Proof of the Fourth Landau’s Problem

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Contents
1 Definition of the Fourth Landau’s Problem 1
2 Algorithm for Proof of the Fourth Landau’s Problem 1

1 Definition of the Fourth Landau’s Problem

Definition: Is the set of primes of the form \((n^2 + 1)\) infinite?

2 Algorithm for Proof of the Fourth Landau’s Problem

The proof of the Fourth Landau’s Problem is a consequence of the Proof of the Legendre’s Conjecture.

Let the odd number be \(y\).

Half of the numbers \((n^2 + 1)\), following \(n^2 = y^2\) are even and correspondingly composite numbers.

The other half \((n^2 + 1)\) follows an even number of \(n^2 = (y + 1)^2\).

Then:

\[
n^2 + 1 = (y + 1)^2 + 1 = y^2 + 2y + 2 = y(y + 2) + 2.
\]

Let’s represent (1) with respect to the odd number \(y_k\) following the given \(y\):

\[
n^2 + 1 = y_k(y_k - 2) + 2.
\]

But the number represented by the expression (2) is the first in one of the two sets of Proof of the Legendre’s Conjecture:

\[
\{y_n \mid y_k(y_k - 2) < y_n < y_k^2, \ y_k \geq 3\}.
\]
According to the **Proof of the Legendre’s Conjecture**, the frequency of the appearance of composite numbers in the entire given segment:

\[
Z_{y_{\text{comp}}} \big( \{ y_{\text{comp}} \} \big) = 33.3\ldots3\% \big( \{ 3y \mid y \geq 3, \ 3y = y_n \} \big) + \\
+ \sum_{m=3} Z_{y_{om}} \bigg( \{ y_{om}y_m \mid y_m \geq y_{om}, \ y_{om}y_m = y_n, \ y_{om} < N_{y_n}, \ \frac{y_m}{3} \notin \mathbb{N}, \ldots, \frac{y_m}{y_{om}(m-1)} \notin \mathbb{N} \} \bigg) < 100\% ,
\]

where: the number of digits represented by (…) in the first term, \( \to \infty \);

\( m \) is the number of a member of a sequence of odd primes;

\( y_{\text{comp}} \) is a composite odd number in a given segment of a sequence of odd numbers \( \{ y \} \);

\( Z_{y_{\text{comp}}} \) is the frequency of the appearance of composite numbers (in %) in a given segment of the sequence \( \{ y \} \);

\( N_{y_n} \) is the number of terms in the set \( (3) \);

\( N_{y_n} = y_k - 1 \);

\( y_m \) is a sequence of odd numbers with the conditions given in the formula.

Therefore, although with increasing \( y_k \) the probability of the appearance of a composite number in the first term of the set \( (3) \) increases, it never reaches 100%.

That is, the set of primes of the form \( (n^2 + 1) \) is infinite.

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