Proof of the Legendre’s Conjecture (Third Landau’s Problem)

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1 Definition of the Legendre’s Conjecture

Definition: Is it true that between \(n^2\) and \((n+1)^2\) there is always a prime number \((y_o)\)?

2 Algorithm for Proof of the Legendre’s Conjecture

Since after 2 the sequence of primes \(\{y_o\}\) enters an infinite sequence of odd numbers \(\{y\}\), the formulation of the Legendre’s Conjecture must be changed to consider this sequence, which we know from the Proof of the Goldbach’s Conjecture.

For this, if \(n^2\) or \((n+1)^2\) are even numbers, they will be replaced by odd numbers \((n^2-1)\) or \(((n+1)^2-1)\), respectively, which does not change the very essence of the question, since these numbers are composite.

Let the odd number \(n^2 = y^2\), then the even number \((n+1)^2 = (y+1)^2 = y^2 + 2y + 1\). Let’s replace this even number in a sequence of odd numbers \(\{y\}\):

\[y^2 + 2y + 1 - 1 = y^2 + 2y = y(y + 2).\]  \hspace{1cm} (1)

Thus, let’s must consider each set:

\[\{y_n \mid y_k^2 < y_n < y_k(y_k + 2), \ y_k \geq 3\}.\]  \hspace{1cm} (2)

The number of terms in each set (2):

\[N_{y_n} = y_k - 1.\]  \hspace{1cm} (3)

But in the sets:

\[\{y_n \mid y_k(y_k - 2) < y_n < y_k^2, \ y_k \geq 3\}\]  \hspace{1cm} (4)

the number of terms is also equal to (3).

It is logical to consider the sets (2) and (4) with respect to sets with equal \(N_{y_n}\). But the segments between \(y_k(y_k - 2)\) and \(y_k^2\), \(y_k^2\) and \(y_k(y_k + 2)\) are segments in a sequence of odd numbers for which the Formula
of Disjoint Sets of Odd Numbers of **Proof of the Goldbach’s Conjecture** is valid. For the entire sequence of odd numbers \( \{y\} \), it has the form of the following expression:

\[
\left(0.0\ldots 01\%(1) + 33,3\ldots 3\%(\{3y\}) + \sum_{n=3}^{n \to \infty} Z_{y_{on}} \left(\{y_{on}y_n \mid \frac{y_n}{3} \notin \mathbb{N}, \ldots, \frac{y_n}{y_{o(n-1)}} \notin \mathbb{N}\}\right)\right) \to 100% ,
\]

where: the number of digits represented by (…) in the first two terms \( \to \infty \);

\( n \) is the number of a member of a sequence of odd primes;

\( y_n \) is a sequence of odd numbers with the conditions given in the formula;

\( y_{o(n-1)} \) is the prime number in sequence of primes just before \( y_{on} \);

\( Z_{y_{on}} \) is the frequency of appearance of the given set (in %) in the sequence \( \{y\} \).

For the segments (3) of the sets (2) and (4) Formula of Disjoint Sets of Odd Numbers (5) takes the following form, where the percentage of the sets remains unchanged:

\[
Z_{y_{comp}} (\{y_{comp}\}) = 33,3\ldots 3\%(\{3y \mid y \geq 3, \ 3y = y_n\}) + \sum_{m=3}^{m \to \infty} Z_{y_{om}} \left(\{y_{om}y_m \mid y_m \geq y_{om}, \ y_{om}y_m = y_n, \ y_{om} < N_{y_n}, \ \frac{y_m}{3} \notin \mathbb{N}, \ldots, \frac{y_m}{y_{o(m-1)}} \notin \mathbb{N}\}\right),
\]

where: the number of digits represented by (…) in the first term, \( \to \infty \);

\( m \) is the number of a member of a sequence of odd primes;

\( y_{comp} \) is a composite odd number in a given segment of a sequence of odd numbers \( \{y\} \); \( Z_{y_{comp}} \) is the frequency of the appearance of composite numbers (in %) in a given segment of the sequence \( \{y\} \);

\( N_{y_n} \) is the number of terms in the sets (2) or (4);

\( N_{y_n} = y_k - 1 \);

\( y_m \) is a sequence of odd numbers with the conditions given in the formula.

But since in the whole sequence of odd numbers \( \{y\} \) the frequency of appearance of known sets according to (5) only tends to 100%, then in the case of (6):

\[
Z_{y_{comp}} (\{y_{comp}\}) < 100%.
\]

That is, between \( n^2 \) and \( (n + 1)^2 \) there is always a prime number.

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