S- Anti Fuzzy M-Semigroup

Farhan Dakhil Shyaa

Department of Mathematics University of Al-Qadisiyah, College of Education, Al-Qadisiyah, Iraq
Farhan.Shyaa@qu.edu.iq, farhan_math1@yahoo.co.uk

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Abstract

In this paper, we define the concept of a smarandache anti fuzzy M-semigroup (S-Anti Fuzzy M-Semigroup) and some elementary properties about this concept are discussed.

Key words: Fuzzy sets, semigroup, M-semigroup

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Introduction

In 1965 Zadeh introduced the concept of fuzzy set[1], in 1971 Rosenfeld formulated the term of fuzzy subgroup[2]. In 1994 W.X.Gu, S.Y.Li and D.G.Chen studied fuzzy groups and gave some new concepts as M-fuzzy groups [3]. In 2002 W.B.Vasantha introduced the concepts of smarandache semigroups[4]. Smarandache fuzzy semigroups are studied in 2003 by W.B.Vasantha[5]. In 2011 H.R.Yassein and M.O.Karim introduced the concept of a smarandache M – semigroup (S-M-semigroup) and studied some basic properties [6].

In this paper, the concept of Smarandache anti-fuzzy M semigroup are given and its some elementary properties are discussed

1- Preliminaries

Definition (1.1): Let G be a group. A fuzzy subset μ of a group G is called anti fuzzy subgroup of the group G if:

1- \( \mu(xy) \leq \max\{\mu(x), \mu(y)\} \) for every \( x, y \in G \).

2. \( \mu(x)=\mu(x^{-1}) \) for every \( x \in G \). [7]

Definition (1.2): A semigroup H with operators is an algebraic system consisting of a semigroup H , set M , and a function defined in the product M×H and having values in H such that , if ma denotes the element in H determined by the element a in H and the element m in M , then m(ab)=(ma)(mb) , a,b \in H and m \in M then H is M – semigroup [3].

We shall usually use the phrase "G is an M-group” to a group with operators.

Definition (1.3): If \( \mu \) is a fuzzy set of G and \( t \in [0,1] \) then \( \mu_t = \{ x \in G | \mu(x) \leq t \} \) is called a t-level set \( \mu \) [6].

Definition (1.4): Let G and G’ both be M – groups , f be a homomorphism from G onto G’ , if f(mx)=mf(x) for every m \in M , x \in X , then f is called a M – homomorphism [5].

Definition (1.5): Let S be a semigroup , S is said to be a smarandache semigroup (S – semigroup ) if S has a proper subset P such that P is a group under the operation of G [5].

Definition (1.6): Let G be any group. A mapping \( \mu: G \rightarrow [0,1] \) is a fuzzy group if (1) \( \mu(xy) \geq \min \{ \mu(x) , \mu(y) \} \) (2) \( \mu(x^{-1}) = \mu(x) \) for all \( x, y \in G \)[1].

Definition (1.7): Let H be M- semigroup . H is said to be a smarandache M – semigroup (S-M-semigroup) if H has a proper subset K such that K is M- group under the operation of H [6].

this S- fuzzy semigroup is denoted by \( \mu_p:P \rightarrow [0,1] \) is fuzzy group.

Definition(1.8) : A group with operators is an algebraic system consisting of a group G , set M and a function defined in the product M×G and having value in G such that , if ma denotes the elements in G determined by the elements a in H and the element m in M , then m(ab)=(ma)(mb) hold for all a,b in G ,m in M [3].

Definition (1.9):Let H be a S - M –semigroup. A fuzzy subset \( \mu :H \rightarrow [0,1] \) is said be smarandache fuzzy M-semigroup if \( \mu \) restricted to at least one subset K of H which is subgroup is fuzzy subgroup[2].
Definition (1.10): Let $S$ and $S'$ be any two $S$-semigroups. A map $\varphi$ from $S$ to $S'$ is said to be $S$- semigroup homomorphism if $\varphi$ restricted to a subgroup $A \subseteq S \rightarrow A' \subseteq S'$ is a group homomorphism [2].

Definition (1.11): Let $H$ and $K$ be any two $S$-$M$-semigroup. A map $\varphi$ from $H$ to $K$ is said to be $S$-$M$- semigroup homomorphism if $\varphi$ restricted to a $M$- subgroup $A \subseteq H \rightarrow A' \subseteq K$ is $M$- homomorphism [6].

Definition (1.12): Let $f$ be a function from a set $X$ to a set $Y$ while $\mu$ is fuzzy set of $X$ then the image $f(\mu)$ of $\mu$ is the fuzzy set $f(\mu): Y \rightarrow [0,1]$ defined by: [7]

$$f(\mu(y)) = \begin{cases} \sup \mu(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

Definition (1.13): Let $f$ be a function from a set $X$ to a set $Y$ while $\mu$ is fuzzy set of $Y$ then the inverse image $f^{-1}(\mu)$ of $\mu$ under $f$ is the fuzzy set $f^{-1}(\mu): X \rightarrow [0,1]$ defined by $f^{-1}(\mu)(x) = \mu(f(x))$ [7].

2-The Main Results

In this section we shall define Smarandache anti fuzzy $M$-semigroup and give some its results.

Definition (2.1): Let $G$ be $M$- group and $\mu$ be anti fuzzy subgroup of $G$ if $\mu(mx) \leq \mu(x)$ for every $x \in G$, $m \in M$, then $\mu$ is said to be anti fuzzy subgroup with operators of $G$, we use the phrase $\mu$ is an $M$- anti fuzzy subgroup of $G$ instead of a fuzzy subgroup with operators of $G$.

Definition (2.2): Let $S$ be an $S$-semigroup. A fuzzy subset $\mu: S \rightarrow [0,1]$ is said to be Smarandache anti fuzzy semigroup (S- anti fuzzy semigroup) if $\mu$ restricted to at least one subset $P$ of $S$ which is a subgroup is anti fuzzy subgroup.

that is for all $x,y \in P \subseteq S$, $\mu(xy^{-1}) \leq \max \{ \mu(x), \mu(y) \}$.

Definition (2.3): Let $H$ be a $S$-$M$-semigroup. A fuzzy subset $\mu: H \rightarrow [0,1]$ is said to be Smarandache anti fuzzy $M$-semigroup if restricted to at least one subset $K$ of $H$ which is anti fuzzy $M$-subgroup

Proposition (2.4): If $\mu$ is $S$-anti fuzzy $M$-semigroup of $S$-$M$-semigroup then:

1) $\mu_k(m(xy)) \leq \max \{ \mu_k(mx), \mu_k(my) \}$
2) $\mu_k(mx^{-1}) \leq \mu_k(x)$

For all $m \in M$, $x,y \in K$

Proof: $\mu$ is $S$- fuzzy $M$-semigroup

Then there exist subset $K$ of $H$ which is $M$-subgroup such $\mu$ restricted of $K$ which is anti-fuzzy

i.e $\mu_k: K \rightarrow [0,1]$, $M$- anti fuzzy subgroup

for all $x,y \in K$, $m \in M$, it is clear that

1) $\mu_k(m(xy)) \leq \mu_k((mx)(my)) \leq \max \{ \mu_k(mx), \mu_k(my) \}$
2) $\mu_k(mx^{-1}) = \mu_k(mx)^{-1} \leq \mu_k(mx)$

Proposition (2.5): Let $G$ be $S$-semigroup, $\mu$ fuzzy set of $G$. Then $\mu$ is an $S$- anti fuzzy $M$-semigroup of $G$ if and only if $f \forall t \in [0,1]$, $\mu_t$ is an $S$-$M$-semigroup $\mu_t \neq \emptyset$. 
**Proof:** It is clear $\mu_t$ is semigroup of $G$ while $\mu_t \neq \emptyset$ holds.

for any $x \in \mu_t$, $m \in M$

$\mu(mx) \leq \mu(x) \leq t$

hence $mx$ in $\mu_t$, hence $\mu_t$ is an $M$- semigroup of $G$.

Since $\mu$ S-anti fuzzy $M$- semigroup $\exists K \subset G$

subgroup $\exists \mu_t : K \rightarrow [0,1]$

fuzzy $M$- subgroup.

$\mu_K = \{ x \in K \mid \mu_K(x) \leq t \}.$

It is clear $\mu_K$ is group. Hence $\mu_t$ S-M- semigroup.

Conversely, Since $\mu_t$ S-M- semigroup then there exists a proper subset $K$ of $G$ such that $K$ is $M$- subgroup.

If there exists $x \in K$, $m \in M$ such that $\mu_K(mx) > \mu_K(x)$.

let $t = \frac{1}{2} (\mu_K(mx) + \mu_K(x))$ then $\mu_K(x) < t < \mu_K(mx)$ $mx \notin \mu_K$, so here emerges a contradiction.

$\mu_K(mx) \leq \mu_K(x)$ always holds for any $x \in K, m \in M$.

$\mu_K$ is $M$- fuzzy subgroup hence $\mu$ is S- anti fuzzy $M$- subgroup. ■

**Proposition (2.6):** Let $H$ and $K$ both be S-M- semigroup and $f$ as S-M- semigroup homomorphism from $H$ onto $K$. if $\mu$ is an S- anti fuzzy $M$- semigroup of $H$ then $f(\mu)$ is an S-anti fuzzy $M$-semigroup of $K$.

**Proof:**

Since $f : H \rightarrow K$ is as S-M- semigroup homomorphism then $f$ restricted to $M$- subgroup.

$A \subset H \rightarrow B \subset K$ is $M$- homomorphism,

$f^{-1}(\mu) : A \rightarrow [0,1]$ such that $A M$-subgroup ,

For any $m \in M, x \in A$

$f^{-1}(\mu)(mx) = \mu_A(f(mx))$

$= \mu_A(m(f(x)) \leq \mu_A(f(x))$

$= f^{-1}(\mu)(x)$

$f^{-1}(\mu)$ is S-anti fuzzy $M$- semigroup ■

**Proposition (2.7):** Let $H$ and $K$ both be S-M- semigroups and $f$ as S-M- semigroup homomorphism from $H$ onto $K$. if $\mu$ is an S- anti fuzzy $M$- semigroup of $H$ then $f(\mu)$ is an S-anti fuzzy $M$-semigroup of $K$.

**Proof:**

Since $f : H \rightarrow K$ is as S-M- semigroup homomorphism then $f$ restricted to $M$- subgroup.

$A \subset H \rightarrow B \subset K$ is $M$- homomorphism

$f(\mu) : B \rightarrow [0,1]$ such that $B M$-subgroup ,

For any $m \in M, y \in B$

$f(\mu)(my) = \sup \mu(x), x \in f^{-1}(my)$

$= \{ \sup \mu(x) \mid f(x) = my \leq \sup \mu(mx'), f(mx') = mx \}$

, $mx' \in H$
\[ = \sup \mu(x') \quad mf(x')=my \]

\[ \leq \sup \mu(x') \quad f(x')=y \quad x' \in H \]

\[ = f(\mu)(y) \]

hence \( f^{-1}(\mu') \) is S- anti fuzzy M- semigroup.

**References**


ضد شبه الزمرة S-M الضبابية

فرحان داخل شياع

جامعه القادسيه / كليه التربية / قسم الرياضيات

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المستخلص

في هذا البحث عرفنا البنى الجبرية ضد شبه الزمرة S-M الضبابية ودراسة بعض الخواص الأساسية لها.