Refutation of connexive logic based on Wansing's nightmare

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: A, B, C, D; ~ Not, ¬; + Or; & And; > Imply, greater than, → ; = Equivalent, ↔.

From: Connexive logic. Copyright © 2014 by Heinrich Wansing <Heinrich.Wansing@rub.de>. plato.stanford.edu/entries/logic-connexive/

> The set of all valid formulas is axiomatized by the following set of axiom schemata and rules:

$a1_c$	the axioms of classical positive logical
a2	$\sim\sim A \leftrightarrow A$
a3	$\sim(A \vee B) \leftrightarrow (\sim A \wedge \sim B)$
a4	$\sim(A \wedge B) \leftrightarrow (\sim A \vee \sim B)$
a5	$\sim(A \rightarrow B) \leftrightarrow (A \rightarrow \sim B)$
R1	modus ponens

> **MC** can be faithfully embedded into positive classical logic, whence **MC** is decidable. The classical tautology $\sim(A \rightarrow B) \rightarrow (A \wedge \sim B)$ is, of course, not a theorem of **MC**. Like **C**, **MC** is a paraconsistent logic containing contradictions.

From: Ferguson, T.M.; Omori, H.; Wansing, H. (2016). The tenacity of connexive logic: Preface to the special issue. *FCoLog Journal of Logics and their Applications*. 3:3.293.

> In Omori's research note on Francez' paper ... a deductive calculus including the analogous axiom $\neg(\phi \rightarrow \psi) \leftrightarrow (\neg\phi \rightarrow \psi)$ is introduced by means of an axiomatic proof theory and a corresponding possible worlds semantics.

From: Omori, H. (2016). A note on Francez' half-connexive formula. *IFCoLog Journal of Logics and their Applications*. 3:3.507.

> **Remark 4.** ... (Ax12) [$\sim(A \rightarrow B) \leftrightarrow (\sim A \rightarrow B)$] is replaced by '$\sim(A \rightarrow B) \leftrightarrow (A \rightarrow \sim B)$'.

Connexive logic turns on $\sim(A \rightarrow B) \leftrightarrow (A \rightarrow \sim B)$ (a5.1.1)

~(A>B)=(A>~B) ; `FCNT FCNT FCNT FCNT` (a5.1.2)

or the falsity-weakend $\sim(A \rightarrow B) \leftrightarrow (\sim A \rightarrow B)$ (a5.2.1)

~(A>B)=(~A>B) ; `TTTT NNNN CCCC FFFF` (a5.2.2)

Eqs. a5.1.2 and a5.2.2 as rendered are *not* tautologous. Hence connexive logic is refuted.