Refutation of the definition of mutual information

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We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal.

LET p, q, r: H; A, ; B,Y; ~ Not; & And; \ Not And; + Or; - Not Or; > Imply.

From: Wright, J. (2015). Lecture 18: Quantum information theory and Holevo's bound. cs.cmu.edu/~odonnell/quantum15/lecture18.pdf

"Definition 4.5 (Mutual Information). The mutual information I(X;Y) between two random variables X and Y is I(X;Y) = H(X) + H(Y) - H(X,Y). (1.1)

This is supposed to represent the amount of information one learns about X from knowing what Y is. Since the definition is symmetric in X and Y, it also represents the amount of information one learns about Y from knowing X."

We evaluate the consequent of Eq. 1.1 as a potential theorem.

((p&q)+(p&r))-(p&(q&r)) ; `TTTF TFTF TTTF TFTF` (1.2)

Eq. 1.2 as rendered is *not* tautologous.

We evaluate the definition from another source: en.wikipedia.org/wiki/Mutual_information .

"Mutual information can be equivalently expressed as I(X;Y) ≡

H(X) − H(X|Y) ≡ (2.1)
H(Y) − H(Y|X) ≡ (3.1)
H(X) + H(Y) − H(X,Y) ≡ (4.1)
H(X,Y) − H(X|Y) − H(Y|X). (5.1)

where H(X) and H(Y) are the marginal entropies, H(X|Y) and H(Y|X) are the conditional entropies, and H(X,Y) is the joint entropy of X and Y."

(p&q)-(p&(q\r)) ; `TFTF TFTF TFTF TFTF` (2.2)
(p&r)-(p&(r\q)) ; `TFTF TFTF TFTF TFTF` (3.2)
((p&q)+(p&r))-(p&(q&r)) ; `TTTF TFTF TTTF TFTF` (4.2)
(p&(q&r))-((p&(q\r))-(p&(r\q))) ; `FTFT FTFF FTFT FTFF` (5.2)

Eqs. 2.2 and 3.2 are equivalents and 4.2 and 5.2 are not, but each is *not* tautologous. This means the definition of mutual information as stated is not confirmed and hence refuted.