

Lorentz Force in Rindler spacetime

Sangwha-Yi

Department of Math , Taejon University 300-716

ABSTRACT

In the general relativity theory, we define the force in Rindler space-time We find Lorentz force (electromagnetic force) by electro-magnetic field transformations in Rindler space-time. In the inertial frame, Lorentz force is defined as 4-dimensional force. Hence, we had to obtain 4-dimensional force in Rindler space-time.

PACS Number:04,04.90.+e,03.30, 41.20

Key words:General relativity theory,

Lorentz force

Rindler spacetime,

Electro-magnetic field transformation,

e-mail address:sangwha1@nate.com

Tel:010-2496-3953

1. Introduction

In the general relativity theory, our article's aim is that we find Lorentz force by electro-magnetic field transformations in Rindler space-time.

In inertial frame, Lorentz 4-force is

$$F^0 = m_0 \frac{d}{dt} \left(\frac{cdt}{d\tau} \right) = q \frac{\vec{u}}{c} \cdot \vec{E} \quad (1)$$

$$\vec{F} = m_0 \frac{d}{dt} \left(\frac{d\vec{x}}{d\tau} \right) = q \left[\vec{E} + \frac{\vec{u}}{c} \times \vec{B} \right], \quad \vec{u} = \frac{d\vec{x}}{dt} \quad (2)$$

We want to obtain the Lorentz 4-force in Rindler space-time. Hence, we define the force in Rindler space-time.

$$F_{\xi}^0 = m_0 \frac{d}{d\xi^0} \left(\frac{cd\xi^0}{d\tau} \right)$$

$$\vec{F}_{\xi} = m_0 \frac{d}{d\xi^0} \left(\frac{d\vec{\xi}}{d\tau} \right) \quad (3)$$

Or,

$$F_{\xi}^{\mu} = m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^{\mu}}{d\tau} \right) \quad (4)$$

Hence, Lorentz 4-force is in inertial frame

$$F^{\alpha} = m_0 \frac{d}{dt} \left(\frac{dx^{\alpha}}{d\tau} \right) \quad (5)$$

In this time, Minkowski force is in inertial frame or in Rindler space-time[13].

$$f^{\alpha} = m_0 \frac{d}{d\tau} \left(\frac{dx^{\alpha}}{d\tau} \right) = m_0 \frac{d^2 x^{\alpha}}{d\tau^2} \quad (6)$$

$$f_{\xi}^{\mu} = m_0 \frac{d}{d\tau} \left(\frac{d\xi^{\mu}}{d\tau} \right) = m_0 \frac{d^2 \xi^{\mu}}{d\tau^2} \quad (7)$$

2. Electro-magnetic Force in Rindler space-time

Minkowski force is

$$f^{\alpha} = m_0 \frac{d^2 x^{\alpha}}{d\tau^2} = m_0 \frac{d}{d\xi^0} \left(\frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \frac{d\xi^{\mu}}{d\tau} \right) \frac{d\xi^0}{d\tau}$$

$$= m_0 \frac{d}{d\xi^0} \left(\frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \right) \frac{d\xi^{\mu}}{d\tau} \frac{d\xi^0}{d\tau} + m_0 \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \frac{d}{d\xi^0} \left(\frac{d\xi^{\mu}}{d\tau} \right) \frac{d\xi^0}{d\tau}$$

$$= m_0 \frac{d}{d\xi^0} \left(\frac{\partial x^\alpha}{\partial \xi^\mu} \right) \frac{d\xi^\mu}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^\mu \frac{\partial x^\alpha}{\partial \xi^\mu} \frac{d\xi^0}{d\tau}$$

$$F_\xi^\mu = m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^\mu}{d\tau} \right) \quad (8)$$

Hence,

$$\begin{aligned} f^0 &= m_0 \frac{cd^2t}{d\tau^2} = m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^0} \right) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^0 \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} \\ &\quad + m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^1} \right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^1 \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \end{aligned} \quad (9)$$

$$\begin{aligned} f^1 &= m_0 \frac{d^2x}{d\tau^2} = m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^0} \right) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^0 \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} \\ &\quad + m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^1} \right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^1 \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} \end{aligned} \quad (10)$$

$$F_\xi^0 = m_0 \frac{d}{d\xi^0} \left(\frac{cd\xi^0}{d\tau} \right), \quad F_\xi^1 = m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^1}{d\tau} \right)$$

Therefore,

$$\begin{aligned} &F_\xi^0 \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} + F_\xi^1 \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\ &= f^0 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^0} \right) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} - m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^1} \right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} = A \end{aligned} \quad (11)$$

$$\begin{aligned} &F_\xi^0 \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} + F_\xi^1 \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\ &= f^1 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^0} \right) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} - m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^1} \right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} = B \end{aligned} \quad (12)$$

If we represent by the matrix,

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} & \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\ \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} & \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} \end{pmatrix} \begin{pmatrix} F_\xi^0 \\ F_\xi^1 \end{pmatrix} \quad (13)$$

In this time, Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c} \right), \quad x = \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c} \right) - \frac{c^2}{a_0}, \quad y = \xi^2, \quad z = \xi^3$$

(14)

So,

$$\frac{\partial t}{\partial \xi^0} = \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2}, \quad \frac{\partial x}{\partial \xi^0} = \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2}$$

$$\frac{\partial t}{\partial \xi^1} = \sinh\left(\frac{a_0 \xi^0}{c}\right), \quad \frac{\partial x}{\partial \xi^1} = \cosh\left(\frac{a_0 \xi^0}{c}\right) \quad (15)$$

Hence,

$$Det = \left(\frac{\partial t}{\partial \xi^0} \frac{\partial x}{\partial \xi^1} - \frac{\partial t}{\partial \xi^1} \frac{\partial x}{\partial \xi^0}\right) \left(\frac{d\xi^0}{d\tau}\right)^2 = \left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)^2 \quad (16)$$

Therefore,

$$\begin{pmatrix} F_\xi^0 \\ F_\xi^1 \end{pmatrix} = \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)^2} \begin{pmatrix} \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} & -\frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\ -\frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} & \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \quad (17)$$

Hence,

$$F_\xi^0 = \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)^2} \left[A \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} - B \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \right]$$

$$= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \frac{\partial x}{\partial \xi^1} \left[f^0 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^0}\right) c \left(\frac{d\xi^0}{d\tau}\right)^2 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^1}\right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} \right]$$

$$- \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \frac{\partial t}{\partial \xi^1} \left[f^1 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^0}\right) c \left(\frac{d\xi^0}{d\tau}\right)^2 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^1}\right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} \right] \quad (18)$$

As,

$$F_\xi^1 = \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)^2} \left[-A \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} + B \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} \right]$$

$$= -\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \frac{\partial x}{\partial \xi^0} \left[f^0 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^0}\right) c \left(\frac{d\xi^0}{d\tau}\right)^2 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^1}\right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} \right]$$

$$+ \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \frac{\partial t}{\partial \xi^0} \left[f^1 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^0}\right) c \left(\frac{d\xi^0}{d\tau}\right)^2 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^1}\right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} \right] \quad (19)$$

In this time,

$$\begin{aligned}\frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^0} \right) &= \frac{d\xi^1}{d\xi^0} \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \\ \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^0} \right) &= \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \\ \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^1} \right) &= \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c}, \quad \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^1} \right) = \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c}\end{aligned}\quad (20)$$

Therefore, Eq(18) is by Eq(15),Eq(20). Lorentz force F_ξ^0 is in Rindler space-time.

$$\begin{aligned}F_\xi^0 &= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \cosh\left(\frac{a_0 \xi^0}{c}\right) \\ &\times [f^0 - m_0 \left\{ \frac{d\xi^1}{d\xi^0} \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \right\} c \left(\frac{d\xi^0}{d\tau}\right)^2 \\ &- m_0 \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau}] \\ &- \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\ &\times [f^1 - m_0 \left\{ \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \right\} c \left(\frac{d\xi^0}{d\tau}\right)^2 \\ &- m_0 \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau}]\end{aligned}\quad (21)$$

Therefore, Eq(19) is by Eq(15),Eq(20).Lorentz force F_ξ^1 is in Rindler space-time.

$$F_\xi^1$$

$$\begin{aligned}
&= -\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} \\
&\quad \times \left[f^0 - m_0 \left\{ \frac{d\xi^1}{d\xi^0} \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \right\} c \left(\frac{d\xi^0}{d\tau}\right)^2 \right. \\
&\quad \left. - m_0 \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} \right] \\
&\quad - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \\
&\quad \times \left[f^1 - m_0 \left\{ \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \right\} c \left(\frac{d\xi^0}{d\tau}\right)^2 \right. \\
&\quad \left. - m_0 \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} \right] \tag{22}
\end{aligned}$$

In this time, the transformation of electromagnetic field is[12]

$$\begin{aligned}
E_x &= E_{\xi^1}, \\
E_y &= E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right), \\
E_z &= E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_x &= B_{\xi^1}, \\
B_y &= B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_z &= B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \tag{23}
\end{aligned}$$

Hence,

$$\begin{aligned}
f^0 &= F^0 \frac{dt}{d\tau} = q \frac{\vec{u}}{c} \cdot \vec{E} \frac{dt}{d\tau} = q \frac{\vec{u}'}{c} \cdot \vec{E}, \vec{u} = \frac{d\vec{x}}{dt}, \vec{u}' = \frac{d\vec{x}}{d\tau} \\
&= q \frac{1}{c} \left[\frac{dx}{d\tau} E_x + \frac{dy}{d\tau} E_y + \frac{dz}{d\tau} E_z \right], \frac{dx}{d\tau} = \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} + \frac{\partial x}{\partial \xi^1} \frac{d\xi^1}{d\tau}
\end{aligned}$$

$$\begin{aligned}
&= \frac{q}{c} \left[\left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^0}{d\tau} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{d\xi^1}{d\tau} \right\} E_{\xi^1} \right. \\
&\quad + \frac{d\xi^2}{d\tau} \left\{ E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \\
&\quad \left. + \frac{d\xi^3}{d\tau} \left\{ E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right] \quad (24)
\end{aligned}$$

$$f^1 = F^1 \frac{dt}{d\tau},$$

$$\begin{aligned}
F^1 &= q \left[E_x + \frac{1}{c} (u_y B_z - u_z B_y) \right], \quad \vec{u} = \frac{d\vec{x}}{dt}, \quad \vec{u}' = \frac{d\vec{x}'}{d\tau} \\
&= q \left[E_x \frac{dt}{d\tau} + \frac{1}{c} \left(\frac{dy}{d\tau} B_z - \frac{dz}{d\tau} B_y \right) \right], \quad \frac{dt}{d\tau} = \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} + \frac{\partial t}{\partial \xi^1} \frac{d\xi^1}{d\tau} \\
&= q \left[\left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} \frac{d\xi^0}{d\tau} + \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{1}{c} \frac{d\xi^1}{d\tau} \right\} E_{\xi^1} \right. \\
&\quad + \frac{1}{c} \left\{ \frac{d\xi^2}{d\tau} (B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)) \right. \\
&\quad \left. \left. - \frac{d\xi^3}{d\tau} (B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)) \right\} \right] \quad (25)
\end{aligned}$$

In Eq(8), f^2 is

$$f^2 = m_0 \frac{d^2 y}{d\tau^2} = F_{\xi^2} \frac{d\xi^0}{d\tau}, \quad F_{\xi^2} = m_0 \frac{d}{dt} \left(\frac{dy}{d\tau} \right) \frac{dt}{d\xi^0} = F^2 \frac{dt}{d\xi^0} \quad (26)$$

Therefore, Lorentz force F_{ξ^2} is in Rindler space-time.

$$\begin{aligned}
F_{\xi^2} &= m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^2}{d\tau} \right) = F^2 \frac{dt}{d\xi^0} \\
&= q \left[E_y + \frac{1}{c} (u_z B_x - u_x B_z) \right] \frac{dt}{d\xi^0} \\
&= q \left\{ E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \\
&\quad \times \left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \frac{1}{c} \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + \frac{1}{c} \left\{ \frac{d\xi^3}{d\tau} B_{\xi^1} \right. \\
&\quad \left. - \left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{d\xi^1}{d\xi^0} \right\} \right\}
\end{aligned}$$

$$\times \{B_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) + E_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})\}} \quad (27)$$

In Eq(8), f^3 is

$$f^3 = m_0 \frac{d^2 z}{d\tau^2} = F_{\xi^3} \frac{d\xi^0}{d\tau}, \quad F_{\xi^3} = m_0 \frac{d}{dt} \left(\frac{dz}{d\tau} \right) \frac{dt}{d\xi^0} = F^3 \frac{dt}{d\xi^0} \quad (28)$$

Therefore, Lorentz force F_{ξ^3} is in Rindler space-time.

$$\begin{aligned} F_{\xi^3} &= m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^3}{d\tau} \right) = F^3 \frac{dt}{d\xi^0} \\ &= q \left[E_z + \frac{1}{c} (u_x B_y - u_y B_x) \right] \frac{dt}{d\xi^0} \\ &= q \left\{ E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \\ &\quad \times \left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \frac{1}{c} \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \\ &\quad + \frac{1}{c} \left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{d\xi^1}{d\xi^0} \right\} \\ &\quad \times \left\{ B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} - \frac{d\xi^2}{d\xi^0} B_{\xi^1} \} \end{aligned} \quad (29)$$

3. Conclusion

We find the electro-magnetic force in uniformly accelerated frame.

Reference

- [1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)
- [2]W.Rindler, Am.J.Phys.**34**.1174(1966)
- [3]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V
- [4]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)
- [5]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)
- [6]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)
- [7]A.Miller, Albert Einstein's Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)
- [8]W.Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg,1966)
- [9][Massimo Pauri](#), [Michele Vallisner](#), "Marzke-Wheeler coordinates for accelerated observers in special relativity":Arxiv:gr-qc/0006095(2000)
- [10]A. Einstein, " Zur Elektrodynamik bewegter K"orper", Annalen der Physik. 17:891(1905)

- [11]J.W.Maluf and F.F.Faria,"The electromagnetic field in accelerated frames":Arxiv:gr-qc/1110.5367v1(2011)
- [12]S.Yi, "Electromagnetic Field Equation and Lorentz Gauge in Rindler Spacetime", The African Review of Physics,**11**;33(2016)
- [13]D.J. Griffith," INTRODUCTION TO ELECTRODYNAMICS",(2nd ed.,Prentice Hall,Inc.1981)