Refutation of the definition of mutual information

We assume the method and apparatus of Meth8/VŁ4 with τautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal.

LET p, q, r: H; A, ; B,Y; ~ Not; & And; \ Not And; + Or; - Not Or; > Imply.


"Definition 4.5 (Mutual Information). The mutual information I(X;Y) between two random variables X and Y is I(X;Y) = H(X) + H(Y) - H(X,Y).

This is supposed to represent the amount of information one learns about X from knowing what Y is. Since the definition is symmetric in X and Y, it also represents the amount of information one learns about Y from knowing X."

We evaluate the consequent of Eq. 1.1 as a potential theorem.

\[(p&q)+(p&r))-(p&(q&r)) \; ; \quad \text{TTF } \text{ TFF } \text{ TTTF } \text{ TFF} \quad (1.2)\]

Eq. 1.2 as rendered is not tautologous.

We evaluate the definition from another source: en.wikipedia.org/wiki/Mutual_information .

"Mutual information can be equivalently expressed as I(X;Y) ≡

\[H(X) - H(X|Y) \equiv \quad (2.1)\]
\[H(Y) - H(Y|X) \equiv \quad (3.1)\]
\[H(X) + H(Y) - H(X,Y) \equiv \quad (4.1)\]
\[H(X,Y) - H(X|Y) - H(Y|X). \quad (5.1)\]

where H(X) and H(Y) are the marginal entropies, H(X|Y) and H(Y|X) are the conditional entropies, and H(X,Y) is the joint entropy of X and Y."

\[(p&q)-(p&(q|r)) \; ; \quad \text{TFTF } \text{TFTF } \text{TFF } \text{TFTF} \quad (2.2)\]
\[(p&r)-(p&(r|q)) \; ; \quad \text{TFTF } \text{TFTF } \text{TFF } \text{TFTF} \quad (3.2)\]
\[((p&q)+(p&r))-(p&(q&r)) \; ; \quad \text{TTTF } \text{TFTF } \text{TTTF } \text{TFF} \quad (4.2)\]
\[(p&(q&r))-((p&(q|r))-(p&(r|q))) \; ; \quad \text{FTFT } \text{FTFF } \text{FTFT } \text{FTFF} \quad (5.2)\]

Eqs. 2.2 and 3.2 are equivalents and 4.2 and 5.2 are not, but each is not tautologous. This means the definition of mutual information as stated is not confirmed and hence refuted.