

Refutation of Levi-identity and AGM postulates

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We assume the method and apparatus of Meth8/VL4 with \top as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

From: Badura, C. (2016). Truth in fiction via non-standard belief revision.
ilic.uva.nl/Research/Publications/Reports/MoL-2016-07.text.pdf

LET p, q, r : $\text{lc_phi } \varphi, \text{lc_psi } \psi, \text{B}$;
 \sim Not; $+$ Or, expansion; $-$ Not Or, contraction; $\& *$, revision operator;
 $=$ Equivalent; $@$ Not equivalent ;
 $>$ Imply, greater than, not.lt.eq \notin ; $\sim(>)$ Not Imply, lt.eq \subseteq ;
 $(p=p) \top$ designated *proof* value; $(p@p) \text{F}$ as contradiction;
 AGM (Alchourròn, Gärdenfors, Makinson) .

Remark: Equations from the text are not reproduced here due to non-portable pdf characters.

$$(r\&p)=((r\sim)p)+p) ; \quad \text{TFTF TTTT TFTF TTTT} \quad (\text{Levi-identity})$$

The AGM postulates for the revision operator(s) are from page 30:

$$\begin{aligned} (r\&p)=(p=p) ; & \quad \text{FFFF FTFT FFFF FTFT} & (1) \\ \sim(p>(r\&p))=(p=p) ; & \quad \text{FTFT FFFF FTFT FFFF} & (2) \\ (r\&p)>(r+p) ; & \quad \text{TTTT TTTT TTTT TTTT} & (3) \\ (\sim p>r)>\sim((r+p)<(r\&p)) ; & \quad \text{TFTF FTFT TFTF FTFT} & (4) \\ (p@p)>(\sim(r\&p)=(p=p)) ; & \quad \text{TTTT TTTT TTTT TTTT} & (5) \\ (p=q)>((r\&p)=(r\&q)) ; & \quad \text{TTTT TTTT TTTT TTTT} & (6) \\ \sim((r\&(p\&q))>((r\&p)+q))=(p=p) ; & \quad \text{FFFF FFFF FFFF FFFF} & (7) \\ [\text{maybe should read } (r\&(p\&q))>((r\&p)+q) ; & \quad \text{TTTT TTTT TTTT TTTT}] \\ (\sim q>(r\&p))>\sim(((r\&p)+q)>(r\&(p\&q))) ; & \quad \text{TTTT TTTF TTTT TTTF} & (8) \end{aligned}$$

Eq. (Levi-identity) as rendered is *not* tautologous, as a basis for the subsequent AGM expressions.

Eqs. 3, 5, 6, and (arguably) 7 are tautologous. However, Eqs. 1, 2, 4, and 8 are not. This refutes the AGM postulates as a basis for fictional logic.