

# GAME OF INTEGRALS

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Abstract

This note presents some trigonometric integrals

Key words and phrases

Trigonometric integrals, number pi, radicals

## SOME TRIGONOMETRIC INTEGRALS

$$\int_0^{\pi} x \sqrt{1 + \sqrt{1 + \cos^4 x}} \sin x \, dx =$$
$$= \frac{\pi}{4} \left\{ 2\sqrt{1 + \sqrt{2}} + 2 \tan^{-1} \left( \sqrt{-1 + \sqrt{2}} \right) + \ln \left( 1 + \sqrt{2} + \sqrt{2(1 + \sqrt{2})} \right) \right\} \quad (1)$$

$$\int_0^{\pi} x \sqrt{2 + \cos^2 x + 2\sqrt{1 + \cos^2 x + \cos^4 x}} \sin x \, dx =$$
$$= \frac{\pi}{4} \left\{ 2\sqrt{3 + 2\sqrt{3}} + 2 \tan^{-1} \left( \sqrt{-1 + \frac{2}{\sqrt{3}}} \right) + \sqrt{3} \ln \left( 1 + \sqrt{3} + \sqrt{3 + 2\sqrt{3}} \right) \right\} \quad (2)$$

$$\int_0^{\pi} x \sqrt{2 - \cos^2 x + 2\sqrt{1 - \cos^2 x + \cos^4 x}} \sin x \, dx = \frac{\pi}{8} \left\{ 4\sqrt{3} + \pi\sqrt{3} + 2 \ln(2 + \sqrt{3}) \right\} \quad (3)$$

$$\int_0^{\pi} x \sqrt{25 + 7 \cos^2 x + 5\sqrt{25 + 14 \cos^2 x + 25 \cos^4 x}} \sin x \, dx =$$
$$= \frac{\pi}{\sqrt{2}} \left\{ 6 + 3 \tan^{-1} \frac{1}{2} + 2 \ln 5 \right\} \quad (4)$$

$$\int_0^{\pi} x \sqrt{2 + \sqrt{2} \cos^2 x + 2\sqrt{1 + \sqrt{2} \cos^2 x + \cos^4 x}} \sin x \, dx =$$
$$= \frac{\pi}{4} \left\{ 2\sqrt{2 + \sqrt{2}} + 2\sqrt{2 + \sqrt{2}} + 2\sqrt{2 - \sqrt{2}} \tan^{-1} \left( \sqrt{-1 + \sqrt{4 - 2\sqrt{2}}} \right) + \right.$$
$$\left. + \sqrt{2 + \sqrt{2}} \ln \left( 1 + \sqrt{2 + \sqrt{2}} + \sqrt{2 + \sqrt{2} + 2\sqrt{2 + \sqrt{2}}} \right) \right\} \quad (5)$$

$$\int_0^{\pi} x\sqrt{-1+\sqrt{1+\cos^4 x}} \sin x dx =$$

$$= \frac{\pi}{4} \left\{ 2\sqrt{-1+\sqrt{2}} + 2 \tan^{-1} \left( \sqrt{-1+\sqrt{2}} \right) - \ln \left( 1 + \sqrt{2} + \sqrt{2+2\sqrt{2}} \right) \right\} \quad (6)$$

$$\int_0^{\pi} x\sqrt{-2-\cos^2 x+2\sqrt{1+\cos^2 x+\cos^4 x}} \sin x dx =$$

$$= \frac{\pi}{4} \left\{ 2\sqrt{-3+2\sqrt{3}} + 2\sqrt{3} \tan^{-1} \left( \sqrt{-1+\frac{2}{\sqrt{3}}} \right) + \ln \left( 1 + \sqrt{3} - \sqrt{3+2\sqrt{3}} \right) \right\} \quad (7)$$

$$\int_0^{\pi} x\sqrt{-2+\cos^2 x+2\sqrt{1-\cos^2 x+\cos^4 x}} \sin x dx = \frac{\pi}{8} \left\{ 4 + \pi - 2\sqrt{3} \ln \left( 2 + \sqrt{3} \right) \right\} \quad (8)$$

$$\int_0^{\pi} x\sqrt{-25-7\cos^2 x+5\sqrt{25+14\cos^2 x+25\cos^4 x}} \sin x dx =$$

$$= \frac{\pi}{2\sqrt{2}} \left\{ 4 + 8 \tan^{-1} \frac{1}{2} - 3 \ln 5 \right\} \quad (9)$$

$$\int_0^{\pi} x\sqrt{-2-\sqrt{2}\cos^2 x+2\sqrt{1+\sqrt{2}\cos^2 x+\cos^4 x}} \sin x dx =$$

$$= \frac{\pi}{4} \left\{ 2\sqrt{-2-\sqrt{2}} + 2\sqrt{2+\sqrt{2}} + 2\sqrt{2+\sqrt{2}} \tan^{-1} \left( \sqrt{-1+\sqrt{4-2\sqrt{2}}} \right) \right. \quad (10)$$

$$\left. - \sqrt{2-\sqrt{2}} \ln \left( 1 + \sqrt{2+\sqrt{2}} + \sqrt{2+\sqrt{2}+2\sqrt{2+\sqrt{2}}} \right) \right\}$$

## References

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