Partition into triangles revisited

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July 13, 2018

Abstract

We show that if one has ever loved reading Prasolov’s books, then one can move on reading our recent article [3] and several words following to deduce that partitioning a graph into triangles is not an easy problem.

I. Proposition

A classical result claims that three-dimensional matching (3DM) is NP-complete. This was proved in [1] Chap. 3, pp.50-52.

Proposition: 3DM ∼P Partition into Triangles

Proof: At the end of this article, we capture a concise picture scanned from G&J book. Given a 3DM instance, we construct our graph as follows. The vertex set is the same as in the hypergraph of the given instance. For each triple \{ a, b, c \} in the given instance, we put three edges \((a, b), (b, c), (a, c)\). By a careful scrutiny the contents in the picture, we can conclude that if one can pick any triangle in our newly constructed graph, it must also be a triple in the given 3DM instance. (Hint: Consider three cases of \(ab, ss, gg\) in the picture) Q.E.D.

II. Conclusion

As long as we do the research on a well-known conjecture, we should recall our mathematical nature from Kvant, Prasolov-style of doing mathematics, similar to mathematics of [2] back to those beautiful days.

References


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The construction is completed by means of one large “garbage collection” component $G$, involving internal elements $e_{i,k} \in X$ and $e_{i,k} \in Y$, $1 \leq k \leq m(n-1)$, and external elements of the form $\bar{u}_{i,j} \in W$. It consists of the following set of triples:

$$G = \{(u_{i,j}, z_{i,j}, e_{i,k}), (\bar{u}_{i,j}, z_{i,j}, e_{i,k}) : 1 \leq k \leq m(n-1), 1 \leq i \leq n, 1 \leq j \leq m\}$$

Thus any matching $M' \subseteq M$ will have to contain exactly one triple from $G$. This can only be done, however, if some $u_{i,j}$ (or $\bar{u}_{i,j}$) for a literal $u_{i,j}$ ($\bar{u}_{i,j}$) does not occur in the triples in $T \cap M'$, which will be the case if and only if the truth setting determined by $M'$ satisfies clause $\psi$. 

3.3 SIX BASIC NP-COMPLETE PROBLEMS

From the comments made during the description of $M$, it follows immediately that $M$ cannot contain a matching unless $C$ is satisfiable. We now must show that the existence of a satisfying truth assignment for $C$ implies that $M$ contains a matching.

Let $r : U \rightarrow \{T, F\}$ be any satisfying truth assignment for $C$. We construct a matching $M' \subseteq M$ as follows: For each clause $\psi \in C$, let $z_{i,j} \in \{u_{i,j}, \bar{u}_{i,j}: 1 \leq i \leq n\} \cap \psi$ be a literal that is set true by $r$ (one must exist since $r$ satisfies $\psi$). We then set...