

## Confirmation of the triangle inequality

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We assume the method and apparatus of Meth8/VL4 with  $\top$  as the designated *proof* value,  $\text{F}$  as contradiction,  $\text{N}$  as truthity (non-contingency), and  $\text{C}$  as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET  $p, q, r, s$ : point  $p$ , point  $q$ , point  $r$ ;  $\sim$  Not;  $\&$  And;  $+$  Or;  $-$  Not Or;  
 $>$  Imply, greater than;  $<$  Not Imply, less than;  $=$  Equivalent;  
 $((p-q)=(q-p))$  The absolute value of the distance  $p$  to  $q$  is equivalent to that of  $q$  to  $p$ ;  
 $((p-r)=(r-p))$  The absolute value of the distance  $p$  to  $r$  is equivalent to that of  $r$  to  $p$ ;  
 $((q-r)=(r-q))$  The absolute value of the distance  $q$  to  $r$  is equivalent to that of  $r$  to  $q$ .

From: [en.wikipedia.org/wiki/Triangle\\_inequality](http://en.wikipedia.org/wiki/Triangle_inequality)

"[T]he triangle inequality states that for any triangle, the sum of the lengths of any two sides must be greater than or equal to the length of the remaining side." (1.0)

We rewrite Eq. 1.0 as contradiction  $\text{F}$  implies tautology  $\top$ .

"If  $qr$  is not greater than  $pq$  with  $pr$ , then both  $qr$  is greater than  $pq$  and  $qr$  is greater than  $pr$ ." (1.1)

$\sim(((q-r)=(r-q))>(((p-q)=(q-p))+((p-r)=(r-p)))) >$   
 $((((q-r)=(r-q))>((p-q)=(q-p)))\&(((q-r)=(r-q))>((p-r)=(r-p)))) ;$   
TTTT TTTT TTTT TTTT (1.2)

Eq. 1.2 as rendered is tautologous, hence confirming the triangle inequality.

**Remark:** This exercise indirectly speaks to the fact that the vector space is *not* bivalent.