

Shortest refutation of prenex normal form

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We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET $p, q: \phi$ lc_phi, ψ lc_psi; \sim Not; $\&$ And; $>$ Imply; $=$ Equivalent;
 $\#$ necessity, for all or every, $\forall, \forall x$; $\%$ possibility, for one or some, $\exists, \exists x$.

Remark: For clarity, we ignore the x variable below, as it were.

From: en.wikipedia.org/wiki/Prenex_normal_form

"The [implication] rules for removing quantifiers from the antecedent are:

$$(\forall x\phi)\rightarrow\psi \text{ is equivalent to } \exists x(\phi\rightarrow\psi), \quad [1.1.1.1]$$

$$(\#p>q)=\%(p>q); \quad \text{TTTT TTTT TTTT TTTT} \quad (1.1.1.2)$$

$$(\exists x\phi)\rightarrow\psi \text{ is equivalent to } \forall x(\phi\rightarrow\psi). \quad [1.1.2.1]$$

$$(\%p>q)=\#(p>q); \quad \text{TTNN TTNN TTNN TTNN} \quad (1.1.2.2)$$

The [implication] rules for removing quantifiers from the consequent are:

$$\phi\rightarrow(\exists x\psi) \text{ is equivalent to } \exists x(\phi\rightarrow\psi), \quad [1.2.1.1]$$

$$(p>\%q)=\%(p>q); \quad \text{TTTT TTTT TTTT TTTT} \quad (1.2.1.2)$$

$$\phi\rightarrow(\forall x\psi) \text{ is equivalent to } \forall x(\phi\rightarrow\psi). \quad [1.2.2.1]$$

$$(p>\#q)=\#(p>q); \quad \text{NTNT NTNT NTNT NTNT} \quad (1.2.2.2)$$

Eqs. 1.1.2.2 and 1.2.2.2 as rendered are *not* tautologous. Hence rules for the implication operator refute the prenex normal form.