

Refutation of translation invariance in superposition calculus

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We assume the method and apparatus of Meth8/VL4 with \top as the designated *proof* value, \mathbf{F} as contradiction, \mathbf{N} as truthity (non-contingency), and \mathbf{C} as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET $p, q, r, s: A, B, X, Y;$
 \sim Not; $+$ Or; $\&$ And; $>$ Imply, greater than, \rightarrow ; $<$ Not Imply, less than;
 $\#$ necessity, \forall , for all or every; $\%$ possibility, \exists , for one or some.

From: en.wikipedia.org/wiki/Knuth-Bendix_completion_algorithm

[I]f a rewriting system is used to calculate minimal representatives then the order $<$ should also have the property ... called translation invariance.

$$A < B \rightarrow XAY < XBY \text{ for all words } A, B, X, Y \quad (1.1)$$

$$((p < q) > (((r \& p) \& s) < ((r \& q) \& s))) \& \#((p \& q) \& (r \& s));$$

FFFF FFFF FFFF FFFN (1.2)

If we distribute the universal quantifier over each word, and remove the quantified consequent in Eq. 1.2, then:

$$(\#p < \#q) > (((\#r \& p) \& \#s) < ((\#r \& q) \& \#s)); \quad \text{TCTT TCTT TCTT TTTT} \quad (1.3)$$

Eqs. 1.2 and 1.3 are *not* tautologous. Eq. 1.2 is nearly contradictory, excepting one non-contingency value \mathbf{N} . Eq. 1.3 is closest to tautology, excepting the three contingency values \mathbf{C} . Hence the property of translation invariance is refuted.