutrosophic vague soft multiset $(H,C)_1$ by keeping the first and second parts and apply the method in step 3 to the third part to get $(H,C)_2$.
5. Apply MA to the third neutrosophic vague soft multiset part in $(H,C)_2$ to get the decision S_{k_3}.
6. The decision is $(S_{k_1}, S_{k_2}, S_{k_3})$.

We will apply this algorithm to the neutrosophic vague soft multiset model to solve a decision making problem in the following example.

Example 4.1. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$, and $U_3 = \{v_1, v_2\}$, be the sets of "houses", "cars", and "hotels" respectively. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = condominium, e_{U_1,2} = apartment, e_{U_1,3} = duplex\},$$

$$E_{U_2} = \{e_{U_2,1} = compact, e_{U_2,2} = sedan, e_{U_2,3} = sporty\}$$ and

$$E_{U_3} = \{e_{U_3,1} = uptown, e_{U_3,2} = downtown, e_{U_3,3} = suburb\}.$$

Let $A=\{a_1=(e_{U_1,1},e_{U_2,1},e_{U_3,1}),\, a_2=(e_{U_1,2},e_{U_2,3},e_{U_3,1}),\, a_3=e_{U_1,3},e_{U_2,3},e_{U_3,3}\}$ and
$B=\{b_1=(e_{U_1,1},e_{U_2,1},e_{U_3,1}),\, b_2=(e_{U_1,1},e_{U_2,2},e_{U_3,1}),\, b_3=(e_{U_1,2},e_{U_2,3},e_{U_3,1})\}$.

Suppose a person wants to choose objects from the sets of given objects with respect to the sets of choice parameters. Let there be two observations (F,A) and (G,B) by two experts Y_1 and Y_2 respectively. Suppose

$(F,A)=$

$$\Bigg\{\Bigg(a_1,\Bigg(\Bigg\{\frac{h_1}{[0.7,0.8],[0.2,0.6],[0.2,0.3]},\frac{h_2}{[0.4,0.5],[0.2,0.8],[0.5,0.6]},\frac{h_3}{[0.2,0.3],[0.3,0.3],[0.7,0.8]}\Bigg\},\Bigg\{\frac{c_1}{[0.5,0.6],[0.8,0.8],[0.4,0.5]},$$
$$\frac{c_2}{[0.1,0.2],[0.8,0.9],[0.8,0.9]},\frac{c_3}{[0.7,0.9],[0.7,0.8],[0.1,0.3]}\Bigg\}\Bigg\{\frac{v_1}{[0.6,0.8],[0.2,0.3],[0.2,0.4]},\frac{v_2}{[0.5,0.7],[0.8,0.9],[0.3,0.5]}\Bigg\}\Bigg)\Bigg),$$

$$\Bigg(a_2,\Bigg(\Bigg\{\frac{h_1}{[0.3,0.5],[0.1,0.4],[0.5,0.7]},\frac{h_2}{[0.3,0.8],[0.4,0.5],[0.2,0.7]},\frac{h_3}{[0.4,0.8],[0.2,0.3],[0.2,0.6]}\Bigg\},\Bigg\{\frac{c_1}{[0.2,0.3],[0.8,0.9],[0.7,0.8]},$$
$$\frac{c_2}{[0.1,0.2],[0.5,0.9],[0.8,0.9]},\frac{c_3}{[0.8,0.9],[0.3,0.5],[0.1,0.2]}\Bigg\}\Bigg\{\frac{v_1}{[0.3,0.5],[0.5,0.6],[0.5,0.7]},\frac{v_2}{[0.3,0.5],[0.7,0.9],[0.5,0.7]}\Bigg\}\Bigg)\Bigg),$$

$$\Bigg(a_3,\Bigg(\Bigg\{\frac{h_1}{[0.4,0.8],[0.3,0.4],[0.2,0.6]},\frac{h_2}{[0.3,0.5],[0.4,0.6],[0.5,0.7]},\frac{h_3}{[0.1,0.3],[0.4,0.5],[0.7,0.9]}\Bigg\},\Bigg\{\frac{c_1}{[0.2,0.7],[0.4,0.5],[0.3,0.8]},$$

$(G,B)=$

$$\Bigg\{\Bigg(b_1,\Bigg(\Bigg\{\frac{h_1}{[0.2,0.9],[0.1,0.3],[0.1,0.8]},\frac{h_2}{[0.1,0.4],[0.5,0.7],[0.6,0.9]},\frac{h_3}{[0.3,0.8],[0.2,0.4],[0.2,0.7]}\Bigg\},\Bigg\{\frac{c_1}{[0.4,0.9],[0.4,0.5],[0.1,0.6]},$$
$$\frac{c_2}{[0.5,0.6],[0.5,0.9],[0.4,0.5]},\frac{c_3}{[0.1,0.7],[0.6,0.7],[0.3,0.9]}\Bigg\}\Bigg\{\frac{v_1}{[0.8,0.9],[0.1,0.4],[0.1,0.2]},\frac{v_2}{[0.7,0.8],[0.4,0.6],[0.2,0.3]}\Bigg\}\Bigg)\Bigg),$$

$$\Bigg(b_2,\Bigg(\Bigg\{\frac{h_1}{[0.4,0.5],[0.1,0.4],[0.5,0.6]},\frac{h_2}{[0.2,0.3],[0.2,0.5],[0.7,0.8]},\frac{h_3}{[0.4,0.5],[0.6,0.6],[0.5,0.6]}\Bigg\},\Bigg\{\frac{c_1}{[0.7,0.8],[0.5,0.9],[0.2,0.3]},$$
$$\frac{c_2}{[0.5,0.5],[0.2,0.4],[0.5,0.5]},\frac{c_3}{[0.4,0.7],[0.4,0.5],[0.3,0.6]}\Bigg\}\Bigg\{\frac{v_1}{[0.2,0.5],[0.5,0.9],[0.5,0.8]},\frac{v_2}{[0.4,0.6],[0.8,0.9],[0.4,0.6]}\Bigg\}\Bigg)\Bigg),$$

$$\Bigg(b_3,\Bigg(\Bigg\{\frac{h_1}{[0.3,0.6],[0.2,0.6],[0.4,0.7]},\frac{h_2}{[0.5,0.8],[0.2,0.5],[0.2,0.5]},\frac{h_3}{[0.3,0.4],[0.2,0.3],[0.6,0.7]}\Bigg\},\Bigg\{\frac{c_1}{[0.4,0.6],[0.4,0.4],[0.4,0.6]},$$
$$\frac{c_2}{[0.3,0.5],[0.2,0.3],[0.5,0.7]},\frac{c_3}{[0.2,0.8],[0.4,0.4],[0.2,0.8]}\Bigg\}\Bigg\{\frac{v_1}{[0.2,0.3],[0.2,0.3],[0.7,0.8]},\frac{v_2}{[0.3,0.5],[0.7,0.9],[0.5,0.7]}\Bigg\}\Bigg)\Bigg)\Bigg\}.$$

By using the basic neutrosophic vague union, we have
$(H,D)=$

$$\Bigg\{\Bigg(d_1,\Bigg(\Bigg\{\frac{h_1}{[0.7,0.9],[0.1,0.3],[0.1,0.3]},\frac{h_2}{[0.4,0.5],[0.2,0.7],[0.5,0.6]},\frac{h_3}{[0.3,0.8],[0.2,0.3],[0.2,0.7]}\Bigg\},\Bigg\{\frac{c_1}{[0.5,0.9],[0.4,0.5],[0.1,0.5]},$$
$$\frac{c_2}{[0.5,0.6],[0.5,0.9],[0.4,0.5]},\frac{c_3}{[0.7,0.9],[0.6,0.7],[0.1,0.3]}\Bigg\}\Bigg\{\frac{v_1}{[0.8,0.9],[0.1,0.3],[0.1,0.2]},\frac{v_2}{[0.7,0.8],[0.4,0.6],[0.2,0.3]}\Bigg\}\Bigg)\Bigg),$$

$$\left(d_2, \left(\left\{\frac{h_1}{[0.3,0.6],[0.1,0.4],[0.4,0.7]}, \frac{h_2}{[0.5,0.8],[0.2,0.5],[0.2,0.5]}, \frac{h_3}{[0.4,0.8],[0.2,0.3],[0.2,0.6]}\right\}, \left\{\frac{c_1}{[0.4,0.6],[0.4,0.4],[0.4,0.6]}, \frac{c_2}{[0.3,0.5],[0.2,0.3],[0.5,0.7]}, \frac{c_3}{[0.8,0.9],[0.3,0.4],[0.1,0.2]}\right\} \left\{\frac{v_1}{[0.3,0.5],[0.2,0.3],[0.5,0.7]}, \frac{v_2}{[0.3,0.5],[0.7,0.9],[0.5,0.7]}\right\}\right)\right),$$

$$\left(d_3, \left(\left\{\frac{h_1}{[0.4,0.8],[0.3,0.4],[0.2,0.6]}, \frac{h_2}{[0.3,0.5],[0.4,0.6],[0.5,0.7]}, \frac{h_3}{[0.1,0.3],[0.4,0.5],[0.7,0.9]}\right\}, \left\{\frac{c_1}{[0.2,0.7],[0.4,0.5],[0.3,0.8]}, \frac{c_2}{[0.7,0.9],[0.2,0.7],[0.1,0.3]}, \frac{c_3}{[0.7,0.8],[0.1,0.8],[0.2,0.3]}\right\} \left\{\frac{v_1}{[0.3,0.7],[0.2,0.5],[0.3,0.7]}, \frac{v_2}{[0.5,0.8],[0.7,0.9],[0.2,0.5]}\right\}\right)\right),$$

$$\left(d_4, \left(\left\{\frac{h_1}{[0.4,0.5],[0.1,0.4],[0.5,0.6]}, \frac{h_2}{[0.2,0.3],[0.2,0.5],[0.7,0.8]}, \frac{h_3}{[0.4,0.5],[0.6,0.6],[0.5,0.6]}\right\}, \left\{\frac{c_1}{[0.7,0.8],[0.5,0.9],[0.2,0.3]}, \frac{c_2}{[0.5,0.5],[0.2,0.4],[0.5,0.5]}, \frac{c_3}{[0.4,0.7],[0.4,0.5],[0.3,0.6]}\right\} \left\{\frac{v_1}{[0.2,0.5],[0.5,0.9],[0.5,0.8]}, \frac{v_2}{[0.4,0.6],[0.8,0.9],[0.4,0.6]}\right\}\right)\right)\Bigg\}.$$

Now we apply MA to the first neutrosophic vague soft multiset part in (H,D) to take the decision from the availability set U_1 and find the values of $T_*(x_i) = \bar{T}(x_i) - \bar{F}(x_i)$ for interval truth-membership part $\hat{T}_{A_{NV}}(x_i) = [\bar{T}(x_i), T^+(x_i)]$, where $T^+(x_i) = 1 - \bar{F}(x_i)$, $\forall x_i \in U_1$, $F_*(x_i) = \bar{F}(x_i) - \bar{T}(x_i)$ for interval falsity-membership part $\hat{F}_{A_{NV}}(x_i) = [\bar{F}(x_i), F^+(x_i)]$, where $F^+(x_i) = 1 - \bar{T}(x_i)$, $\forall x_i \in U_1$ and take the arithmetic average $I_*(x_i)$ of the end points of the interval indeterminacy-membership part $\hat{I}_{A_{NV}}(x_i) = [\bar{I}(x_i), I^+(x_i)], \forall x_i \in U_1$. Then find the values of $T_*(x_i) + I_*(x_i) - F_*(x_i)$, $\forall x_i \in U_1$. The tabular representation of the first resultant neutrosophic vague soft multiset part will be as in Table 1.

Table 1. Tabular representation: U_1 −neutrosophic vague soft multiset part of (H, D).

U_1	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
h_1	1.4	0.05	0.75	0.05
h_2	0.25	0.95	0.1	-0.65
h_3	0.45	0.65	-0.75	0.4

The comparison table for the first resultant neutrosophic vague soft multiset part will be as in Table 2.

Table 2. Comparison table: U_1 −neutrosophic vague soft multiset part of (H, D).

U_1	h_1	h_2	h_3
h_1	4	3	2
h_2	1	4	2
h_3	2	2	4

Next we compute the row-sum, column-sum, and the score for each h_i as shown in Table 3.

Table 3. Score table: U_1 −neutrosophic vague soft multiset part of *(H, D)*.

U_1	row-sum (r_i)	column-sum (t_i)	Score (s_i)
h_1	9	7	2
h_2	7	9	-2
h_3	8	8	0

From Table 3, it is clear that the maximum score is 2, scored by h_1. Now we redefine the neutrosophic vague soft multiset *(H, D)* by keeping all values in each row where h_1 is maximum and replacing the values in the other rows by zeros to get $(H, D)_1$.

$(H,D)_1 =$

$$\left\{\left(d_1,\left(\left\{\frac{h_1}{[0.7,0.9],[0.1,0.3],[0.1,0.3]},\frac{h_2}{[0.4,0.5],[0.2,0.7],[0.5,0.6]},\frac{h_3}{[0.3,0.8],[0.2,0.3],[0.2,0.7]}\right\},\left\{\frac{c_1}{[0.5,0.9],[0.4,0.5],[0.1,0.5]},\frac{c_2}{[0.5,0.6],[0.5,0.9],[0.4,0.5]},\frac{c_3}{[0.7,0.9],[0.6,0.7],[0.1,0.3]}\right\}\left\{\frac{v_1}{[0.8,0.9],[0.1,0.3],[0.1,0.2]},\frac{v_2}{[0.7,0.8],[0.4,0.6],[0.2,0.3]}\right\}\right)\right),\right.$$

$$\left(d_2,\left(\left\{\frac{h_1}{[0.3,0.6],[0.1,0.4],[0.4,0.7]},\frac{h_2}{[0.5,0.8],[0.2,0.5],[0.2,0.5]},\frac{h_3}{[0.4,0.8],[0.2,0.3],[0.2,0.6]}\right\},\left\{\frac{c_1}{[0,0],[1,1],[1,1]},\frac{c_2}{[0,0],[1,1],[1,1]},\frac{c_3}{[0,0],[1,1],[1,1]}\right\}\left\{\frac{v_1}{[0,0],[1,1],[1,1]},\frac{v_2}{[0,0],[1,1],[1,1]}\right\}\right)\right),$$

$$\left(d_3,\left(\left\{\frac{h_1}{[0.4,0.8],[0.3,0.4],[0.2,0.6]},\frac{h_2}{[0.3,0.5],[0.4,0.6],[0.5,0.7]},\frac{h_3}{[0.1,0.3],[0.4,0.5],[0.7,0.9]}\right\},\left\{\frac{c_1}{[0.2,0.7],[0.4,0.5],[0.3,0.8]},\frac{c_2}{[0.7,0.9],[0.2,0.7],[0.1,0.3]},\frac{c_3}{[0.7,0.8],[0.1,0.8],[0.2,0.3]}\right\}\left\{\frac{v_1}{[0.3,0.7],[0.2,0.5],[0.3,0.7]},\frac{v_2}{[0.5,0.8],[0.7,0.9],[0.2,0.5]}\right\}\right)\right),$$

$$\left.\left(d_4,\left(\left\{\frac{h_1}{[0.4,0.5],[0.1,0.4],[0.5,0.6]},\frac{h_2}{[0.2,0.3],[0.2,0.5],[0.7,0.8]},\frac{h_3}{[0.4,0.5],[0.6,0.6],[0.5,0.6]}\right\},\left\{\frac{c_1}{[0,0],[1,1],[1,1]},\frac{c_2}{[0,0],[1,1],[1,1]},\frac{c_3}{[0,0],[1,1],[1,1]}\right\}\left\{\frac{v_1}{[0,0],[1,1],[1,1]},\frac{v_2}{[0,0],[1,1],[1,1]}\right\}\right)\right)\right\}.$$

Now we apply MA to the second neutrosophic vague soft multiset part in *(H, D)₁*. to take the decision from the availability set U_2 and find the values of $T_*(x_i)=\bar{T}(x_i)-\bar{F}(x_i)$ for interval truth-membership part $\hat{T}_{A_{NV}}(x_i)=[\bar{T}(x_i),T^+(x_i)]$, where $T^+(x_i)=1-\bar{F}(x_i)$, $\forall x_i\in U_2$, $F_*(x_i)=\bar{F}(x_i)-\bar{T}(x_i)$ for interval falsity-membership part $\hat{F}_{A_{NV}}(x_i)=[\bar{F}(x_i),F^+(x_i)]$, where $F^+(x_i)=1-\bar{T}(x_i)$, $\forall x_i\in U_2$ and take the arithmetic average $I_*(x_i)$ of the end points of the interval indeterminacy-membership part $\hat{I}_{A_{NV}}(x_i)=[\bar{I}(x_i),I^+(x_i)],\forall x_i\in U_2$. Then find the values of $T_*(x_i)+I_*(x_i)-F_*(x_i)$, $\forall x_i\in U_2$. The tabular representation of the second resultant neutrosophic vague soft multiset part will be as in Table 4.

Table 4. Tabular representation: U_2 −neutrosophic vague soft multiset part of $(H,D)_1$.

U_2	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
c_1	1.25	-1	0.25	-1
c_2	0.9	-1	1.65	-1
c_3	1.85	-1	1.45	-1

The comparison table for the second resultant neutrosophic vague soft multiset part will be as in Table 5.

Table 5. Comparison table: U_2 −neutrosophic vague soft multiset part of $(H,D)_1$.

U_2	c_1	c_2	c_3
c_1	4	3	2
c_2	3	4	3
c_3	4	3	4

Next we compute the row-sum, column-sum, and the score for each c_i as shown in Table 6.

Table 6. Score table: U_2 −neutrosophic vague soft multiset part of $(H, D)_1$.

U_1	row-sum (r_i)	column-sum (t_i)	Score (s_i)
c_1	9	11	-2
c_2	10	10	0
c_3	11	9	2

From Table 6, it is clear that the maximum score is 2, scored by c_3.

Now we redefine the neutrosophic vague soft multiset $(H, D)_1$ by keeping all values in each row where c_3 is maximum and replacing the values in the other rows by zeros to get $(H, D)_2$.

$(H,D)_2 =$

$$\left\{\left(d_1,\left(\left\{\frac{h_1}{[0.7,0.9],[0.1,0.3],[0.1,0.3]},\frac{h_2}{[0.4,0.5],[0.2,0.7],[0.5,0.6]},\frac{h_3}{[0.3,0.8],[0.2,0.3],[0.2,0.7]}\right\},\left\{\frac{c_1}{[0.5,0.9],[0.4,0.5],[0.1,0.5]},\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.5,0.6],[0.5,0.9],[0.4,0.5]},\frac{c_3}{[0.7,0.9],[0.6,0.7],[0.1,0.3]}\right\}\left\{\frac{v_1}{[0.8,0.9],[0.1,0.3],[0.1,0.2]},\frac{v_2}{[0.7,0.8],[0.4,0.6],[0.2,0.3]}\right\}\right)\right),$$

$$\left(d_2,\left(\left\{\frac{h_1}{[0.3,0.6],[0.1,0.4],[0.4,0.7]},\frac{h_2}{[0.5,0.8],[0.2,0.5],[0.2,0.5]},\frac{h_3}{[0.4,0.8],[0.2,0.3],[0.2,0.6]}\right\},\left\{\frac{c_1}{[0,0],[1,1],[1,1]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0,0],[1,1],[1,1]},\frac{c_3}{[0,0],[1,1],[1,1]}\right\}\left\{\frac{v_1}{[0,0],[1,1],[1,1]},\frac{v_2}{[0,0],[1,1],[1,1]}\right\}\right)\right),$$

$$\left(d_3,\left(\left\{\frac{h_1}{[0.4,0.8],[0.3,0.4],[0.2,0.6]},\frac{h_2}{[0.3,0.5],[0.4,0.6],[0.5,0.7]},\frac{h_3}{[0.1,0.3],[0.4,0.5],[0.7,0.9]}\right\},\left\{\frac{c_1}{[0.2,0.7],[0.4,0.5],[0.3,0.8]},\right.\right.\right.$$

$$\left.\left.\left.\frac{c_2}{[0.7,0.9],[0.2,0.7],[0.1,0.3]},\frac{c_3}{[0.7,0.8],[0.1,0.8],[0.2,0.3]}\right\}\left\{\frac{v_1}{[0.3,0.7],[0.2,0.5],[0.3,0.7]},\frac{v_2}{[0.5,0.8],[0.7,0.9],[0.2,0.5]}\right\}\right)\right),$$

$$\left(d_4,\left(\left\{\frac{h_1}{[0.4,0.5],[0.1,0.4],[0.5,0.6]},\frac{h_2}{[0.2,0.3],[0.2,0.5],[0.7,0.8]},\frac{h_3}{[0.4,0.5],[0.6,0.6],[0.5,0.6]}\right\},\left\{\frac{c_1}{[0,0],[1,1],[1,1]},\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{c_2}{[0,0],[1,1],[1,1]},\frac{c_3}{[0,0],[1,1],[1,1]}\right\}\left\{\frac{v_1}{[0,0],[1,1],[1,1]},\frac{v_2}{[0,0],[1,1],[1,1]}\right\}\right)\right)\right\}.$$

Now we apply MA to the third neutrosophic vague soft multiset part in $(H, D)_2$ to take the decision from the availability set U_3 and find the values of $T_*(x_i)=\overline{T}(x_i)-\overline{F}(x_i)$ for interval truth-membership part $\hat{T}_{A_{NV}}(x_i)=[\overline{T}(x_i),T^+(x_i)]$, where $T^+(x_i)=1-\overline{F}(x_i)$, $\forall x_i\in U_3$, $F_*(x_i)=\overline{F}(x_i)-\overline{T}(x_i)$ for interval falsity-membership part $\hat{F}_{A_{NV}}(x_i)=[\overline{F}(x_i),F^+(x_i)]$, where $F^+(x_i)=1-\overline{T}(x_i)$, $\forall x_i\in U_3$ and take the arithmetic average $I_*(x_i)$ of the end points of the interval indeterminacy- membership part $\hat{I}_{A_{NV}}(x_i)=[\bar{I}(x_i),I^+(x_i)],\forall x_i\in U_3$. Then find the values of $T_*(x_i)+I_*(x_i)-F_*(x_i)$, $\forall x_i\in U_3$. The tabular representation of the third resultant neutrosophic vague soft multiset part will be as in Table 7.

Table 7. Tabular representation: U_3 −neutrosophic vague soft multiset part of $(H, D)_2$.

U_2	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
v_1	1.6	-1	0.35	-1
v_2	1.5	-1	1.4	-1

The comparison table for the second resultant neutrosophic vague soft multiset part will be as in Table 8.

Table 8. Comparison table: U_3 −neutrosophic vague soft multiset part of $(H,D)_1$.

U_3	v_1	v_2
v_1	4	3
v_2	3	4

Next we compute the row-sum, column-sum, and the score for each v_i as shown in Table 9.

Table 9. Score table: U_3 −neutrosophic vague soft multiset part of $(H,D)_2$.

U_1	row-sum (r_i)	column-sum (t_i)	Score (s_i)
v_1	7	7	0
v_2	7	7	0

From Table 9, it is clear that the maximum score is 0, scored by v_1 and v_2.

Thus from the above results the most suitable decision is (h_1, c_3, v_1) or (h_1, c_3, v_2). To illustrate the advantages of our proposed method using neutrosophic vague soft multiset as compared to that of vague soft multiset as proposed by Alhazaymeh and Hassan (2014a), let us consider Example 4.1 above, where the vague soft multiset can only describe (*F, A*) and (*G, B*) in this problem as follows:

$$(F,A)=\left\{\left(a_1,\left(\left\{\frac{h_1}{[0.7,0.8]},\frac{h_2}{[0.4,0.5]},\frac{h_3}{[0.2,0.3]}\right\},\left\{\frac{c_1}{[0.5,0.6]},\frac{c_2}{[0.1,0.2]},\frac{c_3}{[0.7,0.9]}\right\},\left\{\frac{v_1}{[0.6,0.8]},\frac{v_2}{[0.5,0.7]}\right\}\right)\right),...$$

and

$$(G,B)=\left\{\left(b_1,\left(\left\{\frac{h_1}{[0.2,0.9]},\frac{h_2}{[0.1,0.4]},\frac{h_3}{[0.3,0.8]}\right\},\left\{\frac{c_1}{[0.4,0.9]},\frac{c_2}{[0.5,0.6]},\frac{c_3}{[0.1,0.7]}\right\},\left\{\frac{v_1}{[0.8,0.9]},\frac{v_2}{[0.7,0.8]}\right\}\right)\right),...$$

Note that the neutrosophic vague soft multiset is a generalization of vague soft multiset. Thus as shown in Example 4.1 above, the neutrosophic vague soft multiset can explain the universal *U* in more detail with three membership functions, especially when there are many parameters involved, whereas vague soft multiset can tell us limited information about the universal *U*. It can only handle the incomplete information considering both the truth-membership and falsity-membership values, while neutrosophic vague soft multiset can handle problems involving imprecise, indeterminacy and inconsistent data, which makes it more accurate and realistic than vague soft multiset. Furthermore, vague set is an intutionistic fuzzy set (Atanassov, 1986) which is a generalization of fuzzy set, and hence neutrosophic vague soft multiset can better handle the elements of imprecision and uncertainty compared to soft multiset, fuzzy soft multiset, vague soft multiset and intuitionistic fuzzy multiset.

5. Conclusions

We established the concept of neutrosophic vague soft multiset by applying the theory of soft multiset to neutrosophic vague set. The basic operations on neutrosophic vague soft multiset, namely complement, subset, union, intersection, were defined. Subsequently, the basic properties of these operations pertaining to the concept of neutrosophic vague soft multiset were given and proven. Finally, a generalized algorithm is introduced and applied to the neutrosophic vague soft multiset model to solve a hypothetical decision making problem. This new extension will provide a significant addition to existing theories for handling indeterminacy, and spurs more developments of further research and pertinent applications.

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