

Electromagnetic wave function and equation in Rindler Space-time”

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ABSTRACT

In the general relativity theory, we find the electro-magnetic wave function and equation in Rindler space-time. Specially, this article is that electromagnetic wave equation is corrected by the gauge fixing equation in Rindler space-time.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic wave equation and function in Rindler space-time. This article correct the article "Electromagnetic Field Equation and Lorentz Gauge in Rindler space-time" about the existence proof of electromagnetic wave function and equation.

The Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \quad (1)$$

Therefore,

$$cdt = c \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1$$

$$dx = c \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1, dy = d\xi^2, dz = d\xi^3 \quad (2)$$

Hence,

$$\frac{1}{c} \frac{\partial}{\partial t} = \frac{\partial \xi^0}{\partial t} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial t} \frac{\partial}{\partial \xi^1}$$

$$= \frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1}$$

$$\frac{\partial}{\partial x} = \frac{\partial \xi^0}{\partial x} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x} \frac{\partial}{\partial \xi^1}$$

$$= -\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi^3} \quad (3)$$

Hence,

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2 \left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right), \quad \vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right) \quad (4)$$

2. Corrected electromagnetic wave equation in the Rindler space-time

The electro-magnetic field (\vec{E}, \vec{B}) is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{c\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A} \quad (5)$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler space-time [1].

$$\vec{E}_\xi = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{A}_\xi}{c \partial \xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi$$

$$\text{In this time, } \vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right), \quad \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3}) \quad (6)$$

Hence, Lorentz gauge condition is in Rindler space-time[1],

$$\phi_\xi \rightarrow \phi_\xi - \frac{1}{c} \frac{\partial \Lambda}{\partial \xi^0} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2}, \quad \vec{A}_\xi \rightarrow \vec{A}_\xi + \vec{\nabla}_\xi \Lambda, \quad \Lambda \text{ is a scalar function.} \quad (7)$$

$$A^\mu{}_{;\mu} = \frac{\partial A^\mu}{\partial \xi^\mu} + \Gamma^\mu{}_{\mu\rho} A^\rho \rightarrow \partial_\mu (A^\mu + g^{\mu\nu} \partial_\nu \Lambda) + \Gamma^0{}_{01} (A^1 + \frac{\partial \Lambda}{\partial \xi^1}) \quad (8)$$

Lorentz gauge fix condition is in Rindler space-time[1],

$$\begin{aligned} 0 &= \frac{1}{c} \frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}}{c^2} \frac{a_0}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \\ &\rightarrow \frac{1}{c} \frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}}{c^2} \frac{a_0}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} - \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_\xi^2 \right] \Lambda + \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \\ &= 0 \quad (9) \end{aligned}$$

Hence, the gauge equation is

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_\xi^2 \right] \Lambda - \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0$$

(10)

We can use Eq(10) as an electromagnetic wave equation because we can apply electromagnetic wave function instead of the gauge function Λ to Eq(10) in Rindler space-time. Hence,

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] E_{\xi^1} - \frac{\partial E_{\xi^1}}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] B_{\xi^1} - \frac{\partial B_{\xi^1}}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \quad (11)$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] E_y - \frac{\partial E_y}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] B_y - \frac{\partial B_y}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \quad (12)$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] E_z - \frac{\partial E_z}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] B_z - \frac{\partial B_z}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \quad (13)$$

The electro-magnetic wave function is

$$E_x = E_{x0} \sin \Phi, E_y = E_{y0} \sin \Phi, E_z = E_{z0} \sin \Phi$$

$$B_x = B_{x0} \sin \Phi, B_y = B_{y0} \sin \Phi, B_z = B_{z0} \sin \Phi \quad (14)$$

$$E_{\xi^1} = E_x = E_{x0} \sin \Phi, B_{\xi^1} = B_x = B_{x0} \sin \Phi$$

$$E_{\xi^2} = E_y \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_z \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$= (E_{y0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) - (B_{z0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_{\xi^2} = B_y \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_z \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$= (B_{y0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) + (E_{z0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$\begin{aligned}
E_{\xi^3} &= E_z \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (E_{z0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) + (B_{y0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_{\xi^3} &= B_z \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (B_{z0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) - (E_{y0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right) \quad (15)
\end{aligned}$$

$$\begin{aligned}
\Phi &= \omega \left(t - l \frac{X}{c} - m \frac{Y}{c} - n \frac{Z}{c} \right) \\
&= \omega \left\{ \left(\frac{c}{a_0} + \frac{\xi^1}{c} \right) \left(\sinh\left(\frac{a_0 \xi^0}{c}\right) - l \cosh\left(\frac{a_0 \xi^0}{c}\right) \right) + \frac{c}{a_0} - m \frac{\xi^2}{c} - n \frac{\xi^3}{c} \right\}, \\
&\quad l^2 + m^2 + n^2 = 1 \quad (16)
\end{aligned}$$

3. Conclusion

We find the electro-magnetic wave equation and function in uniformly accelerated frame.

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