Refutation of connexive logic based on Wansing's nightmare

We assume the method and apparatus of Meth8/VŁ4 with \( \top \) as autology as the designated proof value, \( \bot \) as contradiction, \( \mathbb{N} \) as truthty (non-contingency), and \( \mathbb{C} \) as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

\[
\begin{align*}
\text{Let} & \quad p, q, r, s: A, B, C, D; \quad \sim \text{Not}, \sim; \quad + \text{Or}; \quad \& \text{And}; \quad > \text{Imply}, \text{greater than}, \to; \quad = \text{Equivalent}, \leftrightarrow.
\end{align*}
\]

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The set of all valid formulas is axiomatized by the following set of axiom schemata and rules:

\[
\begin{align*}
a1 &\quad \text{the axioms of classical positive logical} \\
a2 &\quad \sim \sim A \leftrightarrow A \\
a3 &\quad \sim (A \lor B) \leftrightarrow (\sim A \land \sim B) \\
a4 &\quad \sim (A \land B) \leftrightarrow (\sim A \lor \sim B) \\
a5 &\quad \sim (A \to B) \leftrightarrow (A \to \sim B) \\
R1 &\quad \text{modus ponens}
\end{align*}
\]

\( \text{MC} \) can be faithfully embedded into positive classical logic, whence \( \text{MC} \) is decidable. The classical tautology \( \sim (A \to B) \to (A \land \sim B) \) is, of course, not a theorem of \( \text{MC} \). Like \( \mathbb{C} \), \( \text{MC} \) is a paraconsistent logic containing contradictions.


In Omori’s research note on Francez’ paper ... a deductive calculus including the analogous axiom \( \sim (\phi \to \psi) \leftrightarrow (\sim \phi \to \psi) \) is introduced by means of an axiomatic proof theory and a corresponding possible worlds semantics.


**Remark 4.** ... (Ax12) \( [\sim (A \to B) \leftrightarrow (\sim A \to B)] \) is replaced by \( \sim (A \to B) \leftrightarrow (A \to \sim B) \).

Connexive logic turns on \( \sim (A \to B) \leftrightarrow (A \to \sim B) \) \quad (a5.1.1)

\[
\neg (A \to B) = (A \to \neg B) \quad \text{FCNT FCNT FCNT FCNT} \quad (a5.1.2)
\]

or the falsity-weakend \( \sim (A \to B) \leftrightarrow (\sim A \to B) \) \quad (a5.2.1)

\[
\neg (A \to B) = (\neg A \to B) \quad \text{TTTT NNNN CCCC FFFF} \quad (a5.2.2)
\]

Eqs. a5.1.2 and a5.2.2 as rendered are *not* tautologous. Hence connexive logic is refuted.