Confirmation of Playfair's axiom

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: p, q, single common line, z; ~ Not; & And;
> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
necessity, for all or every; % possibility, for one or some.

From: en.wikipedia.org/wiki/Playfair's_axiom

Proposition 30 of Euclid reads, "Two lines, each parallel to a third line, are parallel to each other." It was noted [..] by Augustus De Morgan that this proposition is logically equivalent to Playfair's axiom. This notice was recounted [..] by T.L. Heath in 1908. De Morgan's argument runs as follows:

Let *X* be the set of pairs of distinct lines which meet (1.1)

(p&q)>(p=q) (1.2)

and *Y* the set of distinct pairs of lines each of which is parallel to a single common line. (2.1)

((p&q)>((p@q)@r))>(p&q) (2.2)

If *z* represents a pair of distinct lines, then the statement, (3.1)

s=(p&q) ; (3.2)

For all *z*, if *z* is in *X* [Eq. 1.1] then *z* is not in *Y* [Eq. 2.1], (4.1)

(#s&(s<((p&q)>(p=q))))>~(s<(((p&q)>((p@q)@r))>(p&q))) ; (4.2)

is Playfair's axiom (in De Morgan's terms, No *X* is *Y*), and its logically equivalent contrapositive

For all *z*, if *z* is in *Y* [Eq. 2.1] then *z* is not in *X* [Eq. 1.1], (5.1)

(#s&(s<(((p&q)>((p@q)@r))>(p&q))))>~(s<((p&q)>(p=q))) ; (5.2)

is Euclid I.30, the transitivity of parallelism (No *Y* is *X*).
[If Eqs. 3.1, then Eqs. 3.1=4.1.] (6.1)

(s=(p&q)) > (((#s&(s<((p&q)>(p=q))))>~(s<(((p&q)>((p@q)@r))>(p&q)))) = ((#s&(s<(((p&q)>((p@q)@r))>(p&q))))>~(s<((p&q)>(p=q))))) ;
`TTTT TTTT TTTT TTTT` (6.2)

Eq. 5.2 as rendered is tautologous, hence confirming Playfair's axiom.