The Superluminal Phenomenon of Light for The Kerr-Newman Black Hole

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We use the Kerr-Newman metric based on the general relativity to discuss the superluminal phenomenon of light at the black hole. The black hole have the rotation term a and the charge term R_Q with the Schwarzschild radius R_S. The geodesic of light is $ds^2=0$ and the equation for three velocity components (dr/dt, $rd\theta/dt$, $r\sin\theta d\phi/dt$) is obtained in the spherical coordinate (r, θ, ϕ) with the coordinate time t. Then three cases of the velocity of light (dr/dt, 0, 0), (0, $rd\theta/dt$, 0), and (0, 0, $r\sin\theta d\phi/dt$) are discussed in this research. According to our discussions, only the case of (dr/dt, 0, 0) gives the possibility of the occurrence of the superluminal phenomenon for r between R_S and $(R_Q^2+a^2\sin^2\theta/2)/R_S$ at $\sin\theta>0$ when $R_Q\sim R_S$. The calculations of the velocity of light reveal that the maximum speed of light and the range of the superluminal phenomenon are much related to the rotational term a. Generally speaking, the superluminal phenomena for light can possibly occur in these cases that the radial velocity dr/dt is dominant and the other two velocity components are comparably small. When the relative velocity between the reference frame and the black hole is not heavy, these results of the superluminal phenomenon are suitable for the observations by an observer in a reference frame at infinity or very weak gravitation like on Earth.

Keywords: Superluminal phenomenon, black hole, Kerr-Newman metric, phase diagram of Quantum Chromodynamics

I. Introduction

Traditionally, the speed of light is limited in the general relativity with a maximum of c in free space or the measurement on Earth. The superluminal phenomenon [1] is an observation from a reference frame that the speed of particle exceeds this maximum c. It is also called the Faster-than-light (FTL) phenomenon and some laboratory experiment [2] has been reported and some astronomical observations [1,3-6] about this phenomenon have been revealed from the relativistically massive sources. As we know, the constant speed of light in the flat spacetime structure is a well certified phenomenon described by the special relativity. In this theory, such as an electron in the synchrotron accelerator always needs a lot of energy to make its speed very close to c but not exceeding c. It is the relativistic factor that obeys the mass-energy equivalence and the equivalent mass of the electron depends on its speed. Exceeding the speed of light seems not to be able to observe macroscopically on Earth. Nowadays, it continuously attracts some scientists to investigate this FTL phenomenon. When some report reveals this phenomenon, one always wants to explain it by the present theorem or try to break some concept such as the limitation of the speed of light to fit the phenomenon.

Gravitational time delay is another attracted astronomically phenomenon that the speed of light would slow down when light passes through a giant star [7-11]. This reveals that the observation of the light speed is affected by the gravity and the measured speed of light is not constant for an observer in a reference frame. Because the special relativity is based on the Minkowski metric describing a flat spacetime, it is not suitable to explain all the astronomical phenomena. Gravitational time delay is a well-known fact predicted by general relativity, and the place nearby the supermassive star with strong gravity is good for observation. This phenomenon motivates us to think about a question whether it is possible to observe the speed of light exceed *c* near the supermassive planets such as the black hole. It is the astronomical phenomenon and the astronomical observations have indicated some supermassive planets having possibilities to investigate this kind of superluminal phenomenon for massive particles [1,3-6].

In this research, we study this phenomenon for light based on the general relativity with the Kerr-Newman metric [12-14] and the constant speed of light exists in a rest frame with the proper time. Our discussions focus on the black hole and gives some special results for the possible occurrence of this superluminal phenomenon.

II. The Kerr-Newman metric and the speed of light

When we discuss the geodesic of light at the black hole, an appropriate choice is using the Kerr-Newman metric [12-14] because it considers the angular momentum *J* and charges *Q* of a black hole simultaneously. The rotation of a black hole inherits from the previous star and it may be charged because absorbs charged plasma like from the high-temperature accretion clouds or neighboring star. The expression of the Kerr-Newman metric in the spherical coordinate $(r,\ \theta,\ \phi)$ is

$$\begin{aligned} \mathrm{d}s^2 &= -c^2\mathrm{d}\tau^2 \\ &= \left(\frac{\mathrm{d}r^2}{\Delta}+\mathrm{d}\theta^2\right)\rho^2-(c\mathrm{d}t-a\sin^2\theta\mathrm{d}\phi)^2\frac{\Delta}{\rho^2} \\ &+\left((r^2+a^2)\mathrm{d}\phi-ac\mathrm{d}t\right)^2\frac{\sin^2\theta}{\rho^2}, \end{aligned} \tag{1}$$

where ds is the invariant interval, τ is the proper time, t is the coordinate time, $a=J/Mc$ with mass M of the black hole is the rotational term, and

$$\rho^2=r^2+a^2\cos^2\theta. \tag{2}$$

$$\Delta=r^2-rR_S+a^2+R_Q^2. \tag{3}$$

The Schwarzschild radius is $R_S=2GM/c^2$ and G is the gravitational constant.

R_Q^2 $=KQ^2G/c^4$ is the term related to the charge Q and K is the Coulomb's constant. In addition, the coordinate time is the time read by the clock stationed at infinity because the proper time and coordinate time becomes identical [15]. The geodesic of light is ds^2=0, then through deduction we have the velocity of light obeying the following equation at the black hole

$$\frac{\rho^4}{\Delta(\Delta - a^2\sin^2\theta)}\left(\frac{\mathrm{d}r}{dt}\right)^2 + \frac{\rho^4}{r^2(\Delta - a^2\sin^2\theta)}\left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2$$
$$- \frac{(\Delta a^2\sin^2\theta - (r^2+a^2)^2)}{r^2(\Delta - a^2\sin\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right)^2$$
$$- \frac{2ac(-\Delta + (r^2+a^2))\sin\theta}{r(\Delta - a^2\sin^2\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right) = c^2. \quad (4)$$

In Eq. (4), $\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)$, $\left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)$, and $\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right)$ are the three velocity components of light in the spherical coordinate. This way to obtain the velocity of light from ds^2=0 has been used to get the velocity of light in the Schwarzschild metric [16-19]. It reveals that the velocity of light at the black hole is much different from the Minkowski spacetime and the form in Eq. (4) is much complicated and dependent on the spherical coordinate, the mass, the angular momentum as well as the charge of a black hole. In the following, we discuss the possibility of the superluminal phenomenon for each velocity component individually.

Before discussing, there is a basic requirement that the time is real at any reference frame. When we consider the geodesic along the radial direction without including the dϕ term, then it requires the dt^2 term in Eq. (1) having

$$\rho^2 > 0, \quad (5)$$

$$(\Delta - a^2\sin^2\theta) > 0. \quad (6)$$

From Eq. (6), it can be expanded as

$$r^2 - rR_S + R_Q^2 + a^2\cos^2\theta > 0. \quad (7)$$

For any real r, Eq. (7) further requires the condition existing at $r=R_S/2$

$$R_S^2 \le 4(a^2\cos^2\theta + R_Q^2). \quad (8)$$

It is the condition for the black hole at $r=R_S/2$ but at other place $r>0$ the condition is different. Such as at $r=R_S$, it only requires

$$R_Q^2 + a^2\cos^2\theta > 0, \quad (9)$$

and $r>R_S$ Eq. (7) automatically exists till to the place far away from the black hole. Although the event horizon depends on θ, it is convenient to discuss the phenomenon using R_S as a reference position and the event horizon approximates to a spherical surface while a<<R_S and R_Q<<R_S. Because the conditions of Eq. (8) exists for all θ, then it gives the lowest requirement

$$R_S^2 \leq 4R_Q^2 \tag{10}$$

at r=R_S/2 and $\theta=\pi/2$, and Eq. (9) gives

$$R_Q^2 > 0, \tag{11}$$

at r=R_S. This is just the condition of the Kerr-Newman metric for the charged black hole. The other requirement is for the dr^2 term in Eq. (1) that is

$$\Delta > 0. \tag{12}$$

It also gives the most strict condition at r=R_S/2

$$R_S^2 \leq 4(a^2 + R_Q^2). \tag{13}$$

From Eqs. (10) and (13), the minimum rotated condition can be obtained

$$0 \leq |a|. \tag{14}$$

However, at r=R_S, it is similar to Eq. (10) which only requires

$$R_Q^2 + a^2 > 0. \tag{15}$$

The other factor worth mentioning is ρ^2 when it is at the denominator. It will arise a mathematical singularity at r=0 and $\theta=\pi/2$. If the black hole has finite-size nucleus, this singularity will automatically remove because J=0, Q=0 as well as zero gravity at r=0. According to Eqs. (10) and (14), it means that even the massive star is very heavy, the formation of a black hole exists some basic conditions.

In Quantum Chromodynamics (QCD), the asymptotic freedom [20] in the strong interaction permits baryons such as proton and neutron reducing their sizes and increasing their densities under ultra high pressure producing from the gravitation of the neutron start [21]. Although the gravitation causes all particles gather much dense in the neutron start, the gravitational pressure can do work to transfer energy to the electromagnetic interaction and the strong interaction in the neutron start with a finite-size core. The neutron star core has been denoted in the phase diagram of QCD [22-24] many years. Recently, the concept of the compact star has been proposed [25] and it indicates this star of 1.3~1.6 times the mass of the sun having the radius of 8-11 km. It consists of the core of the high-density quark-phase matter and the surface of nuclear matter. When we calculate the Schwarzschild radius of the compact star, it approximates 3.9~4.7 km, about the half radius of the compact star. It means that this compact star can have an equivalent gravitation as a black hole when the core of the radius is half as the original compact star. For the black hole, even the gravitation is much larger than that in the neutron start, but shrinking all mass and charges to a singularity seems to be unphysical and unreasonable because it needs the gravitation to do infinite work. As we know, the gravitational energy as well as the electrostatic energy are both proportional to $1/r$, all mass and charges collecting at the singularity establish infinite energy there. Therefore, it is much reasonable to replace the singularity with the finite-size nucleus at the center of the black hole. After all, the black hole is evolutional from the previous star of the mass M which only has finitely equivalent energy ~ Mc^2.

Except for this singularity at the black hole, there are other singularities in the Kerr-Newman metric [12]. Some conditions are required to avoid these singularities. Because we deal with a physical world and not pure mathematics, it needs us to describe the black hole more reasonably.

III. The Judgement of The Superluminal Requirements From The Velocity Component d*r*/d*t* of Light

According to Eq. (4), when we want to discuss the speed of light in the radial direction, the maximum speed occurs when the other velocity components are zero. It is the convenient way to discuss the superluminal conditions. The rule used here is also applied to discuss other velocity components. So we focus on the dr/dt velocity component to check whether the superluminal phenomenon of light exists or not first. When an observe rests in a reference frame such as on Earth or the place with very weak gravitation, Eq. (1) gives the time relationship between the proper time and the coordinate time

$$\mathrm{d}\tau^2 = \frac{(\Delta - a^2\sin^2\theta)}{\rho^2}\mathrm{d}t^2. \tag{16}$$

According to the equivalence principle in general relativity, the time dilation requires the coefficient of the dt^2 less than one which gives the condition

$$r > R_Q^2/R_S. \tag{17}$$

The range for this requirement also exists between 0 and R_S, and considering Eq. (11) at r=R_S it requires

$$R_S^2 > R_Q^2 > 0. \tag{18}$$

When r>R_S, the time dilation automatically exists because Eq. (18) gives the maximum of R_Q less than R_S. However, it seems that Eq. (17) is not well-defined for the region R_Q^2/R_S >r≥0. It is the reason that we adopt a singularity at the center of the black hole where all mass and charges gather there. When we use the model of a finite-size nucleus in the black hole, the Coulomb's repulsive force as well as the strong interaction make all particles not shrink to a singularity and the problem can be solved by establishing the charge distribution between 0 and R_S. Then R_Q is a function of r and θ related to the totally enclosed charges at (r,θ), that is,

$$R_Q = R_Q(r,\theta). \tag{19}$$

Eqs. (8) and (9) support this assumption. It also means that a is a function of (r, θ) between 0 and R_S which might be due to the distribution of its mass M. From the viewpoint of the rotational movement, Eq. (19) is reasonable for a rotationally charged black hole. It means that the charge distributions in Eq. (19) have to ensure Eq. (17) between 0 and R_S existing and the time dilation is still correct from $r \geq 0$. According to Eq. (18) for $r=\alpha R_S$ with $0<\alpha<1$, Eq. (7) becomes

$$(R_Q^2 + a^2\cos^2\theta)/(\alpha - \alpha^2) > R_S^2. \tag{20}$$

This inequality exists for all θ. For very small a, combing Eq. (17) with Eq. (20) gives

$$\alpha > R_Q^2/R_S^2 > (\alpha - \alpha^2). \tag{21}$$

From Eq. (21), it reveals the minimum and maximum of the charge distribution varying with the radial distance r form $r=0$ to $r=Rs$ as shown in Fig. 1.

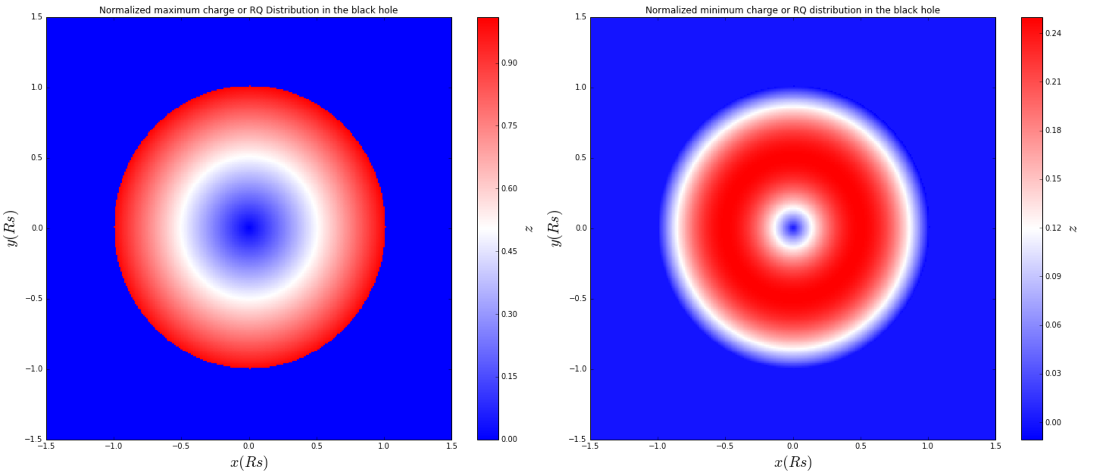

Fig. 1 (a) The minimal distribution of R_Q and (b) the maximal distribution of R_Q varying with the radial distance r for very small a. The color bar is in unit of R_S.

If the superluminal phenomenon occurs, it means $\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right) > c$. Then according to the dr/dt term in Eq. (4), it gives the requirement

$$\frac{\Delta(\Delta - a^2\sin^2\theta)}{\rho^4} > 1. \tag{22}$$

Because $\rho^4 > 0$, it becomes

$$(\Delta^2 - \Delta a^2\sin^2\theta - \rho^4) > 0. \tag{23}$$

Substituting Eqs (2) and (3) into Eq. (23) gives the following relation

$$0 < 2r^2(-rR_S + R_Q^2) + r^2a^2\sin^2\theta + r^2R_S^2 - 2rR_SR_Q^2 + R_Q^4$$
$$+(a^2 + a^2\cos^2\theta)(-rR_S + R_Q^2) + a^4\cos^2\theta\sin^2\theta. \tag{24}$$

Further rearranging Eq. (24), then we have

$$(-rR_S + R_Q^2 + a^2\sin^2\theta/2)(2r^2 - rR_S + R_Q^2 + a^2/2 + 3a^2\cos^2\theta/2)$$
$$> a^4\sin^4\theta/4, \tag{25}$$

or

$$(rR_S - R_Q^2 - a^2\sin^2\theta/2)(2r^2 - rR_S + R_Q^2 + a^2/2 + 3a^2\cos^2\theta/2)$$
$$< -a^4\sin^4\theta/4. \tag{25'}$$

This inequality allows us to discuss the range for occurring superluminal phenomenon. First, the case at θ=0 or π is discussed, then Eq. (23) becomes

$$(-rR_S + R_Q^2)(2r^2 - rR_S + 2a^2 + R_Q^2) > 0. \tag{26}$$

The solutions of Eq. (26) are

$$-rR_S + R_Q^2 > 0 \text{ and} \tag{27a}$$
$$2r^2 - rR_S + 2a^2 + R_Q^2 > 0, \tag{27b}$$

or

$$-rR_S + R_Q^2 < 0 \text{ and} \tag{28a}$$
$$2r^2 - rR_S + 2a^2 + R_Q^2 < 0. \tag{28b}$$

From Eqs. (27a) and (27b), it gives the ranges of *r* that

$$R_Q^2/R_S > r, \tag{29a}$$

$$r < \frac{R_S - [R_S^2 - 8(2a^2 + R_Q^2)]^{1/2}}{4}, \tag{29b}$$

$$r > \frac{R_S + [R_S^2 - 8(2a^2 + R_Q^2)]^{1/2}}{4}, \tag{29c}$$

accompanying with the condition due to the real *r*

$$R_S^2 \geq 8(R_Q^2 + 2a^2). \tag{30}$$

However, Eq. (29a) doesn't satisfy the requirement in Eq. (17), and Eq. (30) obviously violates Eq. (13) at r=R_S/2 so we have to look for the other solution. Then Eqs. (28a) and (28b) give the other ranges

$$R_Q^2/R_S < r, \tag{31a}$$

$$\frac{R_S - [R_S^2 - 8(2a^2 + R_Q^2)]^{1/2}}{4} < r < \frac{R_S + [R_S^2 - 8(2a^2 + R_Q^2)]^{1/2}}{4}, \quad (31b)$$

with the same condition as Eq.(30). Both solutions for *r* cannot give satisfied ranges. To sum up, the discussions from Eqs. (22) to (31) are for the requirements and solutions of v_r^2, not v_r. Then we discuss this phenomenon directly from the expression of the only velocity component (d*r*/d*t*) term obtaining from Eq. (4). This term is

$$v_{r,pole} = \left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{\theta=0,\pi} = \pm c\frac{r^2 - rR_S + a^2 + R_Q^2}{r^2 + a^2}. \quad (32)$$

There are two expressions for (d*r*/d*t*), '+' means light leaving away from the center of the black hole, and '-' means light propagating toward the center of the black hole. So the superluminal solution leaving away the center satisfies the condition $R_Q^2/R_S > r$. However, it still violates the requirement in Eq. (17) and Eq. (18) gives $r<R_S$ in Eq (32). It means that the superluminal phenomenon doesn't happen when light leaves away from the center of the black hole at θ=0 or π in our discussion. The other superluminal solution toward the center has the same *r* condition that the superluminal phenomenon also doesn't happen when light propagates towards the center of the black hole at θ=0 or π in our discussion.

Next, Eq. (24) is discussed for any θ situations. A tricky way to solve Eq. (24) is to define

$$a^4\sin^4\theta/4 = (\alpha a^2)(\beta a^2). \quad (33)$$

Then Eq. (24) can be directly divided into two terms

$$(-rR_S + R_Q^2 + a^2\sin^2\theta/2) \geq \alpha a^2, \quad (34)$$

$$(2r^2 - rR_S + R_Q^2 + a^2/2 + 3a^2\cos^2\theta/2) \geq \beta a^2. \quad (35)$$

Eq. (34) gives the range for the superluminal phenomenon

$$r < [R_Q^2 + a^2(\sin^2\theta/2 - \alpha)]/R_S. \quad (36)$$

When Eq. (36) combines with Eq. (17), the range of *r* for the superluminal phenomenon is given

$$R_Q^2/R_S \leq r < [R_Q^2 + a^2(\sin^2\theta/2 - \alpha)]/R_S. \quad (37)$$

It means that the superluminal phenomenon possibly occurs when this condition in Eq. (37) satisfies. From Eq. (37), it further gives

$$\sin^2\theta/2 - \alpha > 0, \quad (38)$$

or

$$\sin^2\theta/2 > \alpha > 0. \quad (38')$$

From Eq. (38'), the range of α is defined. Then the condition of β is given by

$$\beta > \sin^2\theta/2. \quad (39)$$

From Eq (35), it gives the condition of β between R_Q, a, and R_S for the superluminal phenomenon

$$[8(a^2/2 + 3a^2\cos^2\theta/2 + R_Q^2) - R_S^2]/8 > \beta a^2. \quad (40)$$

In the meanwhile, it also gives the condition of α using Eqs. (33) and (39), that is,

$$2a^4\sin^4\theta/[8(a^2/2 + 3a^2\cos^2\theta/2 + R_Q^2) - R_S^2] < \alpha a^2. \quad (41)$$

From Eqs. (39) and (40), and (38') and (41), they give the ranges for α and β respectively

$$[8(a^2/2 + 3a^2\cos^2\theta/2 + R_Q^2) - R_S^2]/8a^2 > \beta > \sin^2\theta/2, \quad (42)$$

and

$$\sin^2\theta/2 > \alpha > 2a^2\sin^4\theta/[8(a^2/2 + 3a^2\cos^2\theta/2 + R_Q^2) - R_S^2]. \quad (43)$$

Furthermore, comparing the upper limitation with the lower limitation in Eq. (42) gives another condition for the other requirement of R_Q^2 then Eq. (19) at $r=R_S/4$

$$8(2a^2\cos^2\theta + R_Q^2) > R_S^2. \quad (44)$$

This requirement is due to the consideration of the superluminal phenomenon. After discussing above conditions, the upper limitation of r can be obtained. Considering $R_Q \sim R_S$ as Eq. (18), Eq. (37) means that the superluminal phenomena can be observed outer the black hole in the range

$$R_S < r < R_S + a^2\sin^2\theta/2R_S, \quad (45)$$

which is function of θ. An example of the region occurring the superluminal phenomenon for a black hole with $a=2R_S$ and $R_Q=0.999R_S$ is given in Fig. 2(a), where the deep blue region is the spherical region with a radius of R_S and the yellow region means the region for the occurrence of the superluminal phenomena. Here we discuss the region of $r>R_S$ and use R_S as the reference boundary because it might has the case that the event horizon is close to a spherical surface when both $a<<R_S$ and $R_Q<<R_S$. The furthest distance from the center of the black hole in Fig. 2(a) is about $3R_S$ at the equator of $\theta=\pi/2$. All the rotating axes in Figs. 2(a) to 2(d) are parallel to the y-axis. According to Eq. (4) in the case of ($\mathrm{d}r/\mathrm{d}t$, 0, 0), the calculated velocity distribution of light is shown in Fig. 2(b) where the unit of the color bar is c. The velocity distribution matches the region of the superluminal occurrence in Fig. 2(a) and the maximum is about $2.20c$ at $r=R_S$ and $\theta=\pi/2$. When a is increased to $8R_S$ and R_Q is kept at $0.999R_S$, the maximum velocity of light is about $8c$ at $r=R_S$ and $\theta=\pi/2$ as shown in Fig. 2(c). The furthest distance of the superluminal phenomenon is $33R_S$ from the center of the black hole in

Fig. 2(c). For the case of a=20R_S and R_Q=0.999R_S, the maximum speed of light is about 20c at r=R_S and $\theta=\pi/2$ as shown in Fig. 2(d). The furthest distance of the superluminal phenomenon is 201R_S from the center of the black hole in Fig. 2(d). From Figs. 2(c) to 2(d), the occurrences of the high speed of light is centered more and more at the region close to $\theta=\pi/2$. Our discussion is using the Kerr-Newman metric that is a spacetime solution in the general relativity, so considering light bending near the high-speed rotational supermassive black holes, it possibly explains some astronomical observations about the superluminal phenomena from the relativistically massive jet [1,3-6]. This result can be extended to some stars with very high density, large a, and R_Q.

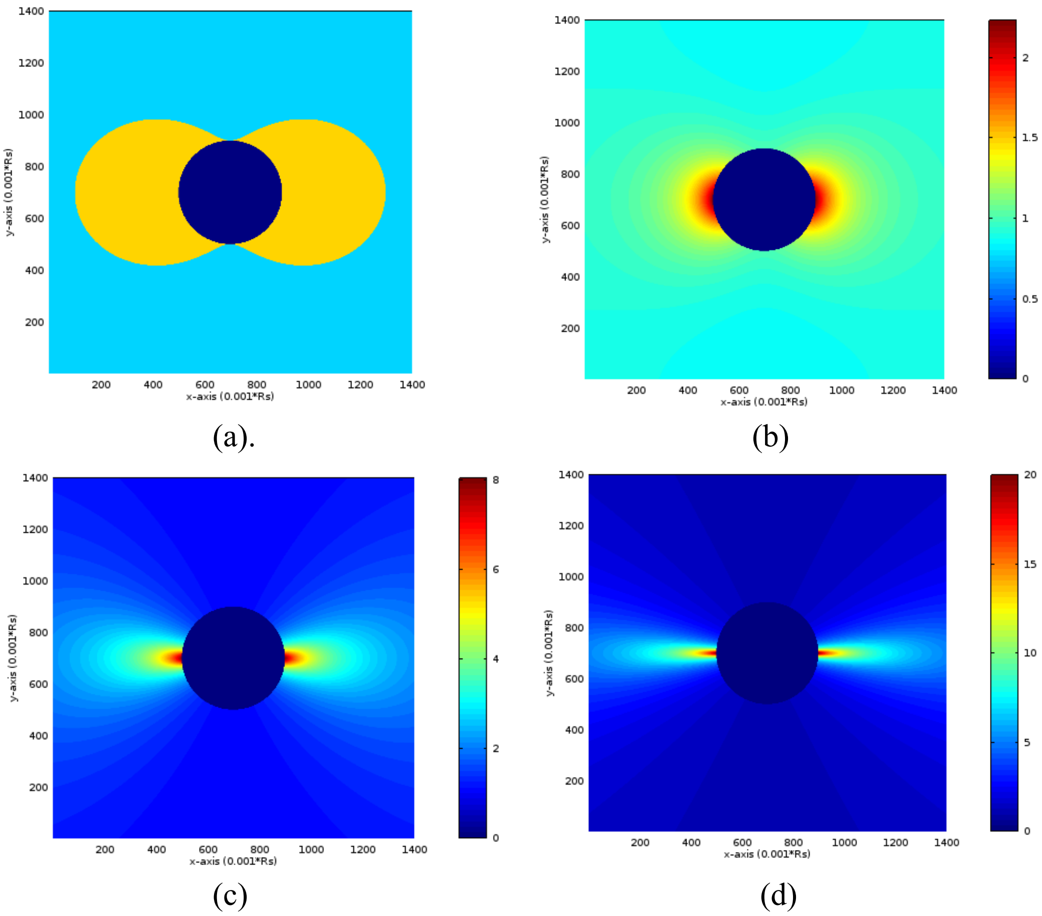

Fig. 2 (a) The superluminal region is denoted by yellow color. The center of the picture is a spherical region with a radius of R_S (deep blue color). In this case, a = 2R_S and R_Q=0.999R_S. The maximum distance for the superluminal phenomenon from the center of the black hole in this case is 3R_S at $\theta=\pi/2$. (b) The speed distribution of light around the black hole with a=2R_S and R_Q = 0.999R_S. (c) The speed distribution of light around the black hole with a=8R_S and R_Q=0.999R_S. The maximum distance of the superluminal phenomenon is 33R_S from the center of the black hole in this case. (d) The speed distribution around the black hole with a=20R_S and R_Q=0.999R_S. The maximum distance of the superluminal phenomenon is 201R_S from the center of the black hole in this case. All these cases the rotational axes are parallel to the y-axis and the color bars show in unit of *c*.

IV. The Judgement of The Superluminal Requirements For The Velocity Component *r*(d$\boldsymbol{\theta}$/d*t*) of Light

The second study case is the velocity component $r\mathrm{d}\theta/\mathrm{d}t$ term in Eq. (4). All the other velocity components are zero. This term is easy to check whether the superluminal phenomenon exists or not. Assuming that it happens, then

$$\frac{r^2(\Delta - a^2\sin^2\theta)}{\rho^4} > 1. \tag{46}$$

Expanding above equation, then we have

$$r^2(-rR_S + R_Q^2) - r^2a^2\cos^2\theta - a^4\cos^4\theta > 0. \tag{47}$$

It can be further rearranged as

$$(-rR_S + R_Q^2 - a^2\cos^2\theta)r^2 > a^4\cos^4\theta. \tag{48}$$

Similar to the discussions of the velocity component $\mathrm{d}r/\mathrm{d}t$, a tricky way is to assume

$$\alpha^2\beta = \cos^4\theta. \tag{49}$$

Then Eq. (48) gives the requirements of r

$$r^2 > \alpha^2a^2, \tag{50}$$

$$-rR_S + R_Q^2 - a^2\cos^2\theta > \beta a^2. \tag{51}$$

Combining Eqs. (51) with (17), and considering the condition of Eq. (8), the range of r for the occurrence of the superluminal phenomenon is given by

$$R_Q^2/R_S < r < (R_Q^2 - a^2\cos^2\theta - \beta a^2)/R_S. \tag{52}$$

Because $\beta \geq 0$, this requirement is not satisfied. Eq. (52) means that in this case of the velocity component $r\mathrm{d}\theta/\mathrm{d}t$ the superluminal phenomenon doesn't occur.

V. The Judgement of The Superluminal Requirements For The Velocity Component *r*sin$\boldsymbol{\theta}$(d$\boldsymbol{\phi}$/d*t*) of Light

The velocity component $r\sin\theta(\mathrm{d}\phi/\mathrm{d}t)$ term is the third case for discussing the possibility of the superluminal phenomenon. All the other velocity components are zero. From Eq. (4), the velocity equation for this case is

$$-\frac{(\Delta a^2\sin^2\theta - (r^2+a^2)^2)}{r^2(\Delta - a^2\sin\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right)^2$$

$$-\frac{2ac\left(-\Delta+(r^2+a^2)\right)\sin\theta}{r(\Delta-a^2\sin^2\theta)}\left(r\sin\theta\frac{d\phi}{dt}\right)=c^2. \tag{53}$$

Next, we replace $r\sin\theta(d\phi/dt)$ with hc, where h is a real value. Then the equation becomes

$$-\frac{(\Delta a^2\sin^2\theta-(r^2+a^2)^2)}{r^2(\Delta-a^2\sin^2\theta)}h^2-\frac{2a\left(-\Delta+(r^2+a^2)\right)\sin\theta}{r(\Delta-a^2\sin^2\theta)}h=1. \tag{54}$$

If the superluminal phenomenon happens, then it means $h>1$. Eq. (54) is the second-order equation in the general form $Ah^2+Bh+C=0$. It requires $0\leq B^2-4AC$ to make sure the real solutions existing. According to this, we have

$$0\leq\frac{\{2a[-\Delta+(r^2+a^2)]\sin\theta\}^2-4[\Delta a^2\sin^2\theta-(r^2+a^2)^2](\Delta-a^2\sin^2\theta)}{r^2(\Delta-a^2\sin^2\theta)^2}. \tag{55}$$

After rearrangement, it gives

$$0\leq\frac{4\left(r^2+a^2-rR_S+R_Q^2\right)(r^2+a^2\cos^2\theta)^2}{r^2(\Delta-a^2\sin^2\theta)^2}, \tag{56}$$

or

$$0\leq\frac{4\Delta\rho^4}{r^2(\Delta-a^2\sin^2\theta)^2}. \tag{56'}$$

Because $\rho^4\geq0$ as well as the denominator $r^2(\Delta-a^2\sin^2\theta)^2\geq0$, it requires $\Delta\geq0$ and $r>0$. The former condition has been shown in Eq. (8). Eq. (56') makes sure that Eq. (54) has real solutions and then we can further discuss whether the superluminal phenomenon exists or not in this case.

In the following, we solve Eq. (54) directly to obtain two solutions of h, that is,

$$h_\pm=\frac{-\frac{2a(-\Delta+(r^2+a^2))\sin\theta}{r(\Delta-a^2\sin^2\theta)}\pm\frac{2(r^2+a^2-rR_S+R_Q^2)^{1/2}(r^2+a^2\cos^2\theta)}{r(\Delta-a^2\sin^2\theta)}}{2\frac{(\Delta a^2\sin^2\theta-(r^2+a^2)^2)}{r^2(\Delta-a^2\sin^2\theta)}}$$

$$=\frac{-ra(-\Delta+(r^2+a^2))\sin\theta\pm r\,(r^2+a^2-rR_S+R_Q^2)^{1/2}(r^2+a^2\cos^2\theta)}{(\Delta a^2\sin^2\theta-(r^2+a^2)^2)}$$

$$=\frac{ra(rR_S-R_Q^2)\sin\theta\pm r\,(r^2+a^2-rR_S+R_Q^2)^{1/2}(r^2+a^2cos^2\theta)}{(r^2+a^2)(r^2+a^2\cos^2\theta)+(rR_S-R_Q^2)a^2sin^2\theta}. \tag{57}$$

It can be further expressed as

$$h_{\pm} = \pm \frac{r\left(r^2 + a^2 - rR_S + R_Q^2\right)^{1/2}}{r^2 + a^2}$$

$$+r\left(rR_S - R_Q^2\right)a\sin\theta \frac{1 \mp \left(r^2 + a^2 - rR_S + R_Q^2\right)^{1/2} a\sin\theta/(r^2 + a^2)}{(r^2 + a^2)(r^2 + a^2\cos^2\theta) + \left(rR_S - R_Q^2\right)a^2\sin^2\theta}. \quad (58)$$

The other two expressions of $h_{\pm}$ are

$$h_{\pm} = \frac{r}{a\sin\theta}$$

$$+\frac{r(r^2 + a^2)(r^2 + a^2\cos^2\theta)}{a\sin\theta} \frac{\pm\left(r^2 + a^2 - rR_S + R_Q^2\right)^{1/2} a\sin\theta/(r^2 + a^2) - 1}{(r^2 + a^2)(r^2 + a^2\cos^2\theta) + \left(rR_S - R_Q^2\right)a^2\sin^2\theta}.$$

(59)

Then the condition is considered whether h can be greater than one or not. Eq. (59) reveals a possible situation for

$$\frac{r}{a\sin\theta} > 1. \quad (60)$$

However, we have discuss it with the second long term in the right-hand side and this velocity component at $\sin\theta=0$ in Eq. (59) is the same as the velocity component $r(\mathrm{d}\theta/\mathrm{d}t)$. It is not easy to deal with so we use the expression in Eq. (58) to judge the occurrence of the superluminal phenomenon. When considering the solution for $h_+ > 1$, the requirement from Eq. (58) is

$$r^2(r^2 + a^2\cos^2\theta)^2\left(r^2 + a^2 - rR_S + R_Q^2\right)$$

$$-[(r^2 + a^2)(r^2 + a^2\cos^2\theta) - (-rR_S + R_Q^2)a\sin\theta(r - a\sin\theta)]^2 > 0. \quad (61)$$

When $\sin\theta\sim 0$, this requirement becomes

$$-a^2(r^2 + a^2)^3 + \left(-rR_S + R_Q^2\right)r^2(r^2 + a^2)^2 > 0. \quad (62)$$

However, both terms in the left-hand side are negative when $r>R_S$, so the superluminal phenomenon doesn't occur at $r>R_S$ when $\sin\theta\sim 0$. Next, we discuss all other cases of $\sin\theta$. Through expanding and rearranging Eq. (61), it gives the requirement

$$\sin\theta > \frac{1}{a(r - a\sin\theta)}\left[\frac{a^2(r^2 + a^2\cos^2\theta)}{2(rR_S - R_Q^2)} + \frac{r^2(r^2 + a^2\cos^2\theta)}{2(r^2 + a^2)}\right.$$

$$\left. + \frac{(rR_S - R_Q^2)a^2\sin^2\theta(r - a\sin\theta)^2}{2(r^2 + a^2)(r^2 + a^2\cos^2\theta)}\right]. \quad (63)$$

The three terms in the right-hand side of Eq. (63) are all positive. According to the geometric inequality in which the first term is equal to the third term here, Eq. (63) can further simplify to the strict condition

$$\sin\theta > \frac{1}{a(r - a\sin\theta)}\left[\frac{a^2(r^2 + a^2\cos^2\theta)}{2(rR_S - R_Q^2)} + \frac{r^2(r^2+a^2\cos^2\theta)}{2(r^2 + a^2)} + \frac{(rR_S - R_Q^2)a^2\sin^2\theta(r - a\sin\theta)^2}{2(r^2 + a^2)(r^2 + a^2\cos^2\theta)}\right]$$

$$\geq \frac{1}{a(r - a\sin\theta)}\left[\frac{a^2\sin\theta(r - a\sin\theta)}{(r^2 + a^2)^{\frac{1}{2}}} + \frac{r^2(r^2+a^2\cos^2\theta)}{2(r^2 + a^2)}\right]. \tag{64}$$

It means the mostly possible place for the superluminal phenomenon in this case at $\sin\theta = 1$. It also requires $r > a\sin\theta$. This requirement needs the three terms in right-hand side to be small enough. When we look at the pre-factor in the right-hand side of Eq. (64), it gives the minimum value when

$$a = \frac{r}{2\sin\theta}. \tag{65}$$

Using $\sin\theta = 1$ and combing the pre-factor, it gives the minimum

$$\frac{a\sin\theta}{(r^2 + a^2)^{1/2}} + \frac{1}{a(r - a\sin\theta)}\frac{r^2(r^2+a^2\cos^2\theta)}{2(r^2 + a^2)} \geq \frac{1}{\sqrt{5}} + \frac{8}{5} > 1. \tag{66}$$

It means that Eq. (66) doesn't satisfy Eq. (63) because $\sin\theta \leq 1$ and the superluminal phenomenon doesn't occur in this discussions of the velocity component $r\sin\theta(\mathrm{d}\phi/\mathrm{d}t)$ when $r > R_S$.

VI. Discussion

Above discussions show that only the case of the velocity of ($\mathrm{d}r/\mathrm{d}t$, 0, 0) for light can possibly occur the superluminal phenomenon at $\theta > 0$. The maximum speed of light is much related to the rotational term a and the charged term R_Q of a black hole. The other two cases of the velocities of (0, $r\mathrm{d}\theta/\mathrm{d}t$, 0) and (0, 0, $r\sin\theta\mathrm{d}\phi/\mathrm{d}t$) for light don't have the possibility of the superluminal phenomenon. However, light can have at least one velocity component when closing or leaving a black hole. Generally speaking, the superluminal phenomenon also possibly occur in these cases of ($\mathrm{d}r/\mathrm{d}t$, $r\mathrm{d}\theta/\mathrm{d}t$, 0), ($\mathrm{d}r/\mathrm{d}t$, 0, $r\sin\theta\mathrm{d}\phi/\mathrm{d}t$), or ($\mathrm{d}r/\mathrm{d}t$, $r\mathrm{d}\theta/\mathrm{d}t$, $r\sin\theta\mathrm{d}\phi/\mathrm{d}t$). In those cases, the radial velocity component is dominant for the occurrences of the superluminal phenomena.

VII. Conclusion

The superluminal phenomenon is an attracted research. This phenomenon can be discussed based on the general relativity with a given spacetime structure. In this research, the Kerr-Newman metric is chosen for describing the spacetime structure at the black hole. The Kerr-Newman metric considers both a and R_Q terms that all kinds of the black hole at present knowledge are included. Because the black hole possesses strong gravity, it is a good astronomical example for studying the superluminal phenomenon. According to the Kerr-Newman metric, the geodesic as well as the velocity components of light can be established. In order to study this phenomenon, three velocity components are independently discussed, and they are (dr/dt, 0, 0), (0, rdθ/dt, 0), and (0, 0, $r$sinθdϕ/dt). From our analysis, only the case of (dr/dt, 0, 0) has the possibility of the occurrence of the superluminal phenomenon between R_S and $[R_Q^2 + (a^2\sin^2\theta)/2]/R_S$ at $\sin\theta > 0$ when $R_Q \sim R_S$. The result reveals that the superluminal phenomenon can possibly happen outer the black hole from the observer in a reference frame. The maximum speed of light and the range of the superluminal phenomenon are much related to the rotational term a and the charged term R_Q of a black hole. Generally speaking, the superluminal phenomena for light can possibly occur in these cases that the radial velocity dr/dt is dominant and the other two velocity components are comparably small or zero. Furthermore, the superluminal phenomenon here just means the results of the measurements from an observer in a reference frame like on Earth. This conclusion can be also applied on some stars with very high density, large a, and big R_Q.

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