The Balan-Killing Manifolds

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Abstract
We define here a differential equation over the metric of a Spin manifold, calling Balan-Killing metrics the solutions of this equation.

1 The Spin manifolds
A Spin manifold $[F]$ admits a spinor bundle by reduction of the tangent bundle with the orthogonal group to the spinor bundle with the group spin. The spinors $\psi$ admit a multiplication by the vectors $X$, called the Clifford multiplication $X.\psi$.

Definition 1 A Killing spinor is defined by the equation:
\[ \nabla X \psi = \mu X.\psi \]
with $\nabla$ the Levi-Civita connection and $\mu \in \mathbb{R}$.

2 The Balan-Killing metrics
We introduce here the following differential equation over the metric. Let $R^\nabla$ be the spinor curvature.

Definition 2 We call a Balan-Killing manifold, a manifold $M$ with riemannian metric $g$ such that $\exists \mu, \forall X,Y$ vectors and $\forall \psi$ spinor:
\[ R^\nabla(X,Y)\psi = \mu r(X.Y - Y.X).\psi \]
with $r$ is the Clifford multiplication and $\nabla$ is the Levi-Civita connection over the spinors, $r$ is the scalar curvature.

For example, a flat space is a Balan-Killing manifold.

3 The Balan-Killing spinors
Definition 3 We call a Balan-Killing spinor, a spinor $\psi$ which verifies the equation:
\[ d(h(Z.\psi, \psi))(X,Y) = \mu h((X.Y - Y.X).\psi, \psi) \]
h is the hermitian metric over the spinor bundle, $d$ is the differential operator over the forms.

It is a differential equation over the spinors.
References

