Heuristic methods for the calculation of mass for particles and their possible interpretation in terms of diagrammatic expansions.

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Abstract

Heuristic methods for the calculation of mass for leptons, baryons and mesons proposed by Barut and other authors in the 1970s to 1990s are discussed, as well as an extension by the present author. Particles are associated with loops of revolving charge, interpreted by the author either as coherent or incoherent loops of waves. Results are consistent with the kinetic energies obtained for the physically analogous superconducting loop case, treated theoretically by Byers and Yang, which scales as n^2(in which n is a Bohr-Sommerfeld quantum number) and displays periodicity as a function of the amount of trapped magnetic flux inside a loop. From Barut´s model we obtain the mass for the tau-lepton, corresponding to n=4, and for n=3 a "proton" with $m \approx$ 945 Mev/c^2 mass. The masses for other baryons can be obtained by considering the coherence breaking effect of trapped flux on the modulation of the mass-energy behavior as a function of n. We discuss also the interpretation of these calculations in field-theoretic terms as presented by other authors in terms of diagrammatic expansions.

Introduction.

In 1961 Deaver and Fairbank [1]carried out an experiment in which they demostrated that magnetic flux can be trapped into a superconducting ring only in quantized amounts. The theoretical analysis of such experiment was carried out by Byers and Yang [2]in a letter published in the same edition of PRL. The arguments used in [2] were based on the imposition of the continuity of the phase of the wave functions of electron pairs around the ring. Such waves remain "rigid" against magnetic-field effects, and phase coherent (as proposed years earlier by Fritz London). Continuity of phase is describable by the Bohr-Sommerfeld (BS) formula which implies action quantization in a closed path. The phase includes the magnetic vector potential action term, to keep gauge covariance in the presence of trapped magnetic fields inside the ring. Applying the BS condition to the quasiparticles momentum one introduces an integer n for the number of turns around the closed path. It is then straightforward to show that the kinetic energy of the particles is proportional to n^2. In addition, the energy should be periodic in the trapped magnetic flux inside the ring. However, such periodicity ends up imposed in a "ad hoc" fashion by changing the value of n in successive steps to keep the kinetic energy to a minimum as the amount of magnetic trapped flux is increased. Such latter condition is a result of the "rigidity" of the wavefunction with respect to the imposition of the magnetic field, so that a magnetic quantum number is not formally introduced---its introduction is replaced by the change in n. A review paper by R. Parks [3] discusses Byers and Yang´s analysis and also the related experiment of Little and Parks[4], which displays measurable evidence for the n^2 dependence of the energies, from the effect of the magnetic flux upon the superconductor transition temperature of the ring material.

An extension of such concept down to the femtometer scale is possible. Current loops have been used as models for the "point" electron intrinsic trajectories in view of the zitterbewegung motion obtainable from Dirac´s equation [5](note that the loop area would be associated with uncertainty in position and not with the size of the electron itself).

Admitting the possibility of an spontaneous intrinsic motion in their model for leptons, Barut and collaborators considered [6,7]the additional introduction into Dirac´s equation of a convective term with the purpose of accounting for self-energy effects, following previous work by Rosen[8]. The results consistently produced a correct prediction of the muon mass.

Would it be possible to extend the method to other particles besides the muon? Barut noted the following[9]. In the absence of a detailed field-theoretical treatment, an heuristic treatment based upon the semi-classical quantization of self-energy effects in the BS theoretical lines would allow the extrapolation of the method to predict the masses of heavier leptons like the Tau and Delta leptons. They would correspond to higher values of the principal quantum number *n* mentioned earlier.

Barut´s initial heuristic model and necessary corrections.

Barut and collaborators considered self-field effects upon the rest energy of leptons in two ways. One of them[6,7], through an altered version of Dirac´s equation. Such equation would include a convective-like term and its solution produces two possible values for mass. One is the mass of the parent lepton (the one which produces the field), and the other a dressed mass affected by the self-interaction. The parent lepton would be the electron and the dressed lepton the muon(μ). The following formula is obtained(where M_e is the electron mass, and α is the fine-structure constant):

$$M_\mu = M_e(1 + 3/ 2\alpha) \qquad (1)$$

Barut considered also a second, considerably simpler way. Aware of the possibility of introducing quantum conditions into periodic motion of particles without solving Schroedingers equation(the "old" quantum theory method), Barut imposed the BS restrictions on action integrals, which should produce integer numbers of the Planck constant. It is well known that BS ignores the wavelike properties (which would amount to consider only coherent wave motion, with no interference)and therefore does not impose full boundary conditions at the turning points of the periodic motion. Details like the ½ extra factor in the harmonic oscillator energy are left aside in the BS “classical” solution. However, if interference of waves is negligible the BS solution for the energies should be correct away from the ground state energy. Barut therefore considered the motion of a classical particle in a circular orbit, subject to the dipolar magnetic force produced by its own magnetic moment ([9]; cf. its ref. 2, which is actually a footnote). In this case the particle producing the moment is an electron , and the moment is the Bohr magneton $\mu_B= e\hbar/2M_e c$ (CGS units).

Newton´s law results in the expression:

$$Mv^2/R= ev\mu_B/ R^3 c \qquad (2)$$

Here M is the dressed particle mass to be calculated. The BS quantization of action around the circular orbit of radius R results in $(2\pi R)Mv =nh$, and thus:

$$R= n\hbar/Mv \qquad (3)$$

which eliminates R from (2). In the following steps of [9]there is a curious mistake. Barut argues that since v^2 is proportional to n^4 such n^4 dependence would remain in the mass expression. However, the M in the denominator of his final formula <u>should actually be</u> M^2. After correction one obtains:

$$(Mv)^2= 4c^4\hbar^2n^4M_e^2/e^4 \qquad (4)$$

Using $\alpha= e^2/\hbar c$, the fine-structure constant, and neglecting differences between v and c in this intrinsic orbital motion(the zitterbewegung-limit), one immediately obtains:

$$M = (2\, n^2/ \alpha)\, M_e \qquad (5)$$

which is proportional to n squared and not n to the fourth power, and is inversely proportional to α. The actual factor should be 3/2 rather than 2 from Dirac´s equation solution. Such mass should be added to the electron mass as (a large) additional term. One would then recover (1) for the case of the muon, n=1.

Latter work by Barut indicates he noticed that the formula in [9] was incorrect, since he never used it again. In (what probably was) his very last paper as a single author Barut [10] applied his semiclassical method to different arrays of particles forming stable configurations, adopting only electric and magnetic dipolar forces. We will comment on this below.

A proton mass, and Barut´s latter "static" heuristic models.

From eq.(5), we consider each higher order lepton as resulting from such self-energy effects acting upon a bare electron, in spite of Barut´s proposal of accumulating the effects of successive members of the sequence of *n,* which may have a field-theoretical interpretation(see comments below). In this way, for *n*=4 one obtains

$M= M_e+(3/2 \times 16) \times 137\ M_e = 3289\ M_e = 1680$ Mev/c^2.

This is about 90 Mev/c^2 smaller than the observed mass for the Tau lepton. If the muon mass is included as the previous member of the series, the agreement becomes almost perfect[9]. For *n*=2 one obtains the exact eta meson mass if a muon is included. The pion mass might also be obtained for *n*=1 by keeping the factor 2 in (5).

A very interesting result is obtained for *n*=3. In this case:

$M= M_e+(3/2 \times 9) \times 137\ M_e = 1851\ M_e = 945$ Mev/c^2.

This is essentially the proton mass, in spite of the incorrect sign.

How to correct this sign? This calculation can be improved by considering three charged particles(justifying *n*=3 in a classical way) instead of just two, and their dressed combination, as shown by Barut[10]. Curiously, Barut[10] amended his heuristic model by extending it to larger groups of particles, and the n^4 factor of [9] is of course absent. By doing that Barut made possible the application of the model to composite particles like the baryons and mesons. The magnetic dipolar interaction can indeed result in a stable 3-particle configuration of charge = +1 unit. The *n*=3 is then associated with the number of internal unit-charged constituent particles, performing a rigid rotation around a fixed origin. The final result is the same obtained above for the proton[10]. It must be pointed out that according to Barut the constituents of a baryon or meson behave as unit-charged objects, as observed in their decay processes. One should consider that even if fractionary charges are present they might combine forming singly-charged revolving objects.

Incidentally, there actually exists a wealth of experimental data demonstrating the general proportionality of mass of particles with the inverse of the alpha constant. Leptons, mesons, baryons, follow such behavior[11].

Further improvements and field-theoretical interpretation.

Barut is not actually the only author to have noticed the possibility of treating leptons and other particles theoretically in the same footing. Harari, Shupe, and Elbaz and collaborators, went a step further showing how the generation of both leptons and quarks might be treated graphically in quantum-field theory[12-15]. There might exist a full correspondence between leptons and quarks. The n^2 dependence has been proposed in a quark mass formula[15], and associated with an expansion of self-interaction bubble-diagrams. The heuristically-proposed number *n* might be interpreted as the number of bubbles in the graphical expansion. The idea of superposing the effects of successive *n* proposed in [9] can be interpreted as the summation of terms of progressively more complex diagrams.

The heuristic approach therefore seems to describe a first-order, essentially “static” approximation to self-energy contributions to the mass-energy expression. The treatment by Elbaz and collaborators goes beyond such approximation, considering dynamical aspects of multiparticle interaction(the graphical expansion). In view of the results of [11], it seems that in problems such as that of mass determination the static approximation is already capable of producing quantitative results for many particles. Topology ends up introduced into the problem through the different kinds of graphs that might be involved in the expansion. From the complexity of Elbaz´s analysis it seems clear however, that much more detailed experimental data would be necessary to discern between the different models that might be proposed to go beyond the simple heuristic level of the static BS-based solution.

How to improve the heuristic approach? In spite of producing the nucleons mass, what we have called here the static model of Barut[10] cannot cope with the variations observed in the masses of Λ, Σ, Ξ, Ω, particles as compared to the nucleons value. In terms of diagrams one might argue that other kinds of diagrams should be added besides the bubbles, but which ones? In heuristic terms the answer seems to come from our comparison with the Byers-Yang treatment of the kinetic energy of currents trapped in a superconductor ring. The energy should be periodic depending on the amount of trapped flux in the ring. In a recent paper the author has found such kind of periodicity by plotting the rest mass of baryons against a number of flux quanta obtained from magnetic moment and mass data for baryons[16]. As proposed in [16], the way of reaching beyond the static conditions in Barut´s treatment seems to be to allow the rigid multi-particle structures to oscillate whilst rotating. This implies the consideration of propagating interfering (incoherent) waves around the current loop, in contrast to the coherent circulation of waves which might be associated with rigid rotation of point like particles.

Conclusion.

This work has presented a detailed analysis of what seems to be the most sophisticated heuristic models for mass calculation in the literature, namely the models proposed by Barut some 25-40 years ago. Such undertaking is worthwhile in view of the extensive amount of data for baryons and mesons which behaves quantitatively as proportional to the inverse of the alpha-constant, as predicted by Barut´s treatment[11]. We have extended a bridge between such models and previous work [2] on the energies trapped in superconducting rings. Recent work by the author has shown evidence not only for the inverse alpha-dependence of mass, but also for the periodicity of mass-energy as a funcion of the trapped magnetic flux inside the region covered by a particles constituents motion, which looks like a fingerprint of self-magnetic field effects on the mass problem. Such trapped flux modulates the mass-energies giving rise to the observed range of mass values (with the nucleons mass as the baseline) observed in the families of baryons. Such conclusion is quantitatively supported[16]. Translated into field-theoretical terms the values of the quantum number *n* have been proposed to represent the number of bubble diagrams, which take account of self-interaction terms. Those early studies propose there should exist a full correspondence between leptons and quarks[12-15]. The full translation of the heuristic ideas into field-theoretical terms seems possible, but theories based upon QCD and proposed in the past 25 years have completely dominated the literature. Leptons and quarks are treated as completely diverse objects, as well as the strong interaction being treated by a completely different formalism as compared with electromagnetism. This makes difficult the interpretation of the results discussed in [11], as well as understanding several other pieces of evidence relating mass to the square root of spin[17], which would require the recognition of the importance of the magnetic self-field effects discussed in the present work.

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