Interval neutrosophic sets and topology
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Abstract
Purpose – In 2005, Smarandache generalized the Atanassov’s intuitionistic fuzzy sets (IFSs) to neutrosophic sets (NS), and other researchers introduced the notion of interval neutrosophic set (INSs), which is an instance of NS, and studied various properties. The notion of neutrosophic topology on the non-standard interval is also due to Smarandache. The purpose of this paper is to study relations between INSs and topology.
Design/methodology/approach – The paper investigates the possible relations between INSs and topology.
Findings – Relations on INSs and neutrosophic topology.
Research limitations/implications – Clearly, the paper is confined to IFSs and NSs.
Practical implications – The main applications are in the mathematical field.
Originality/value – The paper shows original results on fuzzy sets and topology.
Keywords Set theory, Topology, Cybernetics, Fuzzy logic
Paper type Research paper

1. Introduction
In various recent papers, Smarandache (2002, 2003, 2005) generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs).

The notion of IFSs defined by Atanassov (1983, 1986) has been applied by Çoker (1997) for study intuitionistic fuzzy topological spaces (IFTS). This concept has been developed by many authors (Bayhan and Çoker, 2003; Çoker, 1996, 1997; Çoker and Eş, 1995; Eş and Çoker, 1996; Gürçay et al., 1997; Hanafy, 2003; Hur et al., 2004; Lee and Lee, 2000; Lupiáñez, 2004a, b, 2006a, b, 2007; Turanh and Çoker, 2000).

Smarandache also defined the notion of neutrosophic topology on the non-standard interval (Smarandache, 2002).

One can expect some relation between the intuitionistic fuzzy topology (IFT) on an IFS and the neutrosophic topology. We show in (Lupiáñez, 2008) that this is false. Indeed, an IFT is not necessarily a neutrosophic topology.

Also (Wang et al., 2005) introduced the notion of interval neutrosophic set (INSs), which is an instance of NS and studied various properties. We study in this paper relations between INSs and topology.

2. Basic definitions
First, we present some basic definitions. For definitions on non-standard analysis (Robinson, 1996):

Definition 1. Let \( X \) be a non-empty set. An IFS \( A \), is an object having the form \( A = \{ x, \mu_A, \gamma_A \mid x \in X \} \) where the functions \( \mu_A : X \to I \) and \( \gamma_A : X \to I \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of non-membership (namely \( \gamma_A(x) \)) of each element \( x \in X \) to the set \( A \), respectively, and \( 0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \) for each \( x \in X \) (Atanassov, 1983).
Definition 2. Let $X$ be a non-empty set, and the IFSs $A = \{ <x, \mu_A, \gamma_A > | x \in X \}, B = \{ <x, \mu_B, \gamma_B > | x \in X \}$. Let:

- $\bar{A} = \{ <x, \gamma_A, \mu_A > | x \in X \}$;
- $A \cap B = \{ <x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B > | x \in X \}$; and
- $A \cup B = \{ <x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B > | x \in X \}$ (Atanassov, 1988).

Definition 3. Let $X$ be a non-empty set. Let $0_\omega = \{ <x, 0, 1 > | x \in X \}$ and $1_\omega = \{ <x, 1, 0 > | x \in X \}$ (Coker, 1997).

Definition 4. An IFT on a non-empty set $X$ is a family $\tau$ of IFSs in $X$ satisfying:

- $0_\omega, 1_\omega \in \tau$;
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$; and
- $\cup G_j \in \tau$ for any family $\{G_j | j \in J\} \subset \tau$.

In this case the pair $(X, \tau)$ is called an IFTS and any IFS in $\tau$ is called a intuitionistic fuzzy open set in $X$ (Coker, 1997).

Definition 5. Let $T, I, F$ be real standard or non-standard subsets of the non-standard unit interval $]-1, 1[$, with:

- $\sup T = t_{\sup}, \inf T = t_{\inf}$;
- $\sup I = i_{\sup}, \inf I = i_{\inf}$; and
- $\sup F = f_{\sup}, \inf F = f_{\inf}$ and $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}, n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}$. $T, I, F$ are called neutrosophic components. Let $U$ be an universe of discourse, and $M$ a set included in $U$. An element $x$ from $U$ is noted with respect to the set $M$ as $x(T, I, F)$ and belongs to $M$ in the following way: it is $t$% true in the set, $i$% indeterminate (unknown if it is) in the set, and $f$% false, where $t$ varies in $T$, $i$ varies in $I$, $f$ varies in $F$. The set $M$ is called a NS (Smarandache, 2005).

Remark. All IFS is a NS.

Definition 6. Let $X$ be a space of points (objects) with generic elements in $X$ denoted by $x$. An INS $A$ in $X$ is characterized by thuth-membership function $T_A$, indeteminacy-membership function $I_A$ and falsity-membership function $F_A$. For each point $x$ in $X$, we have that $T_A(x), I_A(x), F_A(x) \in [0, 1]$ (Wang et al., 2005).

Remark. All INS is clearly a NS.

Definition 7.

- An INS $A$ is empty if $\inf T_A(x) = \sup T_A(x) = 0, \inf I_A(x) = \sup I_A(x) = 1, \inf F_A(x) = \sup F_A(x) = 0$ for all $x \in X$.
- Let $0 =< 0, 1, 1 >$ and $1 =< 1, 0, 0 >$ (Wang et al., 2005).

Definition 8. Let $C_N$ denote a neutrosophic complement of $A$.

Then $C_N$ is a function $C_N : N \rightarrow N$ and $C_N$ must satisfy at least the following three axiomatic requirements:

1. $C_N(0) = 1$ and $C_N(1) = 0$ (boundary conditions);
2. let $A$ and $B$ be two INSs defined on $X$, if $A(x) \leq B(x)$, then $C_N(A(x)) \geq C_N(B(x))$, for all $x$ in $X$ (monotonicity); and
3. let $A$ be an INSs defined on $X$, then $C_N(C_N(A(x))) = A(x)$, for all $x$ in $X$ (involutivity) (Wang et al., 2005).
Proposition 1. Let $I_N$ denote a neutrosophic intersection of two INSs $A$ and $B$. Then $I_N$ is a function $I_N : N \times N \rightarrow N$ and $I_N$ must satisfy at least the following four axiomatic requirements:

1. $I_N(A(x), 1) = A(x)$, for all $x$ in $X$ (boundary condition).
2. $B(x) \leq C(x)$ implies $I_N(A(x), B(x)) \leq I_N(A(x), C(x))$, for all $x$ in $X$ (monotonicity).
3. $I_N(A(x), B(x)) = I_N(B(x), A(x))$, for all $x$ in $X$ (commutativity).
4. $I_N(A(x), I_N(B(x), C(x))) = I_N(I_N(A(x), B(x)), C(x))$, for all $x$ in $X$ (associativity) (Wang et al., 2005).

Definition 9. Let $IN$ denote a neutrosophic intersection of two INSs $A$ and $B$. Then $IN$ is a function $IN : N \times N \rightarrow N$ and $IN$ must satisfy at least the following four axiomatic requirements:

1. $IN(A(x), 0) = A(x)$, for all $x$ in $X$ (boundary condition).
2. $B(x) \leq C(x)$ implies $IN(A(x), B(x)) \leq IN(A(x), C(x))$, for all $x$ in $X$ (monotonicity).
3. $IN(A(x), B(x)) = IN(B(x), A(x))$, for all $x$ in $X$ (commutativity).
4. $IN(A(x), IN(B(x), C(x))) = IN(IN(A(x), B(x)), C(x))$, for all $x$ in $X$ (associativity) (Wang et al., 2005).

Definition 10. Let $UN$ denote a neutrosophic union of two INSs $A$ and $B$. Then $UN$ is a function $UN : N \times N \rightarrow N$ and $UN$ must satisfy at least the following four axiomatic requirements:

1. $UN(A(x), 0) = A(x)$, for all $x$ in $X$ (boundary condition).
2. $B(x) \leq C(x)$ implies $UN(A(x), B(x)) \leq UN(A(x), C(x))$, for all $x$ in $X$ (monotonicity).
3. $UN(A(x), B(x)) = UN(B(x), A(x))$, for all $x$ in $X$ (commutativity).
4. $UN(A(x), UN(B(x), C(x))) = UN(UN(A(x), B(x)), C(x))$, for all $x$ in $X$ (associativity) (Wang et al., 2005).

3. Results

Proposition 1. Let $A$ be an IFS in $X$, and $j(A)$ be the corresponding INS. We have that the complement of $j(A)$ is not necessarily $j(\bar{A})$.

Proof. If $A = \{x, \mu_A, \gamma_A\}$, $j(A) = \{\mu_A, 0, \gamma_A\}$.

Then:

- for $0 = \{x, 0, 1\}$ is $j(0) = j(\{x, 0, 1\}) = \{0, 0, 1\} = \{0, 0, 1\}$; and
- for $1 = \{x, 1, 0\}$ is $j(1) = j(\{x, 1, 0\}) = \{1, 0, 0\} = \{1\}$

Thus, $1 = \bar{0}$ and $j(1) = 1 \neq C_N(j(\bar{0}))$ because $C_N(1) = 0 \neq j(\bar{0})$.

Definition 11. Let us construct a neutrosophic topology on $NT = ]0, 1[^1$, considering the associated family of standard or non-standard subsets included in $NT$, and the empty set which is closed under set union and finite intersection neutrosophic. The interval $NT$ endowed with this topology forms a neutrosophic topological space (Smarandache, 2002).

Proposition 2. Let $(X, \tau)$ be an IFTS. Then, the family of INSs $\{j(U) | U \in \tau\}$ is not necessarily a neutrosophic topology.

Proof. Let $\tau = \{1, 0, A\}$ where $A = (x, 1/2, 1/2)$ then $j(1) = 1, j(0) = (0, 0, 1) \neq \emptyset$ and $j(A) = (1/2, 0, 1/2)$. Thus, $\{j(1), j(0), j(A)\}$ is not a neutrosophic topology, because the empty INS is not in this family.

References


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