# A Derivation of Fluidic Maxwell-Proca Equations for Electrodynamics of Superconductors and Implication to Chiral Cosmology model

Victor Christianto*[1], Florentin Smarandache[2], Yunita Umniyati[3]

[1] Malang Institute of Agriculture (IPM), Indonesia. Email: victorchristianto@gmail.com

[2] Dept. Mathematics & Sciences, Univ. New Mexico, Gallup, USA. Email: florentin.smarandache@laposte.net

[3] Head of Physics Lab, Dept. Mechatronics, Swiss-German University, BSD City, Tangerang, Indonesia. Email: yunita.umniyati@sgu.ac.id

## Abstract

In a rather old paper, Mario Liu described a hydrodynamic Maxwell equations. While he also discussed potential implications of these new approaches to superconductors, such a discussion of electrodynamics of superconductors is made only after Tajmar's paper. Therefore, in this paper we present for the first time a derivation of fluidic Maxwell-Proca equations. The name of fluidic Maxwell-Proca is proposed because the equations were based on modifying Maxwell-Proca and Hirsch's theory of electrodynamics of superconductor. It is hoped that this paper may stimulate further investigations and experiments in superconductor. It may be expected to have some impact to cosmology modeling too, for instance we consider a hypothetical argument that photon mass can be origin of gravitation. Then, after combining with the so-called chiral modification of Maxwell equations (after Spröessig), then we consider chiral Maxwell-Proca equations as possible alternative of gravitation theory. Such a hypothesis has never considered in literature to the best of our knowledge.

**Key Words**: Hirsch theory, London equations, hydrodynamics Maxwell equations, Proca equations, electrodynamics of superconductor, chiral medium, chiral gravitation theory.

## 1. Introduction

According to J.E. Hirsch, from the outset of superconductivity research it was assumed that no electrostatic fields could exist inside superconductors and this assumption was incorporated into conventional London electrodynamics.[2] Hirsch suggests that there are difficulties with the two

*Correspondence: Victor Christianto, http://researchgate.net/profile/Victor_Christianto
Email: victorchristianto@gmail.com

London equations. To summarize, London's equations together with Maxwell's equations lead to unphysical predictions.[1] Hirsch also proposes a new model for electrodynamics for superconductors. [1][2]

In this regard, in a rather old paper, Mario Liu described a hydrodynamic Maxwell equations.[3] While he also discussed potential implications of these new approaches to superconductors, such a discussion of electrodynamics of superconductors is made only after Tajmar's paper. Therefore, in this paper we present for the first time a derivation of fluidic Maxwell-Proca-Hirsch equations. The name of Maxwell-Proca-Hirsch is proposed because the equations were based on modifying Maxwell-Proca and Hirsch's theory of electrodynamics of superconductor.

Therefore, the aim of the present paper is to propose a version of fluidic Maxwell-Proca model for electrodynamics of superconductor. It is hoped that this paper may stimulate further investigations and experiments in particular for fractal superconductor. It may be expected to have some impact to cosmology modeling too, which will be discussed in the last section.

## 2. Hirsch's proposed revision of London's equations

According to J.E. Hirsch, from the outset of superconductivity research it was assumed that no electrostatic fields could exist inside superconductors and this assumption was incorporated into conventional London electrodynamics.[2] Hirsch suggests that there are difficulties with the two London equations. Therefore he concludes that London's equations together with Maxwell's equations lead to *unphysical* predictions.[1] However he still uses four- vectors J and A according to Maxwell's equations:

$$\Box^2 A = -\frac{4\pi}{c} J, \tag{1}$$

and

$$J - J_0 = -\frac{c}{4\pi\lambda_L^2}(A - A_0). \tag{2}$$

Therefore, Hirsch proposes a new fundamental equation for electrodynamics for superconductors as follows: [1]

$$\Box^2 (A - A_0) = \frac{1}{\lambda_L^2}(A - A_0), \tag{3a}$$

where

- London penetration depth $\lambda_L$ is defined as follows:[2]

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2}, \tag{3b}$$

- And d'Alembertian operator is defined as: [1]

-

$$\Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial_t^2} \ . \tag{3c}$$

Then he proposes the following equations: [1]

$$\Box^2 (F - F_0) = \frac{1}{\lambda_L^2}(F - F_0), \tag{4}$$

and

$$\Box^2 (J - J_0) = \frac{1}{\lambda_L^2}(J - J_0), \tag{5}$$

where F is the usual electromagnetic field tensor and $F_0$ is the field tensor with entries $\vec{E}_0$ and 0 from $\vec{E}$ and $\vec{B}$ respectively when expressed in the reference frame at rest with respect to the ions.

In the meantime, it is known that Proca equations can also be used to described electrodynamics of superconductors, see [4]-[8]. The difference between Proca and Maxwell equations is that Maxwell equations and Lagrangian are based on the hypothesis that the photon has zero mass, but the Proca's Lagrangian is obtained by adding mass term to Maxwell's Lagrangian.[17] Therefore, the Proca equation can be written as follows:[17]

$$\partial_\mu F^{\mu\nu} + m_\gamma{}^2 A_\nu = \frac{4\pi}{c} J^\nu , \tag{6a}$$

where $m_\gamma = \frac{\omega}{c}$ is the inverse of the Compton wavelength associated with photon mass. [17] In terms of the vector potentials, equation (6a) can be written as [17]:

$$(\Box + m_\gamma) A_\mu = \frac{4\pi}{c} J_\mu . \tag{6b}$$

Similarly, according to Kruglov [15] the Proca equation for a free particle processing the mass m can be written as follows:

$$\partial_\nu \varphi_{\mu\nu}(x) + m^2 \varphi_\mu(x) = 0, \tag{7}$$

Now, the similarity between equations (1) and (6b) are remarkable with exception that equation (1) is in quadratic form. Therefore we propose to consider a modified form of Hirsch's model as follows:

$$(\Box^2 - m_\gamma^2)(F - F_0) = \frac{1}{\lambda_L^2}(F - F_0), \tag{8a}$$

and

$$(\Box^2 - m_\gamma^2)(J - J_0) = \frac{1}{\lambda_L^2}(J - J_0). \quad (8b)$$

The relevance of the proposed new equations in lieu of (4)-(5) should be verified by experiments with superconductors [16]. For convenience, the equations (8a)-(8b) can be given a name: Maxwell-Proca-Hirsch equations.

## 3. Fluidic Maxwell-Proca Equations

In this regard, in a rather old paper, Mario Liu described a hydrodynamic Maxwell equations.[3] While he also discussed potential implications of these new approaches to superconductors, such a discussion of electrodynamics of superconductors is made only after Tajmar's paper. Therefore, in this section we present for the first time a derivation of fluidic Maxwell-Proca-Hirsch equations.

According to Blackledge, Proca equations can be written as follows [7]:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \kappa^2 \phi, \quad (9)$$

$$\nabla \cdot \vec{B} = 0, \quad (10)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (11)$$

$$\nabla \times \vec{B} = \mu_0 j + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \kappa^2 \vec{A}, \quad (12)$$

where:

$$\nabla \phi = -\frac{\partial \vec{A}}{\partial t} - \vec{E}, \quad (13)$$

$$\vec{B} = \nabla \times \vec{A}, \quad (14)$$

$$\kappa = \frac{mc_0}{\hbar}. \quad (15)$$

Therefore, by using the definitions in equations (9)-(12), and by comparing with hydrodynamic Maxwell equations of Liu [3, eq. 2], now we can arrive at fluidic Maxwell-Proca equations, as follows:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \kappa^2 \phi, \quad (16)$$

$$\nabla \cdot \vec{B} = 0, \quad (17)$$

$$\dot{B} = -\nabla \times E - \nabla \times \left( \hat{\beta} \nabla \times H_0 \right), \tag{18}$$

$$\varepsilon_0 \mu_0 \dot{E} = \nabla \times B - \mu_0 j - \kappa^2 A - \left( \hat{\sigma} E_0 + \rho_e v + \hat{\gamma} \nabla T \right) - \nabla \times \left( \hat{a} \nabla \times E_0 \right), \tag{19}$$

where:

$$\nabla \phi = -\frac{\partial \vec{A}}{\partial t} - \vec{E}, \tag{20}$$

$$\vec{B} = \nabla \times \vec{A}, \tag{21}$$

$$\kappa = \frac{m c_0}{\hbar}. \tag{22}$$

Since according to Blackledge, the Proca equations can be viewed as a *unified wavefield* model of electromagnetic phenomena [7], therefore we can also regard the fluidic Maxwell-Proca equations as a *unified wavefield* model for electrodynamics of superconductor.

Now, having defined fluidic Maxwell-Proca equations, we are ready to write down fluidic Maxwell—Proca equations using the same definition, as follows:

$$(\Box_\alpha^2 - \kappa^2)(F - F_0) = \frac{1}{\lambda_L^2}(F - F_0), \tag{23}$$

And

$$(\Box_\alpha^2 - \kappa^2)(J - J_0) = \frac{1}{\lambda_L^2}(J - J_0), \tag{24}$$

where

$$\Box_\alpha^2 = \nabla^{\alpha 2} - \frac{1}{c^2} \frac{\partial^{\alpha 2}}{\partial_t^{\alpha 2}}. \tag{25}$$

As far as we know, the above fluidic Maxwell-Proca equations have never been presented elsewhere before. Provided the above equations can be verified with experiments, they can be used to describe electrodynamics of superconductors.

As a last note, it seems interesting to remark here that Kruglov [15] has derived a square-root of Proca equations as a possible model for hadron mass spectrum, therefore perhaps equations (23)-(25) may be *factorized* too to find out a model for hadron masses. Nonetheless, we leave this problem for future investigations.

## 4. Towards Chiral Cosmology model

The Maxwell-Proca electrodynamics corresponding to a finite photon mass causes a substantial change of the Maxwell stress tensor and, under certain circumstances, may cause the electromagnetic stresses to act effectively as "negative pressure." In a recent paper, Ryutov, Budker, Flambaum suggest that such a negative pressure imitates gravitational pull, and may produce effect similar to gravitation. In the meantime, there are other papers by Longo, Shamir etc. discussing observations indicating handedness of spiral galaxies, which seem to suggest chiral medium at large scale. However, so far there is no derivation of Maxwell-Proca equations in chiral medium.

In a recent paper, Ryutov, Budker, Flambaum suggest that Maxwell-Proca equations may induce a negative pressure imitates gravitational pull, and may produce effect similar to gravitation.[18]

In the meantime, there are other papers by Longo, Shamir etc. discussing observations indicating handedness of spiral galaxies, which seem to suggest chiral medium at large scale. As Shamir reported:

> "*A morphological feature of spiral galaxies that can be easily identified by the human eye is the handedness—some spiral galaxies spin clockwise, while other spiral galaxies rotate counterclockwise. Previous studies suggest large-scale asymmetry between the number of galaxies that rotate clockwise and the number of galaxies that rotate counterclockwise, and a large-scale correlation between the galaxy handedness and other characteristics can indicate an asymmetry at a cosmological scale*."[24]

However, so far there is no derivation of Maxwell-Proca equations in chiral medium. Therefore, inspired by Ryutov et al.'s paper, in this paper, we present for the first time a possibility to extend Maxwell-Proca-type equations to chiral medium, which may be able to explain origin of handedness of spiral galaxies as reported by M. Longo et al.[20-23]

The present paper is intended to be a follow-up paper of our preceding paper, reviewing Shpenkov's interpretation of classical wave equation and its role to explain periodic table of elements and other phenomena [22][11][13].

## 5. Maxwell-Proca Equations in Chiral Medium

Proca equations can be considered as an extension of Maxwell equations, and they have been derived in various ways, see for instance [4, 6, 7]. It can be shown that Proca equations can be derived from first principles [6], and also that Proca equations may have link with Klein-Gordon equation [7]. However, in this paper I will not attempt to re-derive Proca equations. Instead, I will use Proca equations as described in [6].

It shall be noted, that the relations between flux densities and the electric and magnetic fields depend on the material. It is well-known that for instance all organic materials contain carbon and realize in this way some kind of optical activity. Therefore, Lord Kelvin introduced the notion of the chirality measure of a medium. This coefficient expresses the optical activity of the underlying material. The correspondent constitutive laws are the following:[19]

$$D = \varepsilon E + \varepsilon\beta \text{ rot } E \text{ (Drude-Born-Feodorov laws)}, \quad (26)$$

$$B = \mu H + \mu\beta \text{ rot } H \quad (27)$$

where e = E(t, x) is the electric permittivity, j = p(t, x) is the magnetic permeability and the coefficient β describes the *chirality measure* of the material.[2]

Now, since we want to obtain Maxwell-Proca equations in chiral medium, then eq. (12) should be replaced with eq. (26). But such a hypothetical assertion should be investigated in more detailed.

Since according to Blackledge, the Proca equations can be viewed as a unified wavefield model of electromagnetic phenomena [7], then we can also regard the Maxwell-Proca equations in chiral medium as a further generalization of this unified wavefield picture.

## Concluding remarks

One of our aims with the present paper is to propose a combined version of London-Proca-Hirsch model for electrodynamics of superconductor. Considering that Proca equations may be used to explain electrodynamics in superconductor, the proposed fluidic London-Proca equations may be able to describe electromagnetic of superconductors. It is hoped that this paper may stimulate further investigations and experiments in particular for superconductor. It may be expected to have some impact to cosmology modeling too.

Another purpose is to submit a new model of gravitation based on a recent paper by Ryutov, Budker, Flambaum, who suggest that Maxwell-Proca equations may induce a negative pressure imitates gravitational pull, and may produce effect similar to gravitation. In the meantime, there are other papers by Longo, Shamir etc. discussing observations indicating handedness of spiral galaxies, which seem to suggest chiral medium at large scale
.
However, so far there is no derivation of Maxwell-Proca equations in chiral medium. In this paper, we propose Maxwell-Proca-type equations in chiral medium, which may also explain (albeit hypothetically) origin of handedness of spiral galaxies as reported by M. Longo et al.

It may be expected that one can describe handedness of spiral galaxies by chiral Maxwell-Proca equations. This would need more investigations, both theoretically and empirically.

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