

# The behavior of primes

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## Abstract

In this paper, we propose the axiomatic regularity of prime numbers.

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## 1 Introduction

In 1859, Riemann [Rie59] showed a deep connection between non-trivial zeros of the Riemann zeta-function and the prime numbers. Our motivation is to axiomatize the structure of primes.

## 2 Results

These below are some patterns of number.

Let  $t_n$  denote the  $n$ th triangular number. Then

$$t_n = \binom{n+1}{2} \quad n \geq 1,$$

where  $\binom{n}{k}$  is the binomial coefficients.

Let  $F_n$  be the  $n$ th Fibonacci number. Then

$$F_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}},$$

where  $n$  is a positive integer.

Let  $B_n$  be the  $n$ th Bernoulli number. Then

$$B_n = (-1)^{n+1} n \zeta(1 - n),$$

where  $\zeta(1 - n)$  is the Riemann zeta-function.

**Postulate 2.1** (Peano Postulates). Given the number 0, the set  $\mathbf{N}$ , and the function  $\sigma$ . Then:

1.  $0 \in \mathbf{N}$ .
2.  $\sigma : \mathbf{N} \rightarrow \mathbf{N}$  is a function from  $\mathbf{N}$  to  $\mathbf{N}$ .
3.  $0 \notin \text{range}(\sigma)$ .
4. The function  $\sigma$  is one-to-one.
5. If  $I \subset \mathbf{N}$  such that  $0 \in I$  and  $\sigma(n) \in I$  whenever  $n \in I$ , then  $I = \mathbf{N}$ .

We define  $1 = \sigma(0)$ ,  $2 = \sigma(1)$ ,  $3 = \sigma(2)$ , etc. Next, we propose the fundamental properties of prime numbers.

**Postulate 2.2.** Given a prime number  $p$ ,  $\chi(n)$  denotes the number of third positive divisor of  $n$ ,  $\sigma(n)$  denotes the sum of positive divisors of  $n$ , and  $\Delta(n)$  denotes the number of positive divisors of  $n$  besides 1 and  $n$ . Then:

1.  $2 \leq p$ .
2.  $4 \nmid p$ .
3.  $(-1)^{\chi(p)} = 1$ .
4.  $3 \leq \sigma(p)$ .
5.  $\Delta(p) = 0$ .

## References

- [Rie59] B. Riemann. Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse. *Monatsber. Akad. Berlin*, pages 671–680, 1859.