

The generalized Seiberg-Witten equations

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Abstract

We show a set of equations which generalizes the Seiberg-Witten equations

1 The Seiberg-Witten equations

The Seiberg-Witten equations are the following ones [F] [M]:

$$\begin{aligned}\mathcal{D}_A(\psi) &= 0 \\ F_+(A) &= -\frac{1}{4}\omega(\psi)\end{aligned}$$

2 The generalization of the SW equations

We consider two spinors ψ, ϕ and we define [F] the coupled Seiberg-Witten equations (A, A', f, ϕ, ψ) :

$$\begin{aligned}\mathcal{D}_A(f\psi) &= \mathcal{D}_{A'}((1/f)\phi) = 0 \\ F_+(A) &= -\frac{1}{4}\omega(\psi) \\ F_+(A') &= -\frac{1}{4}\omega(\phi) \\ f^*A &= (1/f)^*A'\end{aligned}$$

A, A' are connections $f : M \rightarrow S^1$. If $f = 1$, then we have the Seiberg-Witten equations.

3 The invariants of Seiberg-Witten generalized

We have to prove compactness of the moduli spaces and to define the invariants of Seiberg-Witten over them.

References

- [B] N.Berline, E.Getzler, M.vergne, "Heat kernels and Dirac operators", Springer-Verlag, 1992.
- [F] T.Friedrich, "Dirac operators in Riemannian Geometry", Graduate Studies in Mathematics vol 25, AMS, 2000.
- [K] M.Karoubi, "Algèbres de Clifford et K-théorie", Ann.Scient.Ec.Norm.Sup. 4 ser. 1 (1968), 161-270.
- [M] J.Morgan, "The Seiberg-Witten equations and applications to the topology of smooth four-manifolds", Mathematical Notes, Princeton University Press, 1996.
- [W] E.Witten, "Monopoles and four-manifolds", Math.Res.Lett. 1(1994), 769-796.