## SIMULATION MODELLING IN TRAINING FUTURE TEACHERS OF MATHEMATICS

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**Abstract.** Issues of simulation modeling are currently being given more and more attention in training specialists from different fields. In particular, in pedagogical universities it can be actively used during classes in teaching methods of various subjects. Simulation modeling involves the replacement of a real process by its model, reflecting the most significant characteristics of a real process from the point of view of the goals set [1]. Therefore, the objective of this paper is to evaluate the effectiveness of the use of simulation modeling in the process of professional training of future school teachers of mathematics. Pedagogical experiment, observation and modeling were used as research methods. In order to learn the skills of conducting lessons before starting pedagogical practice at school, the student of a pedagogical university needs to acquire the initial skills of conducting lessons, and it is desirable to get this experience in conditions close to real as much as possible [2]. In ordinary practice, the future teachers conduct their first test classes for “studying students" - their groupmates. But, since the school material of the lesson is too familiar to these "students" and their behavior is too predictable, the pedagogical practice carried out in this form is fare from effective, since it has nothing to do with the real school lesson. Bringing this lesson to reality, in our opinion, can be done in two ways. For example, a teacher selects material for the lesson that would be similar to the material in the school textbook, but is either unfamiliar or little known to the audience, which plays the role of schoolchildren, or conducts classes for virtual students, replacing the “studying students” - groupmates - with simulators. The paper shows the effective use of a virtual simulator for the training of future teachers of mathematics.

**Keywords:** simulation methods in education, training of future teachers of mathematics, educational technologies, simulation modeling, pedagogical practice.

**Introduction.** Our interest in this topic arose after listening to the report by Frederic Castel (ESPE de l'académie de Reims au sein de l'Université de Reims's insight: Frédéric CASTEL, directeur adjoint de l'ESPE de l'académie de Reims, en charge des sites de Chaumont et Troyes.), the director of the branches of the Pedagogical Institute at the University of Reims in the cities of Troyes and Chaumont. Frederic Castel made his presentation at a conference dedicated to the 25th anniversary of the Faculty of Mathematics and Informatics of the Pedagogical University of the city of Naberezhnye Chelny in 2015. He spoke about the experience of French scientists in creating a virtual classroom and the first results in the use of their proposed methodology. In our work, we consider other areas of simulation modeling in training future teachers of mathematics.

Issues of simulation modeling are currently being given more and more attention in training specialists from different fields [3]. In particular, in pedagogical universities it can be actively used during classes in teaching methods of various subjects [4-7]. Simulation modeling involves the replacement of a real process by its model, reflecting the most significant characteristics of a real process from the point of view of the goals set. In order to learn the skills of conducting lessons before starting pedagogical practice at school, the student of a pedagogical university needs to acquire the initial skills of conducting lessons, and it is desirable to get this experience in conditions close to real as much as possible [8].

Modern students of Russian universities, regardless of the chosen specialty, have great problems with oral presentation of the material due to the lack of the ability to correctly express any thoughts. This, of course, is a consequence of the fact that in the present school they are little taught to speak verbally, for example, to verbally prove the theorems. In addition, they have no experience of taking oral examinations. Therefore, it is easier for the trainee to silently solve the problem and write it down on the board, which, of course, cannot satisfy the majority of the students. In addition, when they come to practice in school, students find themselves in an environment that is often far from being friendly to them, where, along with tasks from the subject area, circumstances require the solution of pedagogical problems. Due to these reasons, the approach to "classroom" preparation of students for pedagogical practice should be very serious.

The student conducts his/her first test lessons for “students" - his/her groupmates. But, since the school material of the lesson is too familiar to these "students" and their behavior is too predictable, the pedagogical practice carried out in this form is far from effective, since it has nothing to do with the real school lesson. "Students" are familiar with all the material that the "teacher" represents to them, and, of course, this cannot but affect the reaction of "students" during the test lesson, influences their behavior, which in turn reduces the effectiveness of such training lessons for a student who plays the role of a teacher.

**Methods.** In order to bring such test lessons closer to reality, we can go in two ways. One of the ways is to select material for the lesson that would be similar to the material in the school textbook, but unfamiliar, or little known to the audience, which plays the role of schoolchildren. For example, geometry lessons are not based on the material of Euclidean geometry, which is studied at school, but on a material reflecting similar questions of non-Euclidean geometry, for example, Lobachevsky geometry. In this case, we apply a kind of imitation technology, which consists in teaching students of own group not based on school material, but on material similar to school one.

We may also include new theorems of Lobachevsky geometry in the educational process that are analogous to known theorems from Euclidean geometry, for example, such as those considered in papers [9, 10].

On the other hand, learning of how to conduct the lesson itself is the most important task.

Another possibility to simulate real school lessons when conducting test lessons with student is the use of olympiad problems.

Another way of approaching the lesson of the trainee to reality is to replace the groupmate-“students" with simulators. That is, the student conducts test lessons in a computer class, where the role of students is played by computers with the programmed behavior of students. Such a lesson can already be conducted on the material of school textbooks, since these "students" will now play a role according to the pre-set scenario, which has as its goal the verification of all possible skills of the trainee [11].

In this paper, we will consider in more detail the second method of approaching the lesson of the trainee to reality. Obviously, several important issues come to the fore. First, the reaction of such “students" may require the teacher's efforts to explain or interpret some theoretical nuances or questions with the solution of concrete examples (the trainee is not sure in advance which questions and problems in understanding the lesson material will arise with the "student"). Secondly, the simulator, in order to approach the reality of the lesson, has also the pre-set behavioral "anomalies" of such "students", i.e. it allows also checking both the methodical preparation of the student and his/her pedagogical knowledge and skills. Thirdly, this allows assessing the level of preparedness of future teachers of mathematics on the example of solving olympiad problems in geometry with the help of a simulator.

Objective of this work is to assess the effectiveness of the use of simulation modeling in the process of training future school teachers of mathematics and compare their level of preparedness for the future profession, depending on the course they study, and also with the professional level of working teachers.

Pedagogical experiment, observation and modeling were used as research methods. Let's consider in more detail one of the possible scenarios of the simulator operation on the example of the geometric task block (see Figure 1).

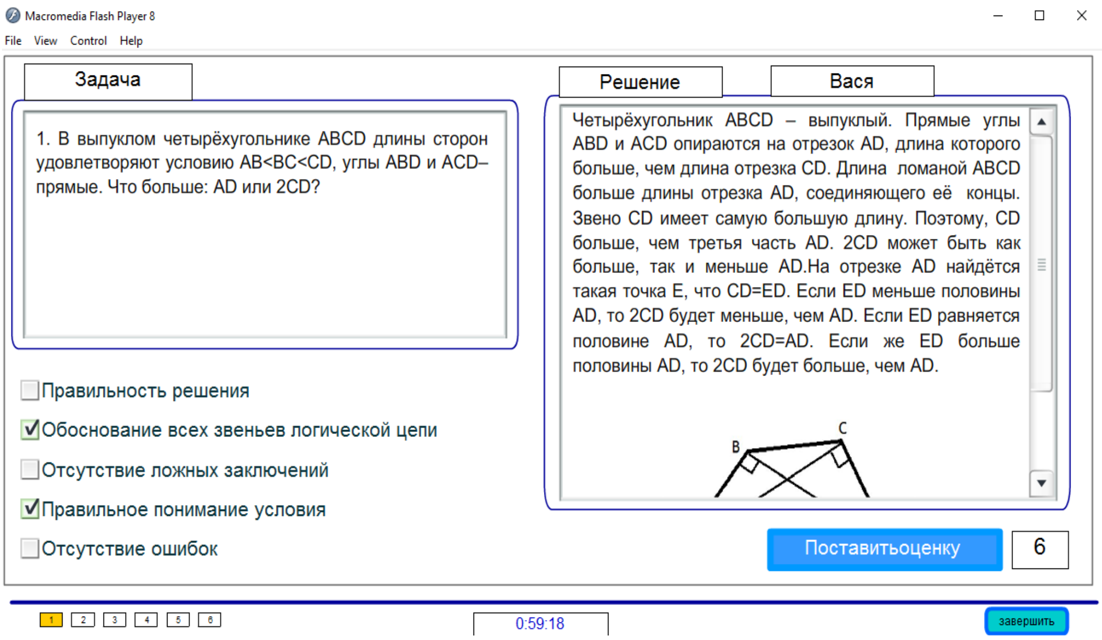


Fig. 1. Working with a geometric task block on the simulator

A virtual class consists of 6 "students" with the names assigned. The trainee is asked to consistently evaluate the solutions to each of the six tasks by all the "students", i.e. check out 36 solutions. Let us give an example of one such task with six "solutions".

Task. The triangle ABC has the value of the angle A equal to $75^\circ$, the value of the angle B is $45^\circ$. Is it true that the points symmetric to the orthocenter H (the point of intersection of heights) relative to the sides of the triangle ABC are the vertices of a right-angled triangle?

**Solution 1.**

*The points symmetric to the orthocenter of the triangle ABC lie on the circumscribed circle. The triangle MNK is similar to the triangle XYZ with the similarity factor 2. The heights of the triangle ABC are the bisectors of the orthic triangle XYZ. Because of their similarity, they will be bisectors for the triangle MNK. Therefore, the arc MB will be equal to the arc BN. The right-angled triangle BXC has the angle B equal to* $45^\circ$*. Hence, the angle CSB is also equal* *to* $45^{\circ}$*. If O is the center of the circle, then the angle MOV is right, since it rests on the same arc as the inscribed angle XCB=MSW. The angle BON is also right, since it is equal to the angle MOV. It follows that the center O lies on the segment MN, that is, MN is the diameter. The angle MKN, resting on it, is right.*

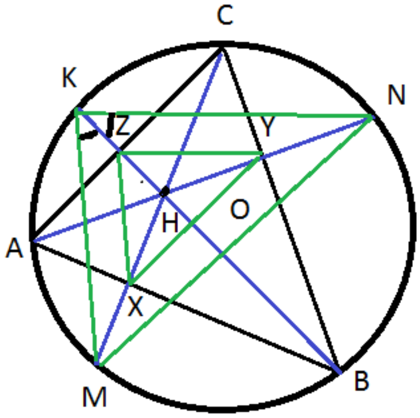


Fig. 2. Solution No.1

**Answer: yes.**

**Solution 2.**

*Let* $A_1\ B_1C_1$ *be an orthic triangle, and* $A_2, B_2, C_2$ *be the points symmetric to the orthocenter H relative to the sides of the triangle ABC. These points lie on the circumscribed circle of the triangle ABC.*

$B_2||A_1\ B_1$, $B_2C_2||B_1C_1$*. Consequently, the angle* $B_2B$ *is equal to the angle* $A_1\ B_1B$*, the angle* $B_2B$ *is equal to the angle*$C_1\ B_1B$*. Since these angles are equal to each other, the central angles* $A_2OB$ *u* $C_2OB$ *will also be equal. If the angle B is equal to* $45^{\circ}$*, then the angle* $BAA_1$ *is also equal to* $45^{\circ}$*, since* $BA_1$ *is the height of the triangle ABC. Therefore, the angle* $BB_2A_2(= BAA_2)$ *is also equal to* $45^{\circ}$*. Similarly, the angle* $BB_2C_2(= BCC_2)$ *is* $45^{\circ}$*. The sum of these angles will be a right angle.*

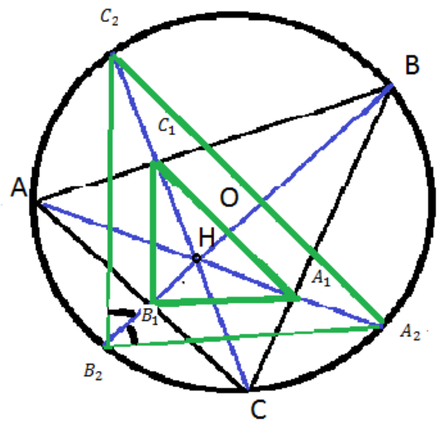


Fig. 3. Solution No.2

***Answer: Yes.***

**Solution 3.**

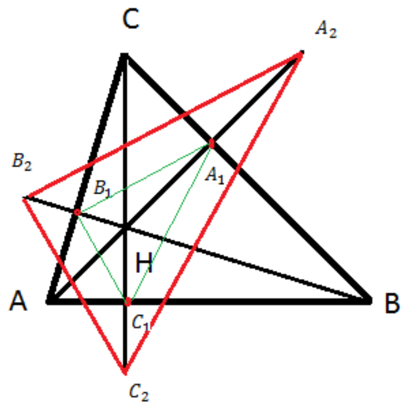


Fig. 4. Solution No.3

*Let* $A_1B_1C_1$ *be an orthic triangle,* $A_2B_2\ C_2$ *be a triangle with vertices symmetric to the orthocenter H relative to the sides. The quadrangle* $AB_1HC_1$ *is inscribed, since it has two opposite right angles. Angles* $C_1B_1A$ *and* $C_1HA$ *are equal (as leaning on the same arc). Angles* $C_1HA$ *and* $CHA_1$ *are equal as vertical. Similarly, in the inscribed quadrilateral* $CA_1HB_1$*, the arc* $CA_1$ *supports the angles* $CHA_1$ *and* $CB_1A_1$*. Hence, the angles* $C_1B_1A$ *and* $CB_1A_1$ *are equal. Consequently, the angles* $CB_1H$ *and* $A_1B_1H$ *are also equal. Each of these angles is equal to the angle B. Hence, the angle* $C_1B_1A_1$ *is equal to* $45^\circ + 45^\circ = 90^\circ$*. The angle* $C_2B_2A_2$ *is equal to it. Consequently, the triangle* $A_2B_2\ C_2$ *is rectangular.*

***Answer: yes.***

***Solution 4.***

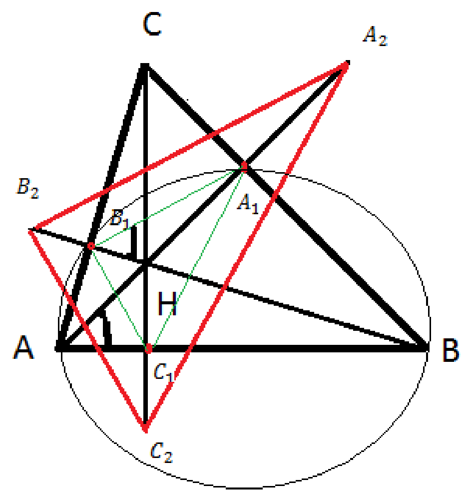


Fig. 5. Solution No.4

*The quadrangle* $AB_1A_1B$ *is inscribed. The circumscribed circle is constructed on AB as on a diameter. Angles* $A_1B_1B$ *and* $A_1AB$ *are equal (lean on the arc* $A_1B$*).*

*Since* $AA_1$ *is the height, then in a rectangular triangle* $AA_1B$ *with angle B of* $45^\circ$ *degrees, the angle* $A_1AB$ *will also be equal to* $45^\circ$*. That is, the angle* $A_1B_1B$ *will be equal to* $A_1B_1B$*. By analogy, the angle* $C_1B_1B$ *will be equal to the angle C. Its value will be equal to* $180^\circ - 45^\circ - 75^\circ = 60^\circ$*.*

*That is, the angle* $A_1B_1C_1$ *will be equal to* $105^\circ$*. The triangle* $A_2B_2\ C_2$ *is homothetic to the triangle* $A_1B_1C_1$*. Consequently, the angle* $A_2B_2\ C_2$ is also equal to $105^\circ$. Therefore, it is obtuse.

**Answer: no.**

**Solution 5.**

*The triangle ABC has the angles equal to* $75^{\circ}, 45^{\circ}$ *and* $60^{\circ}$. *A triangle, which vertices are symmetric to the orthocenter relative to the sides, will be similar to this one. Its angles will be equal to the angles of the triangle ABC. Hence, it will be acute-angled.*

**Answer: no.**

**Solution 6.**

*Let XYZ be an orthic triangle, M, N, K be points symmetric to the orthocenter with respect to the sides. These points must lie on the circumscribed circle of the triangle ABC. The line MC is the height of the triangle ABC. So, it divides the angles YXZ and NMK in half. The angle KBC is equal to the angle KMC (arc KC). Its value is half the angle KMN. Similarly, the value of the angle ABK is equal to half the angle KNM. Hence, the value of the angle ABC is equal to half of the sum of the angles KMN and KNM. The angle ABC is*

$$180^{\circ} - 45^{\circ} - 75^{\circ} = 60^{\circ}.$$

*Hence, the sum of the angles KMN and KNM is equal to* $180^{\circ} - 120^{\circ} = 60^{\circ}$. *The angle MKN is equal to* $120^{\circ}$..

*Similarly, the angle KNM will be equal to*

$$180^{\circ} - 75^{\circ} = 105^{\circ}.$$

*There is an obtuse angle, so the triangle MNK is obtuse.*

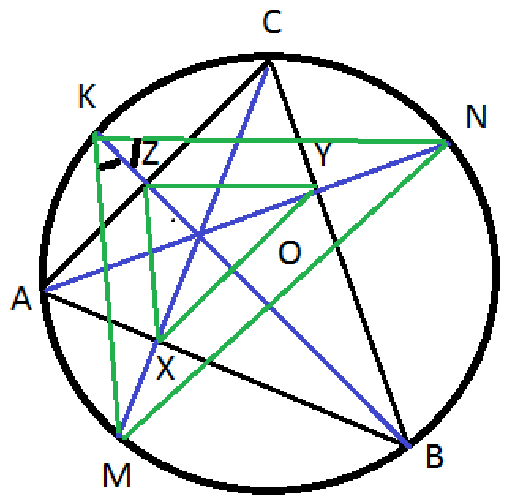


Fig. 6. Solution No.5

***Answer: no.***

The solution of each problem is evaluated according to several particular criteria, such as the presence of logical errors, unreasonable conclusions, technical errors associated with incorrect calculations or inattention. Each solution requires its total assessment. According to this general assessment, the student's rating is determined, based on which the program assigns him/her the solution of the following task. Thus, the student who got the highest score from the "teacher" for the first task "gives out" one of the best solutions to the second task, and vice versa, the student who got the smallest score would be the author of one of the worst solutions for the second time. Similarly, the program works with the following tasks. Solutions of the first task are assigned to students in a pseudo-random way. These features of the algorithm are not reported to students. The work of the "teacher" is evaluated by the coincidence of the overall assessment of each task and by the coincidence of the particular criteria pre-set in the program. The program has an option of limiting the test time by the teacher. At the end of the test, the results of all students and the final score earned by the teacher are displayed on the screen. With the correct work of the teacher, the results of the students in the described algorithm should be stabilized, i.e. “well-advanced" and “slow-advanced" should be identified. In addition, the total score given to the teacher immediately provides information about the quality of checking the student's performance (see Fig. 7).

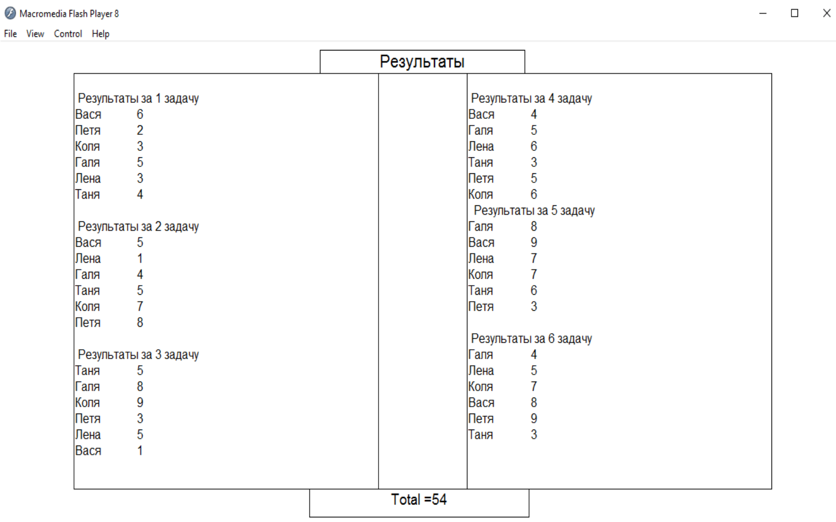

Fig. 7. Total score output screen

**Results.** The tasks in the block selected by us are chosen in such a way that the information from elementary mathematics not always studied in detail at school is solidified simultaneously. For example, the problems of a combination of circles and polygons use the property of the degree of a point with respect to a circle, and some properties of the orthic triangle. The same problem can be solved by different "students" both by analytical and synthetic methods. The drawings to the same task made by different "students" also vary. But in general, this way of testing a student is aimed at identifying and developing, mainly, his/her methodological preparedness in the subject. It assumes the possibility of passing the test simultaneously by a group of students. The proposed approaches were tested in preparation for the pedagogical practice of mathematics students at Naberezhnye Chelny Pedagogical University and Elabuga Institute of KFU. The simulator was also tested by the students of the advanced training courses at Elabuga Institute of KFU. Some results of testing are shown in Table 1.

Table 1. Results of testing students and teachers on the simulator

| Test group | Mean value (%) of correct general estimates of works | Mean value (%) of correctly defined partial criteria |
|---|---|---|
| 2nd year students | 52 | 33 |
| 5th year students (Double Master Degree Program) | 73 | 48 |
| Postgraduate course attenders | 89 | 86 |

**Discussion**. It is worth noting that teachers in general show higher performance on the simulator, but the time limitation significantly worsens their results. while the same time limitation insignificantly affects the results of the students.

We believe that it is necessary to develop a cycle of such simulator programs for secondary school mathematics courses. The scenario described in this paper can be subjected to several modifications, leading to the identification of already psychological characteristics of the future teacher. For example, with the same number of "students" in the class, we increase the bank of solutions. Then, even with relatively successful test results, some teachers can help all "students" become straight-A students, and other teachers can make them C-level students.

**Summary.** The conducted experiment showed the effectiveness and consistency of the application of the simulation method in the process of training future teachers of mathematics. The virtual simulators used during the training allow us to assess not only the students' preparedness for future pedagogical activity, but also to determine the quality of the teacher's work. In general, it should be noted that the use of virtual subject simulators contributes to strengthening the pedagogical potential of students, as well as improving the teaching skills of teachers.

**Conclusions**. As the results of the experiment show, the use of the simulator helps to quickly and effectively diagnose the preparedness of students for the upcoming pedagogical practice. However, in our opinion, in order to improve the psychological and pedagogical competencies, a simulator is also useful, helping the "teacher" to practice their “introduction" in various problematic pedagogical situations. In order to be ready to react adequately to some contingencies, frequent in the educational process, one needs to have a set of options for action, i.e. reactions to the circumstances. Of course, it is impossible to think ahead about each nuance, but the future teacher should have at least some explicative tools in his/her baggage of knowledge. To create such a simulator, a bank of pedagogical situations and variants of their solutions is created, the source of replenishment of which is student pedagogical practice at school. Thus, we see the prospect of expanding the functional of the simulator, as well as the development of its network version, which would allow for more flexible conduct and evaluation of psychological and pedagogical research.

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